THE USE OF UPPER CHARTS IN FORECASTING

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Abstract: A number of suggestions is made for the use of upper charts in forecasting. It is not intended to be an exhaustive list, but merely a summary of ideas found useful in practice. In particular, the importance of estimating upper divergence and vertical motion, and the value of some acquaintance with characteristic types of evolution of large scale upper flow patterns is emphasised.

1. FORECASTING THE FIELD OF SURFACE PRESSURE

No use of the upper charts can indicate the surface pressure tendencies with anything approaching the accuracy of surface observations. However, short period tendencies are known to be unreliable predictors of future events, and a knowledge of the processes aloft associated with surface pressure change is often an advantage.

There are a number of empirical rules available such as those of Müller-Annen (1950,1952) (summarised in the same issue of this Magazine) and Scherhag (1948).

Müller-Annen's rules The direction of displacement of the 24 hour rise and fall centres and qualitative change in intensity are often well indicated by these rules. Frequently the distance of travel presents a problem which cannot readily be resolved. The use of the 1000-500 mb. thickness change has not been tried; it might prove useful if available when required. The Guilbert Grossmann rules are often found to apply in this connection especially with the more rapidly moving "warm air" types.

Scherhag's rules proved somewhat disappointing, except -

(a) The Guilbert Grossman rule.
(b) The effect of cold air advection (pressure fall aloft) and warm air advection (pressure rise aloft).

This is fundamental.
(27)

(c) The development of an upper low frequently results in the splitting of the 24 hour fall area: the rapidly weakening part moving southeast, and the stronger part to the northeast. This is essentially the case of the cut-off upper low mentioned by Müller-Annen.

Alternatively, an analytical approach to pressure change problems may be made.

**Barotropic theory** The atmosphere would be barotropic if surfaces of equal density coincided with surfaces of equal pressure; this would mean that on constant pressure charts there would be no isotherms; the "thermal wind" (as measured from one c-p chart to another) would everywhere be zero.

Vorticity can be defined as the limiting value of circulation/area as area approaches zero. It is a vector, but if the velocity considered is horizontal we are dealing with the vorticity component about a vertical axis, and this is scalar. It may be measured relative to the earth's surface (relative vorticity, commonly represented by the symbol $\zeta$) or relative to space (absolute vorticity represented by $\zeta_a$). Relative vorticity is $2 \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ in Cartesian coordinates, where $x, y, u, v$ are measured with reference to the earth's surface (u east component, v poleward component of velocity), or in "natural coordinates" $V/R - \nabla V/\nabla n$ where $V$ is the speed relative to earth's surface, $R$ is the radius of curvature of the streamline (cyclonic positive) and $n$ is measured at right angles and to the right of the wind direction (in the Southern Hemisphere).  

Absolute vorticity $\zeta_a$ can be shown to equal $f + \zeta$, where $f = 2 \Omega \sin \phi$ ( $\phi$ = latitude, $\Omega$ = angular velocity of earth's rotation).

Now, proceeding from Bjerknes' circulation theorem setting $\phi \frac{dp}{\rho} = 0$ (i.e. assuming barotropic conditions) and assuming horizontal flow, it can easily be shown that

$$\frac{1}{\zeta + f} \frac{d}{dt} (\zeta + f) = - \text{div}_2 \vec{V}$$

where $\rho$ = density, $p$ = pressure, $d/dt$ indicates time differentiation following a parcel, and $\text{div}_2 \vec{V}$
indicates the divergence of (horizontal) velocity, which in Cartesian coordinates is
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \]

(If, furthermore, the flow is assumed to be non-divergent, we have \( \zeta \) a constant, i.e., a moving particle conserves its absolute vorticity.)

