

THE FREQUENCY DISTRIBUTION OF DAILY MAXIMUM WIND GUSTS

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Abstract: The skewness of the frequency distribution of maximum values of meteorological elements and the lack of fit of the normal distribution to the observations is well known. Several types of extreme value distributions are available. However, Gumbel's (1954) distribution, when plotted on Extreme Value Probability Paper, appears as a straight line and extrapolation is simple.

Daily maxima of the homogeneous wind records of Brisbane provide 5,162 observations. These are first plotted as a frequency distribution and smoothed. They are then plotted on Extreme Value Probability Paper. The reduced variate y is related to the frequency p by the relation $y = -\log_e (-\log_e p)$. The Gumbel distribution plots as a straight line, the equation of which is $x = u + (1/a)y$, where u is the mode and $1/a$, the slope of the line, is related to the standard deviation of the sample. As the best fit for the points is a straight line, Gumbel's distribution fits quite well. The larger the number of observations the more closely does Gumbel's theory tend to apply.

1. INTRODUCTION

Workers investigating the frequency distribution of maximum values of various meteorological elements have long realised the skewness of the distribution and the complete lack of fit of the normal distribution of the observations. However, they have also been faced with quite a variety of extreme value distributions from which to choose, such as the logarithmic normal, Pearson's Type III, the Incomplete Gamma distribution, Jenkinson's distribution and Gumbel's distribution. Moran (1957) points out that "we have to estimate the shape of the tail of the distribution from a set of observations not lying in this tail. The form of the distribution is not known and any distribution used must be guessed. This may have a considerable effect since the part of the distribution we are interested in is well away from the part where the observations provide some information about the shape. It is therefore easy to construct two different distributions both of which fit the observations closely but for which the tails are of quite different shape.

However, Gumbel (1954) has shown that for "initial" distributions of the exponential type (which includes the log-normal, chi-square, logistic and normal distributions) the distribution of the maximum values will follow his asymptotic theory. An appealing feature of Gumbel's distribution is that, when plotted on Extreme Value Probability paper (such as Fig 3), it appears as a straight line. There is thus no difficulty in regard to extrapolation.

Preparatory to commencing a study of extreme annual maximum wind gusts, the writer was faced with the above considerations and felt that it might be possible to clarify the position somewhat by studying first of all the distribution of daily maximum wind gusts at a station over a fairly long period of time.

2. GENERAL

The wind record for Brisbane provided a set of 5,162 daily maxima from the date of installation of a Dines anemometer in November, 1938, to its replacement by a distant recording anemometer of similar type in January, 1953. The record is thus perfectly homogeneous in respect to site and instrumentation.

Fig. 1 shows the actual frequency distribution of the 5,162 daily maximum gusts. The mode at 21 mph is very pronounced. Also very pronounced are the secondary minima which occur on the odd numbers of mph (indicated by arrows). (Such a feature would not be in evidence today due to the practice of converting into knots). The reason behind this phenomenon, which is very marked at 19, 23, 27, 37, 39 and 43 mph (i.e. integers 3, 7 and 9), appears to lie in the fact that the Dines chart is ruled into even 2 mph divisions. The fact that it shows up over so many years of record indicates a personal failing common to all observers and not to any particular individual. The statistical procedure was therefore adopted of casting half the odd mph frequencies to the even mph below and half to the one above. This resulted in the smoothed relative frequency curve of Fig. 2. The asymmetrical nature of the distribution is immediately apparent. It is markedly positively skewed. As can be seen, the mode occurs around 21 - 22 mph; the mean maximum gust is found to be 24.6 mph; the standard deviation is 7.8 mph. In Gumbel's theoretical distribution the mode is associated with a probability density of 37 per cent whilst the mean is associated with 57 per cent. This gives an idea of the skewness, remembering that for the normal distribution the mean and the mode coincide at 50 per cent.

