THE EFFECT OF THE TROPOPAUSE ON THICKNESS PATTERNS USED

THE CONSTRUCTION OF HIGH LEVEL C.P. CHARTS.

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Observations of the tropopause in the Australian region disclose wide departures from any simple form on most occasions and it will not be possible in this essay to take these into account. Nor will a definition of "tropopause" be given.

Schematically, we may present an idealised troposphere, stratosphere and tropopause, and enquire into their effect on thickness patterns.

Let our co-ordinates be \( x \), measured on an isobaric surface, and \( p \), measured perpendicularly to the isobaric surfaces. Let a surface (the tropopause) intersect the isobaric surfaces and let \( \frac{dp_T}{dx} = \mathcal{O} \), where \( p_T \) is the pressure at the tropopause.

\[
\begin{align*}
\frac{\partial T}{\partial x} & \text{ tropos } = m, & \frac{\partial T}{\partial p} & \text{ stratos } = 0 \\
\frac{\partial T}{\partial p} & \text{ tropos } = p, & \frac{\partial T}{\partial x} & \text{ stratos } = n
\end{align*}
\]

It can be easily shown that \( n = m + \mathcal{O} \).

In our model it is clear that there is no discontinuity of temperature at the tropopause; but as we move isobarically from troposphere to stratosphere, the temperature gradient changes abruptly from \( m \) to \( n \). Hence the tropopause, thus portrayed, is a temperature discontinuity of the first order, unlike a front, which is ideally a discontinuity of zero order [1].
Fig. 4. - Autographic Records at Adelaide 16-9-49 to 17-9-49.
It now remains to find the effect of this gradient discontinuity on the thickness patterns, which are a simple function of (virtual) mean temperature of the layer.

Let \( \bar{Z} \) be the thickness of the layer from \( p_1 \) to \( p_2 \).

Then, if \( p_T \) lies between \( p_1 \) and \( p_2 \),

\[
\bar{Z} = -\frac{R}{g} \int_{p_1}^{p_2} \frac{T}{p} \, dp = -\frac{R}{g} \int_{p_1}^{p_T} \frac{T}{p} \, dp - \frac{R}{g} \int_{p_T}^{p_2} \frac{T}{p} \, dp
\]

and

\[
\frac{d\bar{Z}}{dx} = -\frac{R_n}{g} \int_{p_1}^{p_T} \frac{dp}{p} - \frac{R_n}{g} \int_{p_T}^{p_2} \frac{dp}{p}
\]

\[
= \frac{R_n}{gM} \left( \log p_1 - \log p_T \right) + \frac{R_n}{gM} \left( \log p_T - \log p_2 \right)
\]

Measuring \( x \) in degrees of latitude, and \( Z \) in geo-potential feet, this becomes

\[
\frac{d\bar{Z}}{dx} = A \log \left( \log p_T - \log p_2 \right) + m \left( \log p_1 - \log p_2 \right) \quad (1)
\]

where \( A = \frac{.98R}{gM} \) \text{ 30.48} \quad (R \text{ is gas constant, Mius}

logarithmic modulus, \( g \) is acceleration of gravity.

The expression for \( \frac{d\bar{Z}}{dx} \) when the temperature gradients can be considered constant with height, say \( z = r \), as in troposphere and stratosphere in our model, is

\[
\frac{d\bar{Z}}{dx} = A \frac{r \log p_1}{p_2} \quad (2)
\]

Clearly there is no discontinuity in \( \frac{d\bar{Z}}{dx} \) as \( p_T \) approaches either \( p_1 \) or \( p_2 \), since in each case the value of \( \frac{d\bar{Z}}{dx} \) obtained from equation (1) approaches that obtained from equation (2).
In fact, discontinuities in thickness gradients probably are either rare or non-existent.

We may now allot numerical values and obtain a schematic thickness pattern.

Let \( x \) be measured towards the south, in degrees

\[
\ell = 3 \text{ mb/degree} \\
m = 0.5^\circ \text{ C/degree} \\
q = 0.2^\circ \text{ C/mb (roughly d.a.l.r. at 300 mb)}
\]

Then \( n = m + \ell q = + 0.1^\circ \text{ C/degree} \)

Consider the thickness of layer 300-200 mb.
When this lies wholly in the stratosphere (i.e., south of the tropopause in our model)

\[
\frac{dz}{dx} = 3.8 \text{ g.p. ft/degree}
\]

When the layer is wholly in the troposphere (north of the tropopause)

\[
\frac{dz}{dx} = -19.1 \text{ g.p. ft/degree}
\]