If the real atmosphere behaves approximately as a barotropic one, we can use the foregoing theory as an (admittedly rather rough) approach to the problem of surface pressure change. It can easily be shown that if \( p_0 \) is surface pressure, \( t \) is time, \( \rho \) is density, \( z \) is height, then
\[ \frac{\partial p_0}{\partial t} = - \int_0^\infty g \, \text{div}_2 \rho \, \vec{V} \, dz \]
where \( \text{div}_2 \) represents the horizontal divergence and
\[ \text{div}_2 \rho \vec{V} = \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = \rho \, \text{div}_2 \vec{V} + \vec{V} \cdot \text{grad}_2 \rho \]
Now if, as is the case for a barotropic atmosphere, \( \vec{V} \cdot \text{grad}_2 \rho \approx 0 \), then
\[ \frac{\partial p_0}{\partial t} \approx - \int_0^\infty g \, \rho \, \text{div}_2 \vec{V} \, dz \]
\[ \approx - \int_0^\infty \rho_0 \, \text{div}_2 \vec{V} \, dp \]
\[ \approx \int_0^\infty \frac{1}{\zeta + f} \, \frac{d}{dt} \left( \frac{\zeta + f}{t} \right) \, dp. \]

Studies of the magnitude of upper divergence have shown that almost invariably surface pressure change results from the difference between two large divergences almost equal in magnitude and opposite in sign respectively above and below a level of non-divergence at approximately 600 mbs, and that the upper one determines the sign of the change. Hence if we can use an upper chart, (above 600 mb) for a qualitative estimate of divergence, we may be on our guard for possible developments at surface level. We will not by this means be able to predict developments aloft.

Now \( \text{div}_2 \vec{V} = \frac{1}{\zeta + f} \frac{d}{dt} \left( \frac{\zeta + f}{t} \right) \)
\[ = \frac{1}{\zeta + f} \left( \frac{\partial (\zeta + f)}{\partial t} + \vec{V} \cdot \nabla \left( \frac{\zeta + f}{t} \right) \right) \]
where \( \frac{\partial}{\partial t} \) indicates time differentiation at a point fixed on the earth's surface.
If the upper system is moving without distortion at velocity \( \mathbf{C} \), we can write
\[
\frac{\partial}{\partial t} (\xi + f) = -\mathbf{C} \cdot \nabla (\xi + f)
\]
and hence
\[
- \text{div}_2 \mathbf{V} = \frac{1}{\xi + f} (\mathbf{V} - \mathbf{C}) \cdot \nabla (\xi + f)
\]

If we choose the 300 mb level, as being near the level of maximum divergence, we note that \( \mathbf{C} \) here is frequently quite small compared with \( \mathbf{V} \), and
\[
- \text{div}_2 \mathbf{V} \approx \frac{1}{\xi + f} \mathbf{V} \cdot \nabla (\xi + f).
\]

We now observe that
\[
\mathbf{V} \cdot \nabla f = v \frac{\partial f}{\partial y} = \frac{2v \Omega \cos \phi}{r}
\]
where \( r \) is the earth's radius, and that \( \mathbf{V} \cdot \nabla \xi = V \cos \alpha \text{grad} \xi \) where \( \alpha \) is the angle between \( \mathbf{V} \) and \( \text{grad} \xi \) so that
\[
\mathbf{V} \cdot \nabla \xi = V \frac{\partial \xi}{\partial s} \quad \text{where} \quad \frac{\partial}{\partial s} \text{ indicates differentiation along a streamline}
\]
\[= V \left( \frac{\partial \mathbf{V}}{\partial s} + K \frac{\partial \mathbf{V}}{\partial s} - \frac{\partial^2 \mathbf{V}}{\partial s \partial n} \right)\]
where \( K = \frac{1}{\mathbf{R}} \).

Hence
\[
- \text{div}_2 \mathbf{V} = f + kV - \mathbf{V} \frac{\partial}{\partial n} \left\{ \frac{2v \Omega \cos \phi + V \frac{\partial K}{\partial s}}{r} + K \frac{\partial \mathbf{V}}{\partial s} \right\}
\]

Thus, on this analysis \( \frac{\partial}{\partial t} \frac{\partial \xi}{\partial \phi} \) will be large and negative when the upper chart shows the following features in combination:

(i) \( V \) is large, as in a jet stream or frontal zone.
(30) \[ f + Kv - \frac{\partial V}{\partial n} \text{ is small and positive (if } \xi_a \leq 0 \text{ we have a condition of dynamic instability). Small } \xi_a \text{ is to be found in the Southern Hemisphere just to the left of a jet, or on the warm air side of a frontal zone.} \]

(iii) The bracketed term is large and negative. This will be contributed to by -

(a) a strong south wind component (in the Southern Hemisphere) in low latitudes \( (\cos \phi \text{ large}) \),

(b) large \( V \) and \( \xi_a \text{ negative and large, i.e., where streamline curvature changes rapidly from cyclonic to anticyclonic} \) (shaded area in fig. 1),

(c) \( K \text{ is large and cyclonic, and the wind decreases rapidly downstream, (this is the so called Schorhag divergence term)} \)
or \( K \) is large and anticyclonic, and the wind increases rapidly downstream.

