The occurrence of lenticular clouds to the lee of mountain ranges has long been known. The early Maori navigators called New Zealand "the land of the long white cloud", and in Germany there is a confused folk legend concerning the "Moanzogotl" cloud which forms over the Riesengebirge. Sailplane pilots were attracted to this evidence of vertical motion the first wave soaring flight having been made in the latter region in 1933.

Sailplanes, having a rate of descent through the air of about 3 feet per second, are well suited for the exploration of up-currents greater than this value, and glider pilots have been responsible for most of the observations of flying conditions in standing waves. In 1940 a German pilot reached 37,000 feet to the lee of the Alps, and since the war wave flights to lesser heights have been made in most European countries with suitable hills, but the most pronounced wave conditions occur in North America, where the great mountain chain lying perpendicular to the westerlies of middle latitudes gives rise to a wave motion extending well into the stratosphere. The world absolute altitude record now stands at 42,000 feet, and was made at Bishop, California, in the wave set up by the Sierra Nevada, where the peaks rise to 14,000 feet. An article by Colson (1952) describing the effects of the vertical currents on winds obtained from pilot balloons, contains some excellent photographs of the cloud formations associated with standing waves, as seen both from the ground and from the air. The usual technique of glider pilots during wave flights is to head into wind, remaining almost over one place, seeking the area of the strongest up-current, guided by the position of the lenticular clouds and the readings of the variometer, a sensitive rate of climb indicator. With strong upper winds gliders are often carried slowly backwards into the descending current, experiencing great loss of height before returning to the up-current region. By gaining maximum height in the first wave and then turning downwind, successive wave up-currents can be used, and in this manner, using two waves created by several mountain ridges, Kuettner recently flew a distance of 375 miles in four hours, and was then forced to land only by approaching darkness.

Mathematical theories for the formation of standing waves have been developed by Lyra (1943), Queney (1948) and Scoen (1949). Scoen has also published a more popular account (1951) in which he emphasises that an increase with height of the component of the wind perpendicular to the obstacle is the primo requirement for waves to form, outweighing
Fig. 2 shows the lower portion of the recorder trace of the second flight, and the variations in the time intervals between the switchings are readily apparent. It will also be seen that a repetition of the reference signal at the 20th bar switch contact occurred, showing that the balloon had been forced down, this taking place at the level of an inversion characterized by a sharp upward decrease in relative humidity. The exact humidity structure while passing through the inversion is not recorded, as the reference signal monopolized the transmission at that time.

The rate of ascent of a balloon depends on many factors, among which may be mentioned the temperature lapse rate and the atmospheric turbulence, but it is assumed here that the variations are due to vertical currents, which have been found by subtracting the mean rate of ascent over a deep layer from the instantaneous values observed each minute. Using the theodolite readings made during the first flight, and estimating the winds at higher levels, the position of the balloon at each minute of the second flight has been plotted on an east-west cross-section in Fig. 3, which has a five-fold vertical scale exaggeration. Direction elements of the airflow are shown at each point, as found by compounding horizontal and vertical velocities. Approximate streamlines with an arbitrary spacing have been drawn to fit these direction elements to indicate the general flow pattern. The wavelength appears to be about 8-10 miles, and the maximum vertical amplitude of the motion of a displaced particle some 4,000 feet.

The temperature soundings plotted on a Skew T-log p diagram in Fig. 4 show that a cooling of 1-2 degrees C took place in the interval between the flights in the lower layers, and that a marked change took place in the region of the inversion. The second flight shows a dry adiabatic lapse rate up to 833 mb., where it abruptly changes to the saturated adiabatic lapse rate, corresponding well to the condensation level of an air parcel raised from the surface without mixing with the environment. It is therefore probable that the balloon entered cloud at this level, despite the contrary evidence of the dow-point curve, and showing a lowering of cloud base since the first ascent. At 771 mb., the lapse rate changes abruptly back to the dry adiabatic rate, accompanied by a sharp fall in dow-point. Thus, rather surprisingly, the inversion does not mark the top of the cloud.