When the layer is partly in the troposphere and partly in the stratosphere (as will be the case for a zone 33-1/3\( ^\circ \) wide), the gradient is given by equation (1)

\[
i.e. \quad \frac{dz}{dx} = A \cdot 0.6 \log (1 + \frac{3x}{200}) - 0.088 \quad \text{if } x \text{ is measured from the intersection of the tropopause with the 200 mb surface. This may be integrated either graphically or mathematically, to give } Z \text{ as a function of } x.
\]

Mathematical integration gives

\[
Z = A \left[ 40 \log (1 + \frac{3x}{200}) + x \left( \frac{0.6 \log (1 + \frac{3x}{200})}{200} - 0.549 \right) \right]
\]

where \( Z \) is the value of \( Z \) at \( x = 0 \).

The graph of the complete function \( Z \) is shown in Figure 1, where \( x = 0 \) is taken as 30\( ^\circ \) S and \( Z \) is taken as 8870 g.p. feet, an observed mean mid-summer value. For comparison, observed values of 300-200 thickness as read from mean charts for January (broken line) are plotted also.

A much closer fit to observed values may, of course, be obtained, by allowing the parameters \( \ell, m, n, q \) to vary with \( x \). In this case integration of the equation for \( dz/dx \) with respect to \( x \) is performed graphically. The values of these parameters which give a reasonably close approximation to all observed values for midwinter are as follow:
I. Thickness calculated from simplified model.
II. Thickness read from mean charts, long. 150 E., mid-summer.

Figure 1.

I. Thickness calculated from values of parameters in Table 1.
II. Observed values from mean charts (mid-winter).

Figure 2.

Thickness 300-200 mbs (Winter.)
I. Thickness curve calculated from values of parameters given in Table 1.
II. Thickness curve read from mean charts (mid-winter).
III. Thickness curve read from mean charts (long. 150° E., mid-summer).

Figure 3.

Schematic representation of sloping, corrugated tropopause on 500-200 mb. thickness pattern.

Figure 4.
<table>
<thead>
<tr>
<th>lat. °S</th>
<th>pT (mb)</th>
<th>E (mb/deg)</th>
<th>m (°C/deg)</th>
<th>n (°C/deg)</th>
<th>q (°C/mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>258</td>
<td>1.6</td>
<td>-0.8</td>
<td>-0.5</td>
<td>0.19</td>
</tr>
<tr>
<td>55</td>
<td>246</td>
<td>0.7</td>
<td>-0.5</td>
<td>-0.35</td>
<td>0.21</td>
</tr>
<tr>
<td>45</td>
<td>234</td>
<td>1.7</td>
<td>-0.5</td>
<td>-0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>35</td>
<td>197</td>
<td>5.8</td>
<td>-0.45</td>
<td>+0.9</td>
<td>0.23</td>
</tr>
<tr>
<td>25</td>
<td>138</td>
<td>5.9</td>
<td>-0.4</td>
<td>+1.3</td>
<td>0.29</td>
</tr>
<tr>
<td>15</td>
<td>103</td>
<td>1.2</td>
<td>0</td>
<td>+0.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 2 shows plotted values of $Z$ derived from the above mean figures, compared with values read from the mean charts for July, for longitude 150°E. Figure 3 shows similar calculated values compared with mean chart values. For completeness, observed midsummer values are added also. The basis of the figures in Table I is contained in [2].

It will be noted that, for the purpose of this simple treatment, the "double" tropopause has been smoothed into a single one. This can only be justified by expediency, and a useful fact is thereby neglected, namely, that the lower tropopause is rather closely correlated with both stratospheric and tropospheric temperatures, whereas the upper one evinces no such connection. The real fact is that the tropopauses show wide deviations from the simple structure depicted above and any attempt to force them into a simple pattern suffers from the defect of artificiality. In particular, this anomalous behaviour occurs principally in the zonal belt 40-25°S, where tropopause slopes and temperature gradients are variable and large, and where in winter jet streams are normally found. It must be pointed out that values of $m$ and $q$ in Table I cannot be taken as representative of mean values in the troposphere but only of those in the upper reaches of the troposphere and including portion of the inter-tropopause layer, where there are two. This merely emphasises what should be obvious, namely, that in the real atmosphere at any time, $E$, $m$, $n$, $q$, $p_T$ are functions not only of $x$, but of $p$ and $y$ (longitude) as well.