Passing now to Fig. 3 we see the distribution plotted on Extreme Value Probability paper. Owing to the large number of points, it was necessary to plot the more congested regions as vertical bars with the number of points represented alongside. For instance, corresponding to

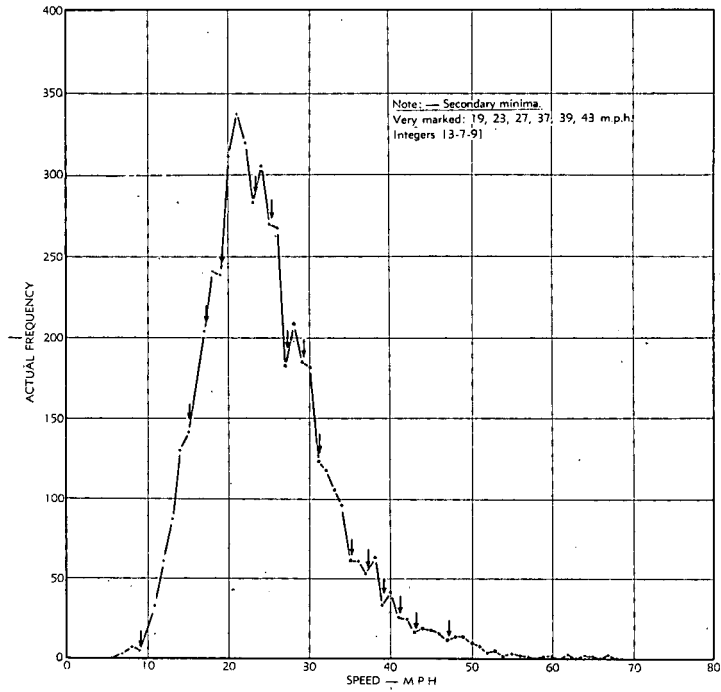


Fig. 1. Daily maximum wind gusts — Brisbane 1938-1953.

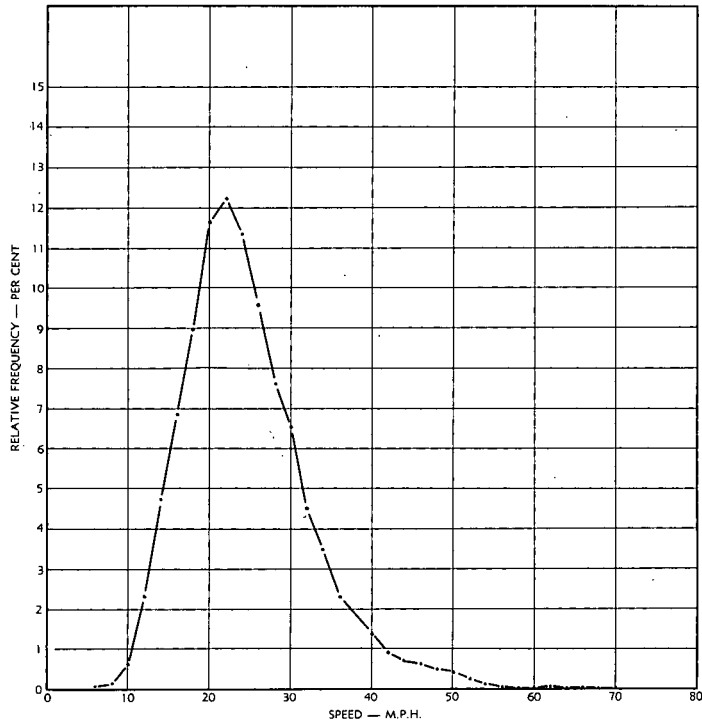


Fig. 2. Maximum daily gust — Relative frequency — Brisbane 1938-1953. 5162 gusts.