Figure 3.

(d) The cyclonic shear decreases rapidly downstream or the anticyclonic shear increases rapidly downstream. This term could be expected normally to reinforce the effects of item (c).

Figure 4.

It is well known that the atmosphere is not barotropic, and that effects other than those mentioned above, must enter into a full consideration of this problem. We have not considered processes of energy transformation which must ultimately determine the whole sequence of events.

We have ignored the effects of density advection, vertical motion and vertical stability. The complete solution even formally is simply not available, but we may consider Priestley's tendency equations, which allow for three dimensional flow. The density advection problem will not be attacked.

**Effect of Vertical Motion:** Priestley (1948) derives an equation for pressure change based on gradient wind.
and on deviations from gradient. As well as terms somewhat similar to those above, there is a term involving vertical velocity advection, the effect of which may be represented by \( \frac{\partial p_0}{\partial t} \)\(_w\) = pressure change due to vertical motion

\[
\frac{1}{F} \int \left. \frac{\rho}{p} \int_0^1 \right. \text{div}_2 \left( \frac{\rho w}{f} \left( \mathbf{k} \times \frac{\partial \mathbf{V}}{\partial z} \right) \right) dp
\]

where \( F \) is a "control factor", \( w \) = vertical velocity component, \( \mathbf{k} \) is unit vertical vector. This contribution to \( \frac{\partial p_0}{\partial t} \) arises through the fact that in rising air there is a horizontal a-gostrophic component of flow from warm to cold, which results in divergence on the warm side and convergence on the cold side of a zone of strong thermal gradients. In the case of conditions favourable for pressure fall considered above, it must be remembered that divergence aloft producing convergence and pressure fall in low levels also results in vertical motion there; in fact, one would expect \( w > 0 \) from the ground to near tropopause level under these conditions, and a contribution to pressure fall from the above integral, in the warm air, if there is a frontal zone or jet. As maximum \( w \) is normally found in the 600-650 mbs level, we should look at isotherms at this level to ascertain where the chief contribution to pressure fall from vertical motion should be.

![Diagram](image)

**Fig. 5**

Figure 5 shows upper isotherms (at say, 700, 600, or 500 mb.) in a region of uniformly ascending air (shaded), showing where to expect surface pressure fall and rise as a result of this motion.

We may summarise the results of the above reasoning thus:
the upper charts give useful indications of likely events on the surface, and in lower latitudes especially, this method has been found to be applicable. It cannot be used in higher latitudes in the Australian area very satisfactorily through lack of soundings south of the Bight.

If the upper systems are moving slowly enough, (small) the 500 mb. chart may be used, but the 300 mb. is perhaps the best. Pressure falls are likely to develop importantly on the surface beneath upper divergence, provided this pattern remains relatively stationary, so that the fall continues for long enough. Upper divergence is to be recognised by strong, diffuent winds, with cyclonic curvatures changing to anticyclonic further downstream. The presence of a jet in abnormally low latitudes is to be regarded as a suspicious circumstance and sometimes precedes such a development by 2-3 days. These developments are strongly damped, or compensated, especially in low latitudes and over land masses by low level convergence; near the coast, where friction is less, isotherms tend to be crowed and more latent heat is available, the development sometimes occurs with great suddenness, with little warning from isallobars. These considerations apply with greatest force to the forecasting of east coast cyclones. It will be seen that in most cases upper divergence is to be found with north to northwest winds aloft, and this provides an explanation for the well known forecasting rule that rain develops with a deep north to northwest stream. Unfortunately the forecasting of the upper charts on which these considerations are based is at yet in its infancy. Nevertheless, the development of the cut off low as described by Müller-Annen, can often be forecast from observations in the Amsterdam Island area. A deep and energetic low developing there and remaining quasistationary is usually followed by strong anticyclonic flow south of say Western Australia and cutting off an upper trough in the eastern Bight or further east.