### A Two Dimensional Picture of Thickness Patterns

It is possible to cater, in our simple model, for a sinusoidal corrugation of the tropopause, but it cannot be expected that this will yield a very realistic pattern owing to the gross over-simplifications made. It is an observed fact [3], that there is a close correlation between tropopause height and troposphere temperature in extra-tropical latitudes, a cold troposphere corresponding to a low warm tropopause and vice versa. Scatter diagrams for a few southern Australian RAOB stations suggest that a rather large fluctuation of the tropopause is 60 mb either side of the mean, and that this...
corresponds to a tropospheric (400 mb) temperature fluctuation of about 5°C. Likewise, the correlation between tropopause temperature and pressure is good, and a mean value of 0.2°C warming for a descent of 60 mb has been taken. This yields, for a wavelength of corrugation of 80° longitude, a mean value of $m_y$ of -0.25°C from ridge of trough and of +0.41°C/deg. long. With a mean value of $f$ from crest to trough, of 3 mb/deg. long, this gives $q = 0.22°C/mb$. Values of $Z - Z_f$ can then be calculated for any point in the field. This has been done for the 300-200 mb, thickness and the results are represented in figure 4, where isopleths are drawn at 200 ft intervals, following current practice in the Central Analysis Section. It is emphasized that this model is schematic only, and particularly in the tropics (north of lat. 25° S) it is invalid, because in those latitudes there is no correlation at all between the height of the tropopause and temperatures, either tropospheric or stratospheric. For this reason figure 6, showing the effect on the normal thickness pattern of the layer 200-100 mb of a tropopause corrugation, is not extended N of lat. 25°S.

Several features of these patterns are noteworthy:—

(1) A cold troposphere corresponding (see e.g. [2]) to an upper tropospheric trough or low, is overlain by a low warm stratosphere, and this has the effect of inverting the temperature field in the stratosphere, i.e.

(2) A thickness trough in the 500-300 mb layer, corresponding to a trough in the tropopause and also to a trough in the 300 mb contours, is overlain by a thickness ridge in the stratosphere. This is shown in the schematic representation in figure 5, and is found to apply generally. It is a useful rule to remember in drawing the 200-100 mb thickness pattern. The result is, of course, one of stratospheric compensation whereby the normally strong W wind and strong pressure gradient in the upper troposphere is rapidly damped with height in the stratosphere and meridional flow is similarly damped.

(3) In the transition layer 300-200 mb, it is to be expected that the tropopause ridge (and upper troposphere ridge) will be accompanied by low values of thickness in the far south, and high values in the north, so that gradients of thickness will be strong in middle latitudes, whereas they should be weak over a tropopause trough (and upper troposphere trough). This reasoning, however, is invalid in winter, since in this season the stratosphere temperatures decrease southwards in higher latitudes, due to radiation from the ozone layer [2].
Schematic representation of effect of sloping corrugated tropopause on 200-100 mb, thickness patterns. Based on mid-winter mean figures.

Isopleths of Tropopause pressure 0800Z. 7/8/51.

Figure 5.

Figure 6.
Figure 7. Relevant temperature traces at 0800Z. 7/8/51. The scale is that of Form F.160.

Figure 8. 500-300 mb. Thickness 0800Z. 7/8/51.
Figure 9.

200-100 mb. Thickness
0800Z. 7/8/51.
tens g.p. feet.

Figure 10.

300-200 mb. Thickness
0800Z. 7/8/51.
tens g.p. feet.
Application to a real situation:

An attempt will now be made to draw 300-200 and 200-100 mb thickness lines for 08Z on 7/6/51, which is of interest in that it preceded by a few hours the development of a deep cyclone near Tasmania.

Figure 6 shows tropopause contours for this situation. In the case of all stations except Parafield, Laverton, Hobart and Macquarie Island, a sharp tropopause did not appear and smoothing has been applied in order to make possible a coherent picture. Figure 7 shows the temperature curves for the four places mentioned above. It will be noted that Parafield's tropopause is complex. It could be expected from the principles outlined above that there would be a maximum of thickness for all layers in the stratosphere in the vicinity of the low tropopause south east of Parafield, and a minimum for all layers in the troposphere. This is reasonably well borne out by the 500-300 mb thickness (mainly troposphere, Fig.8) and the 200-100 mb. thickness (mainly stratosphere, Fig.9). The transition layer 300-200 mb is represented in Fig. 10, but this is more difficult. Figure 4, suggests that the isopleths may be as shown, south of the Continent, but this of course cannot be verified.

Conclusion:

Where the tropopause is sharply defined, its presence has a pronounced effect on the thickness patterns of layers whose boundaries it intersects, such that there is a gradual transition from the thickness gradients appropriate to the stratosphere to those appropriate to the troposphere. A knowledge of this fact, and of the effect of tropopause corrugation, may be of assistance in extrapolating thickness isopleths from an inadequate network of observations.

References.