

HAZINESS ALONG VERTICAL AND OBLIQUE
LINES OF SIGHT

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ABSTRACT: A method is described whereby overall haziness in a vertical direction is measured by recording the elevations at which selected stars disappear to the naked eye.

1. INTRODUCTION

Some idea of the extent to which a given area is subject to haziness in a horizontal direction can be obtained from routine meteorological reports, which include a "visibility distance" as a measure of this factor.

As haze layers are usually stratified, and as the denser haze tends to occur near the surface, horizontal "visibility" is not necessarily an indication of conditions along oblique or vertical lines of sight.

The present paper describes a simple method by which a rough assessment of haziness in a vertical direction can be made in a reasonably quantitative and reproducible manner.

The project was originally carried out to compare overall haziness in several widely separated areas, and to determine day to day fluctuations at each place, with particular reference to observations between the ground and objects above 20,000 ft.

Initial investigations showed that while shallow haze layers occur to very high altitudes, marked variations in vertical visibility are usually due to haze in the lower layers, particularly that in the first 5,000 ft.

This simplifies the problem, since lines of sight to objects at moderate or high altitudes can be expected to pass completely through the greater part of the total atmospheric haze, and further it becomes possible to use objects outside the earth's atmosphere as visibility markers. Stars are suitable for this purpose.

The apparent brightness of a star at any elevation above the horizon depends on the length of the light path through the earth's atmosphere, and the mean extinction coefficient, i.e. the mean degree of light attenuation or scattering per unit length by air molecules and solid particles.

As the length of the light path can be computed for any position of the star, the relative brightness at two points can be used to calculate the mean extinction coefficient which is independent of the line of sight. The relative brightness can be measured photographically, but for the present purpose it is more convenient to work in terms of apparent magnitudes. The two quantities are related, (by definition), in the following manner:-

$$M_1 - M_2 = 2.5 \log E_2 / E_1 \dots\dots\dots(1)$$

where M_1 , M_2 are the magnitudes of any two stars, and E_1 , E_2 the corresponding stellar illuminations at the observer.

The faintest star just visible to the human eye has a magnitude of six, i.e. a brighter star will just disappear when its apparent magnitude falls to six or slightly less. Using this value for M_2 and taking M_1 as the catalogue magnitude of a selected star as listed in the Air Almanac, the change in brightness due to atmospheric conditions can be established.

The method therefore simply consists of following a star of known magnitude with the naked eye until it just disappears in the haze, and recording the "disappearance elevation" by means of a theodolite which is kept trained on the star for this purpose. An ordinary meteorological theodolite is suitable.

It will be seen that the "disappearance elevations" are in themselves a quantitative measure of oblique visibility i.e. they are sufficient to compare "haziness" in different

areas. However it is possible to extract more useful data, and make a rough comparison between haziness in the vertical and horizontal.

Before discussing the manner in which the star observations can be reduced, some terms unfamiliar in normal meteorological work will be introduced.

The stellar illumination "E" which reaches an observer at the earth's surface depends on the stellar intensity "I" and the stellar range "r" according to equation 2 (Bonguer's Law):-

$$E = Ir^{-2}T \quad \dots\dots\dots (2)$$

The transmission factor "T", is introduced to allow for attenuation and scattering along the line of sight.

In determining the catalogue magnitude of a star, "T" is taken as 0.766.

The relation between "T", the length of the light path "R", and the mean extinction coefficient per unit length, " σ ", is :-

$$T = e^{-\sigma R} \quad \dots\dots\dots (3)$$

As molecular scattering varies with air density, it is convenient to reduce the whole atmosphere to a single layer of uniform density. At standard temperature and pressure the equivalent "homogeneous atmosphere" has a depth of about 8 km.

If a star is not in the zenith position, the length of the light path through the homogeneous atmosphere will be a function of the elevation above the horizon, i.e.:-

$$R = 8 f(\theta) \quad \dots\dots\dots (4) \quad \text{where } \theta \text{ is the elevation angle.}$$

For angles larger than 15 degrees, $f(\theta)$ is closely approximated by cosec θ . For smaller angles the corresponding values can be extracted from tables computed by Bemporad (1907), which are available in most astronomical handbooks.

