CORRESPONDENCE

Evaluation of Upper Air Prognosis by Rossby Method

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It was stimulating to read of an attempt to apply a quantitative dynamic forecasting formula to the Australian area. (Karelsky 1957). Although the results were not very encouraging, this theory is historically extremely interesting, as representing an early and bold attempt at solving the large scale forecasting problem. It also stimulated interest in solutions of the dynamical problem, and was in a sense a forerunner of the intensive work in the numerical forecasting field now being witnessed. One might thus be excused for recalling several features of the problem.

One such feature was the emphasis given to the latitudinal variation of the Coriolis parameter, written β by Rossby, as a factor of importance in determining planetary flow patterns of air and water. This had already been pointed out by Ekman (1923). Rossby's result could not be described as "empirical" without straining the language; rather it is ideal and heuristic; an attempt to find how far the assumptions are justified. The parameter β can clearly only be zero at the poles where U and the problem are undefined. It is difficult to excuse confusion on this point. Rossby, of course, admitted that the variations in β and U, the divergence, density gradient, amplitude of disturbances, curvature of the earth and so on were not vanishingly small, but audaciously swept aside these obstacles to focus attention on one aspect. This approach appears to have been justified by subsequent events.

The usefulness of the hitherto (meteorologically) neglected vorticity conservation theorem was recognised: it made possible elimination of the pressure as a variable in the equations of motion. Hence, within the framework of the assumption, the result was valid whether flow was quasi-geostrophic or not. However, it is easy to see, by means of Charney's (1948) considerations, that the type of motion Rossby had in mind must be quasigeostrophic, in all but low latitudes, for

$$\frac{|\text{ageostrophic velocity component}|}{|\text{total velocity}|} \sim \frac{c}{sf} \sim 10^{-1}$$

where c is the velocity of propagation of flow patterns, s is their characteristic dimension, and f the Coriolis parameter.

The effect of curvature of the earth upon Rossby's conclusion was studied by Haurwitz (1940). Wave velocity can be deduced from his (1951) treatment in the Compendium of Meteorology, although his equations 126, 128, 129 (p.417) are incorrectly written, as may be verified by proceeding from equation 122. Assuming that the small oscillations (he, like Rossby, linearized the equations, by introducing the assumption of infinitesimal amplitude) have zero amplitude (nodes) at equator and poles, but no nodes elsewhere, one can derive the result

$$c = U\left(1 - \frac{2}{(m+1)(m+2)}\right) - \frac{R^2\beta}{(m+1)(m+2)}$$

where c is the easterly wave velocity, U the basic current (on the assumption of constant angular velocity everywhere), m the number (integral) of waves round the earth, R the earth's radius and β the Rossby parameter. It can readily be shown from this result that the discrepancy between this and the Rossby formula is least for low latitudes and shorter waves, and becomes quite large in high latitudes.

The following table sets out, for latitude 30°, the speed of waves (m sec^{-1}) according to (a) the spherical earth formula, and (b) the Rossby formula, for various values of U and m. That part of the table representing most frequently observed values of U and m is enclosed by a heavy line, and it will be seen that the discrepancies are rather small in this section.

Neamtan (1946) showed that Haurwitz' result for wave velocity held good in the unlinearized case, provided his κ, constant angular velocity, (Compendium p.417), is given a different interpretation: namely, the <u>mean</u> angular velocity over the earth. In this case a more realistic distribution of undisturbed zonal velocity may be catered for, but the evaluation of U or κ becomes difficult in practice. The results apply to finite amplitude disturbances, and are thus more general than Haurwitz'. They show, too, that the wave velocity is determined by wavelength and by the global mean speed rather than by that over any one zonal strip.

Table showing speed of long waves as derived by

(a) Spherical earth formula

(b) Rossby formula

U m	3		4		5		6		7		8	
($m\ sec^{-1}$)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
5	-35	-62	-22.0	-32.4	-14.3	-19.0	-9.4	-11.6	-6.3	-7.2	-3.9	-4.4
10	-31	-57	-17.3	-27.4	-9.5	-14.0	-4.6	-6.6	-1.4	-2.2	+1.0	+0.6
20	-22	-47	-8.0	-17.4	0	-4.0	+5.0	+3.4	+8.3	+7.8	+10.8	+10.6
30	-13	-37	+1.3	-7.4	+9.5	+6.0	+14.6	+13.4	+18.0	+17.8	+20.6	+20.6
40	-4	-27	+10.6	+2.6	+19.0	+16.0	+24.2	+23.4	+27.7	+27.8	+30.4	+30.6

(U = speed of zonal current,
m = number of waves round the earth)

References

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(R.H. Clarke)
24th March, 1958.

Evaluation of Upper Air Prognosis by Rossby Method

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In his discussion of the Rossby formula (Aust. Met. Mag. 19, p.22) Karelsky claims that

> "Rossby's consideration of φ as a variable in the Cartesian co-ordinate system with the origin fixed at latitude φ is very questionable. φ being considered as the latitude of the origin of the fixed co-ordinate system (which does not change its position) cannot be a function of y in this system".

This seems to represent a misunderstanding of Rossby's reasoning.

Rossby was concerned with the simplest possible model of the zonal circulation, viz. that of an incompressible frictionless fluid in horizontal motion. For this model the equations of motion can be compressed in the vorticity equation

$$d/dt\ (\zeta + f) = 0$$

where $f = 2\omega \sin \Phi$ is the Coriolis parameter and ζ the relative vorticity. The motion was considered in an infinite plane touching the earth in latitude φ.

This plane forms an angle equal to φ with the axis of rotation of the earth, and since the Coriolis parameter represents the component of the earth's vorticity at right angles to the plane it is clear that f must have the same value $2\omega \sin \varphi$ everywhere on the plane. Hence it follows from the vorticity equation that meridional movement will <u>not</u> change the relative vorticity.

However, while a mathematician might have stopped at this trivial result Rossby noted that the essential difference, dynamically speaking, between conditions on the plane and on the earth's surface is the change which occurs in the angle between the zenith direction and the earth's axis during meridional movement on the earth's surface. This feature can be built into the model by assuming a plane made up of strips $dy = a\, d\Phi$ wide (a = radius of the earth) which initially touch the earth each in a different latitude, acquiring thereby as it were their true Coriolis parameter, and are then placed side by side to form the co-ordinate plane. The latter will now be characterised, for small perturbations from the latitude φ, by an approximately linear variation of f with the meridional co-ordinate y.

It seems quite beside the point to raise mathematical objections to the above artifice of a plane endowed with the most essential feature of the earth's surface (or a similar concept put forward by Eliassen and Kleinschmidt in Handbuch der Physik, vol. Geophysik II, p. 77). What matters is that in this manner the model remained sufficiently tractable to yield a closed expression the Rossby formula, describing for the first time the equilibrium relation between the strength of flow and the length of waves in the upper westerlies - quite apart from opening the way to the most important field of modern dynamic meteorology.

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7th May, 1958