The elapsed times from the start of the ascent have been entered alongside some of the points plotted in Fig. 4 and in Fig. 5 (an enlarged portion of Fig. 3), the potential temperatures at these points have been entered along the flight path of the balloon. The path of the balloon closely follows
FIG. 3  WEST-EAST CROSS SECTION THROUGH HOBART SHOWING APPROXIMATE STREAMLINES AT 08252, 13th MARCH, 1952.
the streamlines, which are also lines of constant potential temperature, and therefore the lapse rates recorded are close to the dry adiabatic. The rapid increase in potential temperature as the balloon passes through the inversion occurs, somewhat paradoxically, while the balloon is descending, the streamlines being more sharply distorted than the path of the balloon. Therefore, as has been pointed out by Ludlam (1952), the temperature sounding made under these conditions is not representative of the general air mass, the inversion recorded being unduly sharpened by the cooling by ascent of the air immediately below the inversion, and the warming by descent of that immediately above the inversion in the regions traversed by the radiosonde.

The dewpoint curve in Fig. 4 indicates the likelihood of further cloud formation below the inversion at 570 mb. The clouds at this level would be of small vertical thickness, and would be of lenticular form, following the streamlines over the wave-crests. The lower clouds would be more akin to cumuli in their manner of formation, as once formed as a result of the vertical air motion through the wave, the latent heat of condensation released would give them buoyancy with respect to the airstream at their level. As those clouds would probably be in the form of long rolls perpendicular to the wind direction, the airstream at cloud level would be forced to ascend over them. If moist, this would give rise to a pilous cap over the roll, but in this case it appears that no condensation took place, resulting in a cold cap over the cloud due to the dry adiabatic ascent. It is difficult to imagine that this statically unstable stratification can persist — possibly, by analogy with aerofoil aerodynamics, the acceleration of the airstream over the forward edge of the cloud maintains turbulent flow until the wave-crest has been passed. This may account for the severe turbulence in the trough of a wave described by Turner (1952), and the progressive ruffling of a lenticular cloud commencing from the down-wind edge reported by Austin (1952).

Scorer's criterion for the formation of stationary waves is that there should be two layers, in each of which the parameter $L^2$ is constant, such that the inequality

$$L_{\text{lower}}^2 - L_{\text{upper}}^2 > \frac{\pi^2}{4} L^2$$

is satisfied, where $h$ is the depth of the lower layer, and

$$L^2 = \frac{\beta}{U^2} - \frac{U''}{U}$$

where $U$ is the horizontal component of the wind perpendicular to the obstacle, $U'' = \frac{d^2 U}{dz^2}$, $g$ is the acceleration due to gravity, $\beta = \frac{\partial \Phi}{\partial z}$ and $\Phi$ is the potential temperature.
Fig. 6. Velocity and temperature-height profiles used for calculation of $L$. 
TABLE I.

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<tr>
<th>z/km</th>
<th>U/km</th>
<th>U''/x10^5</th>
<th>β</th>
<th>Δβ/ΔU^2</th>
<th>U''/x10^5</th>
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Table 1 shows the values of $l^2$ in a manner similar to that of Scorer's second paper. The first term on the right-hand side of equation (2) was obtained from the temperature-height curve of the first ascent, slightly smoothed, taking differences over steps of 200 metres. The SOWIN observation of $U$ extends up to 2000 m, and in order to evaluate $U''$, a fourth degree polynomial was fitted to the observed winds, read at 200 m interval from Fig. 6. Above 2000 m, a linear velocity-height profile was postulated, so that the second term in equation (2) is then zero. The bracketed values of $l^2$ in the table are unreliable, as the fit of the polynomial is not good in those regions.

Taking the lower layer of quasi-constant $l^2$ to extend from 1100 to 3100 m, the inequality (1) becomes approximately

$$(1.0 - 0.05) \times 10^{-6} > \frac{\pi}{4} \times 2000^2$$

or

$$9.5 \times 10^{-7} > \frac{6.2 \times 10^{-7}}{}$$

and therefore lee wave formation should be possible, such that, if the values of $l^2$ were truly constant, the wavelength should lie between the limits:

$$\frac{2\pi}{\sqrt{2}h_{\text{ave}}} < L < \frac{2\pi}{\sqrt{2}h_{\text{upper}}}$$

or

$$1.6 \text{ miles} < L < 7.0 \text{ miles}.$$  

Considering that the winds were observed in the already disturbed airstream to the lee of the obstacle and have been somewhat doubtfully extrapolated upwards, and that the values of $l^2$ are not strictly constant in the two layers, there is fair agreement with the estimate made earlier.