The development of low pressure beneath an anticyclonically curved confluent stream, as indicated by the theory presented above is seldom observed in the Australian area, although occasional signs of such a process are to be observed on the northwest coast of West Australia. In general, jet maxima are to be found at or near upper troughs, especially on their eastern side, and confluent jets with anticyclonic curvature, although they probably occur in higher latitudes, are not often observed over Australia.
Bjerknes and Holmboe have given a method of qualitatively assessing divergence on upper charts, which yields results somewhat similar to the above analysis. Subject to qualifications about the speed of translation of the systems, which need not usually concern us above 700 mbs, an asymmetrical low (i.e., one about which the winds are much stronger on one side than on the other) will have divergence effects as illustrated in figure 6, that is, if the low is thought of as moving in the direction of the strongest wind around it, divergence will be found on its forward side, and convergence on its rear side. This will be seen to agree with the barotropic analysis given earlier, but is of less general application.

The translation of "cold pools". When a "cold pool" exists aloft, accompanied by no surface low, there is a tendency for it to be steered by the surface winds, as pointed out by Scherhag. This sometimes assists in the extrapolation of this not uncommon feature. It should be noted that such a feature regularly tends to occur on the northeast rim of a stationary warm high, and is associated with cold air advection round the high. This feature should be carefully watched for development, for "energy dispersion" from a deep upstream trough can result in a weak trough developing into a pronounced upper low and later cyclogenesis on the surface.

It has been observed that when surface cyclogenesis results from a slow moving upper low, the former continues to deepen and move slowly so long as it is approximately beneath the upper divergent area; eventually, however, it frequently lies directly beneath the upper low, and this appears to be a sign of its rapid movement away.
2. ESTIMATING VERTICAL MOTION FROM UPPER CHARTS

The equation of continuity may be written -
\[ \text{div} \rho \vec{V} = -\frac{\partial \rho}{\partial t} + \rho \text{div} \vec{V}_2 + \vec{V}_2 \cdot \nabla \rho + \frac{\partial \rho \vec{w}}{\partial z} \]

where \( \vec{V}_2 \) is the horizontal wind vector, and \( \vec{w} \) the vertical component.

Then
\[ \frac{\partial \rho \vec{w}}{\partial z} = -\frac{\partial \rho}{\partial t} - \vec{V}_2 \cdot \nabla \rho - \rho \text{div} \vec{V}_2 \]

For many purposes the sum of the first two terms on the right-hand side may be neglected, and hence,
\[
(\rho \vec{w})_H \approx \left\{ \begin{array}{l}
H \int_0^H \rho \text{div} \vec{V} \, dz \\
H \frac{1}{\rho_H} \int_{p_0}^{p_H} \text{div} \vec{V}_2 \, dp
\end{array} \right.
\]

where \( p_0 \) is ground pressure, and \( \rho_H \) is pressure, \( w_H \) vertical motion and \( \rho_H \) density, at height \( H \).

Fig. 7 A vertical distribution of divergence.

If, as we have assumed in Fig. 7, the vertical distribution of divergence is

(i) convergence in the lower levels decreasing to nil at about 600 mbs, and
(ii) divergence above 600 mb, decreasing to nil in the stratosphere, then clearly \( w \) reaches a maximum at 600 mb, and decreases to nil in the upper troposphere.

If it is desired to evaluate \( w \) at 600 mbs, we could evaluate the area \( A \) which is \( \int \text{div} \mathbf{V}_p \, dp \) and \( B \) corresponds to 600 mbs, or, since the areas \( A \) and \( B \) are found to be almost equal, we could evaluate area \( B \). We could also ascertain \( w \) roughly by evaluating surface convergence or 300 mb divergence. Hence the divergence at 300 mb, as well as giving an indication of surface pressure change, also indicates roughly the direction and magnitude of vertical motion. This is thought to be very important for forecasting; in fact, widespread rain usually occurs with strong divergence indicated aloft, and the view is here presented that the moisture to be found in the radiosonde traces with north to northwest winds aloft is not so much the result of horizontal advection, as is so often assumed, but of vertical advection due to convergence in relatively moist surface air. This points to the need for emphasis on vertical rather than on horizontal air motion, insofar as it can be ascertained.