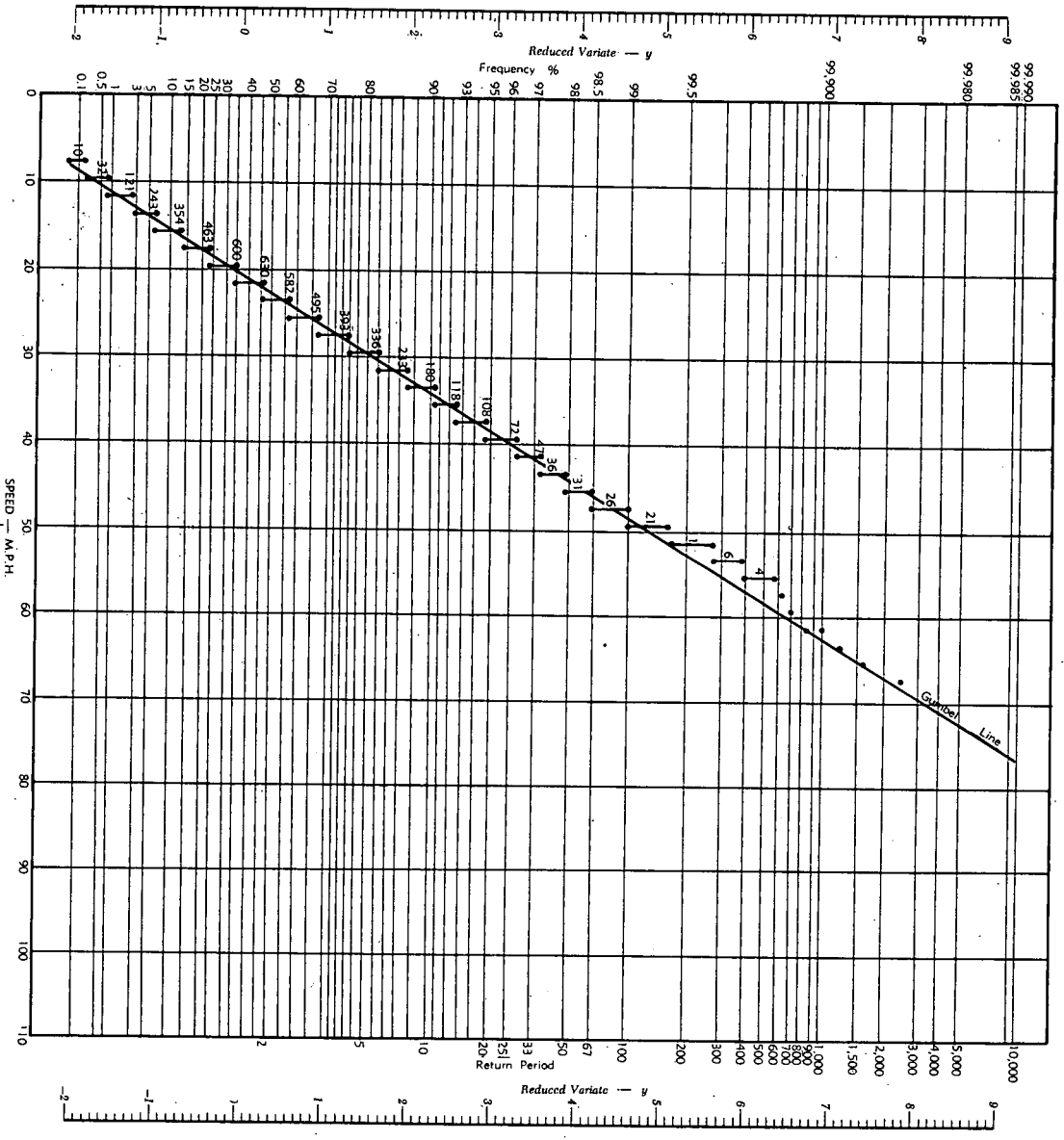


Fig. 3. Extreme value probability - Daily maximum wind gusts - Brisbane

a speed of 22 knots, occupying the frequency 35 to 47 percent, is to be found a vertical bar representing 630 points. The plotting positions of the points are given by $m/(N + 1)$ where m is the rank of the point (working upwards from the lowest value) and N is the total number of observations. The reduced variate y has a vertical scale alongside that of the frequency. The reduced variate is related to the frequency p by the relation.

$$y = -\log_e (-\log_e p)$$

Tables of the iterated natural logarithm, in a convenient form for use in this type of work, are to be found in N.B.S. (1953).

The Gumbel distribution plots as a straight line on this type of probability paper. The equation to the Gumbel line is

$$x = u + (1/\alpha) y$$

Here u is the mode, and $(1/\alpha)$ is the slope of the line; $(1/\alpha)$ is related to the standard deviation of the sample and is therefore a measure of dispersion.

The Gumbel line was calculated for the Brisbane figures and was found to be

$$x = 21.03 + 6.1265 y$$

It is drawn in on Fig. 3. It would be difficult to justify the fitting of any curve other than a straight line to the plotted points, and we can draw the conclusion that Gumbel's distribution fits the daily maxima quite well.

This is not a surprising result in view of the large number of gusts involved in each daily sample, of which the daily maximum is the extreme. In Gumbel's theory, N is the number of extremes, n is the number of observations in each set of the original data yielding one extreme. The larger the value of n , the more closely does Gumbel's theory tend to apply. Here n is the total number of gusts in an average day, which is, of course, quite large.

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