If M_c is the catalogue magnitude of a star, and M_a is its apparent magnitude at some angle of elevation, and if E_c and E_a are the corresponding stellar illuminations at the

observer, then equation (1) becomes:-

$$M_a - M_c = 2.5 \log E_c / E_a.$$

Substituting for "E" from equation (2):-

$$M_a - M_c = \frac{I r^{-2} T_c}{I r^{-2} T_a}$$

For catalogue magnitudes T_c is 0.766, and for T_a we can write: $e^{-8\sigma f(\theta)}$ from equations (3) and (4),

$$\begin{aligned} \text{i.e. } M_a - M_c &= 2.5 \cdot \frac{0.766 I r^{-2}}{e^{-8\sigma f(\theta)} I r^{-2}} \\ &= 2.5 \cdot \log 0.766 - 2.5 \log e^{-8\sigma f(\theta)} \\ &= 20 \sigma f(\theta) \log e - 0.3 \end{aligned}$$

At the "disappearance elevation" the apparent magnitude (M_a) will be 6 or slightly less. Substituting this value in the above expression:-

$$20 \sigma f(\theta) \log e = 6.3 - M_c$$

The mean extinction coefficient can then be calculated by re-arranging in the form:

$$\sigma = \frac{6.3 - M_c}{8.7 f(\theta)} \dots\dots\dots (5)$$

2. RESULTS OF OBSERVATIONS

Observations were attempted each night over a period of ten months, but cloud and other factors limited the total number to 108. Computed values of the mean extinction coefficient, (from equation 5) are summarised in Table 1.

TABLE 1

Month	No of Obs.	Mean extinction coefficient (Average for month)	Range
February	7	0.022	0.016-0.034
March	9	0.024	0.014-0.052
April	14	0.016	0.014-0.022
May	1	0.015	
June	6	0.028	0.022-0.030
July	12	0.020	0.016-0.027
August	8	0.022	0.016-0.025
September	16	0.023	0.017-0.025
October	25	0.025	0.016-0.082
November	9	0.025	0.021-0.028
December	1	0.026	

For pure air the extinction coefficient is 0.0127 per kilometre, corresponding to a vertical extinction of 0.02. The results shown in the above table indicate that oblique visibility is good at Woomera, and that in general the area is comparable with some observatory sites.

The good vertical visibility also confirms that haziness in a horizontal direction, which is not uncommon, is usually confined to a relatively shallow layer near the surface. This might be expected as a temperature inversion is commonly found in the vicinity of 5,000 ft. throughout the year.

3. EQUIVALENT VISIBILITY

There is no simple measure of "visibility" since it depends on a large number of factors such as the size, colour and brightness of the visibility marker, background brightness and contrast, the degree of attenuation and scattering, and the means used to observe the object. Extinction coefficients have been recommended as a suitable measure, but as they are not commonly used in meteorological work, it is of some interest to see what a given extinction coefficient means in terms of the more usual "visibility distance."

To do this it will be necessary to define "visibility distance" in a more rigid manner than that adopted for meteorological purposes, which largely ignores most of the factors outlined above. For the present purpose it is convenient to define the "visibility distance" as "the range at which a large and optically black object is just discernible". There is some difference of opinion concerning the minimum contrast which the human eye can detect, but estimates are usually between 0.02 and 0.04. The former value, which appears to be fairly widely accepted at present, will be adopted here.

The contrast between an object of brightness B_1 against a background of brightness B_2 , is by definition:

$$C = \frac{B_1 - B_2}{B_2} \dots\dots\dots (6)$$

If an object of brightness B_0 at zero distance is moved away from an observer against a background of brightness B_b , then Koschmeider (See Middleton, 1952) has shown that the apparent brightness B_r at a given range "r" can be derived from the following expression:-

$$B_r = B_0 T_r + B_b (1 - T_r)$$

where "T" is the transmission factor. Substituting the appropriate values in equation (6):-

$$C = \frac{B_0 T_r + B_b (1 - T_r) - B_b}{B_b}$$

For an optically black object B_0 is zero, and the above expression reduces to:-

$$C = - T_r$$

or $C = e^{-\sigma V}$

(from equation (3), and expressing the range as the visibility distance "V").

For a threshold contrast of 0.02, this expression becomes:-

$$V = \frac{3.912}{\sigma} \dots\dots\dots (7)$$

From equation (7) it will be seen that for pure air at standard temperature and pressure, ($\sigma = 0.0127$), the corresponding visibility distance would be about 380 km.

The mean extinction coefficients obtained from the star observations must be used with caution, since haze and air density distributions are not uniform along oblique lines of sight, but it is of some interest to substitute the values given in Table 1, to compare the manner in which visibility is restricted in the horizontal and vertical.

The results, which are shown in Table 2, indicate the distance it would be possible to "see" through a fictitious atmosphere with the same extinction coefficient as that found from the star observations.