Sutcliffe's Method

Sutcliffe (1947) has given a method of ascertaining vertical motion; his formula for the relative divergence being

\[
\text{div}_p \mathbf{V}_{500} - \text{div}_p \mathbf{V}_o = - \frac{V'}{r} \frac{\partial}{\partial s} \left( \zeta_0 + \zeta_500 + r \right)
\]

\[
= - \frac{V'}{r} \frac{\partial}{\partial s} \left( 2 \zeta_0 + \zeta_500 + r \right)
\]

where subscript \( p \) indicates divergence measured on a constant pressure surface, \( \mathbf{V}_{500} \) and \( \mathbf{V}_o \) the wind velocity at 500 and 1000 mb respectively, \( V' \) is the "thermal wind" from 1000 to 500 mb, \( \zeta_0 \) is the relative vorticity at 1000 mb, \( \zeta_500 \) that at 500 mb, \( \zeta' \) that of the "thermal wind" in the layer 1000-500 mb, and \( \partial/\partial s \) indicates differentiation following the "thermal wind".

This, by reference to figure 7, can be seen to be an indicator of vertical motion; large positive values of relative divergence will be associated with
strong ascending motion.

Since $V'$ $\approx \bar{V}$ in most cases and $\bar{V}_0$ is normally small compared with $\chi_{500}$, this formula does not differ very greatly from the barotropic one given above; at least it should give similar qualitative results in many situations. It differs essentially in presenting a baroclinic theory, where "solenoids", or isobaric temperature gradients, are taken into account. As in the case of the barotropic theory presented above, the terms in the equation can be evaluated by means of a grid, but this requires considerable time. A purely qualitative treatment, as above, is possible, based on the surface chart and 1000-500 mb. thickness lines. The results arrived at are that ascent is favoured by

(i) strong $V'$, as in a frontal zone,
(ii) low latitude,
(iii) large value of bracketed terms.

(a) strong decrease of surface vorticity along the thickness lines (in the direction of the thermal wind.) This indicates a tendency for the surface vorticity to be advected with the thermal wind. This is the so called "thermal steering term".

(b) strong decrease of thickness vorticity along the thickness lines, for a further analysis of which see the barotropic treatment. This is the so called "thermal development" term.

(c) strong southerly wind component (in southern hemisphere)

Vertical Motion from Constant Pressure Charts

Vertical motion may be evaluated from isotherms and C-P isohyposes. On the assumption that upper flow is isentropic, and that isotherms move relatively slowly, isentropic upflow is indicated when the isotherm-isohypse relation indicates warm air advection, downglide with cold air advection, It is suggested, however, that this method is inferior to the use of the 500 mbs, or especially 300 mbs, chart, as described above, for estimating or forecasting vertical motion.

The distribution of humidity aloft is regarded as a valuable forecasting tool. The effects of advection of a deep moist stream are well known and will not be laboured.
In general the existence of such a stream simply means that upward motion has been occurring. Clearly, continued upglide results in rain, and it is cessation of upglide which accompanies cessation of rain. The amount of rain will depend not only on the water vapour available, but on the processes lifting and condensing it.

3. DEVELOPMENT IN RELATION TO THICKNESS PATTERN

Sutcliffe and Forsdyke (1950) have classified types of 1000-500 mb. thickness patterns and brief notes on these follow: (diagrams have been changed for the southern hemisphere).

(i) Cold thermal trough

(ii) Warm thermal ridge

(iii) Sinusoidal thermal pattern.

Tendency for cyclogenesis at C and anticyclogenesis at A. A cyclone will tend to move through the pattern and intensify at C, while an anticyclone tends to remain at A and intensify.

(complementary to (i))

Gives classical wave cyclone and cold anticyclone. Systems travel under thermal steering like a wave.
(iv) Cyclonic thermal involution. An exaggeration of the sinusoidal pattern and develops therefrom in early states of occlusion. Liable to evolve further with little movement.

(v) Anticyclonic thermal involution. Pattern of well developed and slow moving anticyclone.

(vi) Diffluent thermal jet. Pressure systems often travel in the jet, cyclones swinging out to right, anticyclones to left, at thermal delta.