Apart from the spectacular effects observed in mountainous country, signs of lee waves can often be seen in regions of comparatively low relief, extending to great heights compared with the height of the obstacles initiating the disturbances, as shown by Austin (1952) and Ludlam (1952 b and c). In the Melbourne area, while isolated well-marked lenticular clouds may often be seen over the bold features such as Mount Macedon and Mount Donna Buang, the whole Dandenong Ranges exerts a more profound effect, even to the west of the city, where it has very gentle slopes. With a northerly or northwesterly airstream small lenticular altocumulus or high stratocumulus patches can often be seen forming an hour or two after sunrise in a line along the northern horizon when observed from the city. Sometimes this cloud spreads rapidly southwards to form a continuous shield over the coastal plain, having a clear-cut edge to the north, and corrugations aligned approximately E-W parallel to the edge. This bears a great resemblance to the extensive cloud mass formed to the lee of the eastern Alps.
during a south-Fähn, as described by Pielsticker (1942) in an account of the German investigations into standing waves.

Occasionally the sheet may be very dense with virga falling from it. A good example of this type occurred on the 6th July 1950, when the sheet persisted nearly all day, and the Laverton radiosonde recorded up at 800 mb. in precipitation falling from a very dark patch of cloud to the west of the city, very little rain actually falling in the city itself.

Fig. 7 shows the positions of the principal cloud bars observed about midday on the 17th August 1950, when a thin cloud sheet which had formed in the morning was breaking up. The azimuth and elevation of the ends of the cloud bars were obtained by theodolite, and the plan positions of the clouds found by assuming the height to be 10,000 feet, as indicated by the soundings in Fig. 8. The northermmost clouds had already dispersed at this stage, but other measurements have indicated that the distinct northern edge is usually about 30 miles to the north of the city.

Extensive cloud could, of course, be expected in many northerly synoptic situations, being associated with depressions advancing from the west, but the rapid formation of the cloud shortly after a clear sunrise points to an initial disturbance in the layers close to the ground. This is probably the breakdown of the nocturnal inversion layer, giving rise to the changes in the temperature and velocity-height profiles indicated in Fig. 9. Although the temperature distribution is more favourable for waves at sunrise according to Scarfe's theory, the wind-shear layer is very shallow, and the effective height of the hills in the airstream is lower than is the case after some surface heating has taken place. In winter the clouds formed by wave motion may reduce further heating and these conditions can then persist, giving rise to the cold, dull, windy days so much a feature of Melbourne winters, but in summer the ground warms rapidly so that free convection currents soon disrupt the quasi-laminar flow, and the wave clouds are usually only short lived. The formation of the surface stagnant layer at night would therefore account for the dispersal at night of the clouds described by Ludlam (1952 b and c).

Summing up, it can be said that wave motions can occur to the lee of any pronounced orographic obstacle, especially if it extends some distance perpendicular to the airstream.
Essendon RAWIN 1400 E.S.T. 17th August
Surface  360/20 kts  10,000 ft  340/25 kts
3000 ft  360/20    12,000 ft  340/26
5000 ft  360/20    14,000 ft  340/31 kts
8000 ft  350/36

Fig. 9.
(a) At sunrise

(b) After some surface heating.

Effective height of hill. Stagnant surface layer.
and has a steep face. An increase of windspeed with height over a deep layer is the major condition imposed upon the airstream, and an increasing instability with height also favours waves. For the waves to persist, the air mass must be stable with respect to any clouds formed, and it appears that the major role of the inversions so often observed in association with standing waves is to maintain static stability.

Acknowledgements are due to Mr. V.J. Bahr, who drew attention to the anomalous flights at Hobart and suggested the cause, and to Dr. U. Radok, who carried out the computations of 12.

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