TABLE 2

Month	Mean equivalent visibility	Visibility Range
February	177 Kms	244-116 Kms
March	163	280-75
April	244	280-177
May	261	
June	140	177-130
July	196	244-145
August	177	244-156
September	170	230-156
October	156	244-48
November	156	187-140
December	150	

4. APPLICATION TO SPECIFIC PROBLEMS

The extinction coefficients found from the star observations are mean values for the whole atmosphere. This meets the primary purpose of the present investigation, which was to examine overall haziness, but the extinction coefficients so obtained cannot be applied to objects within the earth's atmosphere without making simplifying assumptions.

For objects at moderate or high altitudes, i.e. above 40,000 ft., it is reasonable to assume that 90 per cent of the total haze will occur at lower levels. A new set of extinction coefficients can be computed on this basis by taking into account the remaining air and haze at higher altitudes.

Observations to lower altitudes are really outside the scope of the present method, but where overall haziness is small, or most of the haze occurs in the lower levels, rough initial assessments can be made. For example, if it is assumed that all the haze occurs in the surface layers, and that it is uniformly distributed as the result of turbulence, the following simple model applies. (This is useful in dealing with marked industrial "smog", since haze in the upper levels is likely to be unimportant compared with that near the surface).

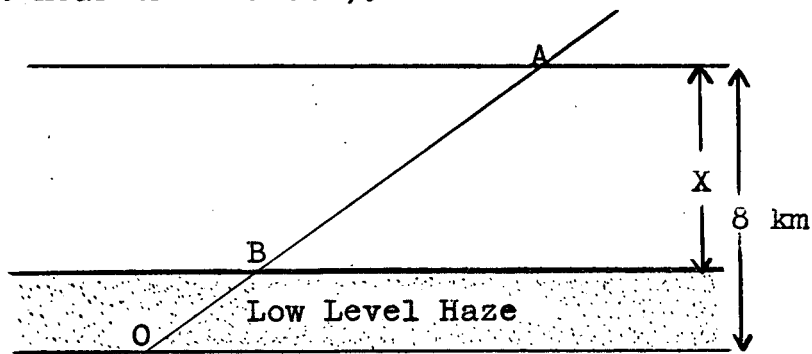


Fig. 1. Along line of sight OA , O is an observer at the ground, B the top of the haze level and A the top of the homogeneous atmosphere of depth X above the haze level.

Let " O " be the observer at the ground, " B " a point at the top of the low level haze, " A " a point at the outer edge of the homogeneous atmosphere (depth 8 km), and " X " the

depth of the uncontaminated air.

If σ_A is the extinction coefficient of pure air, σ_H the extinction coefficient within the haze layer, and σ_0 the mean extinction coefficient along the path OA (as determined from the star observations), then the various quantities are related in the following manner:-

$$X = 8 \frac{\sigma_H - \sigma_0}{\sigma_H - \sigma_A}$$

The depth of the surface haze can usually be estimated from radiosonde soundings, and it is then possible to derive σ_H i.e. the extinction coefficient within the haze. The mean extinction coefficient to any other altitude can be computed in a similar manner.

Rather better approximations can be made by assuming that 80 per cent of the total haze occurs in the lower layers, with the rest uniformly distributed at higher altitudes. It will be noted that if the extinction coefficient within the surface haze is measured by some other method, it would be possible to determine the percentage of haze in the higher levels. Some investigations along these lines have been made, using a "visibility range gauge", and an optically black object for the visibility marker. Initial results have been disappointing, apparently because the minimum contrast which the human eye can detect is larger than 0.02 as some writers have suggested. However, as these matters are better investigated by more precise methods, and are rather outside the scope of the present paper, they will not be discussed further here.

5. CONCLUSION

In conclusion a few comments might be made on the practical side.

- (a) The star extinction method does not seem to be greatly affected by variations in eyesight. Results obtained by six different observers showed maximum differences of about 20 per cent, mainly at very low angles of elevation where significant differences might be expected.

- (b) The method also appears to work equally well with stars of different magnitudes. Ten sets of observations, using Regulus (magnitude 1.3), and Alpherat (magnitude 2.2), gave mean extinction coefficients of 0.0204 and 0.0198 respectively. The maximum difference, which occurred twice, was 0.003.
- (c) As the various equations are very sensitive to small angular changes at low angles of elevation the magnitude of the selected stars should be such that the "disappearance elevations" remain above 5 degrees.

To minimise difficulties in identification by unskilled observers, only stars of first and second magnitude were used in the present investigation, but the resulting angles were undesirably small because of the good visibility conditions on most nights.

References

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| Bemporad, A | 1907 | See Smithsonian Met. Tables, 6th ed., 1949, Table 137. |
| Middleton, W.E.K. | 1952 | Vision through the atmosphere, UNI... Toronto Press, pp 63 - 4, eqn. 4.8. |