(vi) Confluent thermal jet. Systems formed near jet entrance are liable to break away and run rapidly through the strong gradient to be replaced by new developments.
(viii) Diffluent thermal ridge.

Pattern tends to move slowly with systems.

(ix) Confluent thermal ridge.

Developments tend to cause slow or retrograde movement of thickness pattern. Pressure systems so formed tend to break away and move rapidly. Expect "breakaway secondary depressions on warm fronts or warm occlusions".

(x) Diffluent thermal trough

The most certain indication of cyclogenesis. Thickness pattern tends to move with pressure features. Cyclone may deepen greatly with slow movement, and tends to evolve to a slow-moving occluded depression. Do not expect cold outbreak or blocking action.
(xi) Confluent thermal trough

Thickness pattern tends to move slowly or retrograde. Breakaway depressions tend to move rapidly downstream from C. A difficult situation for forecasting. A cold outbreak may occur with a cold cut off low north of the main westerlies.

From (x) and (xi) we may decide the "regular dilemma of forecasting" i.e. whether to forecast a ridge behind the cold trough (if trough is confluent) or no ridge (if trough is diffluent).

4. EFFECTS OF VERTICAL STABILITY ON DEVELOPMENT

Sumner has qualified Sutcliffe's theory by taking vertical stability into account, showing that this strongly damps development, especially in small systems, and especially in low latitudes. Conversely, even slight instability assists development of small centres, especially in low latitudes. Rapid development of a system could be expected once instability is established, and this is of great importance in areas liable to tropical cyclones. In assessing possible developments from upper charts, considerations of stability should always be kept in mind.

5. ROSSBY'S WAVE THEORY

The 500 mb. "waves" in the westerlies are shown to move with a speed given by \[ C = U - \beta \frac{L^2}{4 \pi^2} \] where \( U \) is the basic westerly current, \( \beta = 2 \lambda \cos \varphi / r \) and \( L \) is the wavelength. This formula can rarely, if ever, be used with advantage in Australia, but it is true that:
(a) the stationary wavelength tends to be established when any re-arrangement of planetary flow develops, such as, for example, occurs with strong cyclonic development in one place. At latitude 35°, stationary waves vary in wavelength from 62° longitude for $U = 30$ kts. to 89° longitude for $U = 60$ kts., and about 70° longitude is a frequently observed wavelength.

This tendency, usually very well marked, for stationary conditions to be propagated downstream, is thought to be an important forecasting rule. It explains the success achieved overseas with the use of "constant vorticity trajectories". In particular, the downstream development from a deep quasi-stationary upper trough, of an intense quasi-stationary ridge, is very frequently observed; intensification of the next trough downstream under these conditions is usually also noted within 2-3 days. This illustrates the phenomenon known as "energy dispersion".

(b) A wave which is less than the stationary wavelength downstream from a large quasi-stationary wave tends to be eliminated from the main westerly stream. This may occur either by rapid movement of the wave southeast while filling, or, if the wave is well marked, by the development of a cut-off low equatorward of the main westerly belt. This is characterised by a relatively slow initial movement towards east to northeast. This kind of development, mentioned by Müller-Annen and already referred to, is of rather frequent occurrence, and must be looked for when cyclogenesis in the Amsterdam Island area is intense. The east to northeast movement, frequently with surface cyclogenesis, often causes forecast failures and is a feature to be carefully reckoned with. If the next downstream trough from the deep stationary one is near the stationary position, "energy dispersion" results in its intensification, some 2-3 days after that of the upstream trough (e.g. over Amsterdam). This may be accompanied by retrogression, as when a trough east of Tasmania moves to the west of it, a deep low being near Amsterdam Island.

(c) Short waves move more rapidly than long ones, as seen clearly in any "high index" situation, when rapidly moving troughs succeed one another. Short waves may move through the longer ones, and this gives rise to "energy dispersion" phenomena.
If isohypses and isotherms are about 90° out of phase, rapid wave motion is the rule; if they are in phase, slow eastward movement may be expected if the isotherms show greater amplitude than the isohypses, slow westward movement if the isohypse amplitude is greater.

References:


Sutcliffe, R.C., 1947 Q.J.R. Met. Soc. 73, p. 370.