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THE EVAPORATION OF RAINDROPS AND THE ROLE OF COLLISION WITH CLOUD DROPLETS IN RAIN OF ICE CRYSTAL ORIGIN

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Abstract: Following an analysis of the occurrence of rain at Adelaide, S.A, during 1957-58-59, in relation to cloud characteristics and air mass trajectory, in which possible evaporation effects were indicated, a more detailed consideration is presented of the effects of evaporation in the sub-cloud layer, of accretion of water drops in the warm part of the cloud and of the intensity of rain originating as ice crystals in very thick stratified clouds.

It is suggested that the intensity of rain of ice crystal origin, from very thick stratiform clouds, depends to a large extent on the mass increase of rain drops in the layer of cloud warmer than $0^{\circ}C$, and the decrease by evaporation effects in the layer from cloud base to ground.

A "Dryness Index" is postulated, based on the empirical formula of Kinzer and Gunn (1951), to estimate the shrinkage of drops in the sub-cloud layer, and a "Coalescence Factor" is defined. It is suggested that moderate and heavy intensities of rain occur only when the Dryness Index is less than 5.0, while when the Dryness Index is high only weak intensities, say less than 4 points per hour, occur.

1. INTRODUCTION

The amount of rain during any period is simply the sum of the mass of individual drops reaching the ground in that period. The terminal velocity of drops relative to the surrounding air may be regarded as a function only of drop mass. Relationships between drop size spectrum and intensity of rain have been sought by various workers.

The complicated distributions of rain drop size that occur preclude a single simple relationship. It is suggested, however, that observations of the relationship between drop size spectra and rainfall intensity and also laboratory measurements of evaporation of raindrops, that have been reported by many Authors during the past ten years, will apply to rain at Adelaide.

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In this paper the observations reported by Blanchard and Spencer (1957) are applied to obtain an approximate maximum raindrop size associated with a particular intensity. Although their result is based on the data of "Project Shower" carried out at Hawaii during 1954, it is thought that as a first approximation it will be useful when applied to the particular situations studied at Adelaide.

Occurrences of rain at Adelaide in the six hour period centred on the time of release of the Radiosonde have been examined for the three years 1957, 1958 and 1959 and those cases where rain fell from stratiform cloud extending more than 20,000 feet above the freezing level have been selected for study. Cloud base height was obtained from visual observations, while cloud tops were estimated from the Radiosonde record according to criteria,

(i) Relative Humidity greater than 80% and

(ii) Moist Adiabatic lapse rate,

defining cloudy air.

2. PHYSICAL CONSIDERATIONS

(a) Evaporation of raindrops

To obtain the shrinking rate of rain drops falling through an unsaturated air layer, the well-known formula of Kinzer and Gunn (1951) is used.

$$\frac{dM}{dt} = 4\pi r D(\rho_\infty - \rho_r)\left(1 + \frac{F}{4\pi D/\nu}\sqrt{Re}\right)$$

M is mass of a raindrop of radius r.

ρ_∞, ρ_r are the vapour densities of surrounding air and at the surface of the raindrop respectively.

D is a diffusion coefficient.

F a dimensionless factor which is regarded as constant for $r > 250\mu$

ν is the kinematic viscosity of water in air.

Re is the Reynolds number for flow of air around the drop.

If the variation of evaporation rate due to variation of drop size through a thin layer is ignored,

$$1 + \frac{F}{4\pi D/\nu} Re^{\frac{1}{2}} = f(r) = \text{constant in this layer.}$$

$$\frac{dM}{dt} = 4\pi r D(\rho_\infty - \rho_r) f(r)$$

$$4\pi r^2 \frac{dr}{dt} = 4\pi r D(\rho_\infty - \rho_r) f(r) \quad \text{if density of water is taken} = 1$$

$$r\frac{dr}{dt} = D(\rho_\infty - \rho_r) f(r) \qquad (1)$$

writing $\frac{dz}{dt} = w$

eqn. (1) becomes $$r w \frac{dr}{dz} = D(\rho_\infty - \rho_r) f(r) \qquad (2)$$

If w is taken as constant for the range of maximum drop size in a thin layer and approximated to by an average $\overline{w}$, integrating eqn. (2) gives

$$\frac{1}{2}\overline{w}\left[r_1^2 - r_0^2\right] = D f(r) \left(\overline{\rho_\infty - \rho_r}\right)(z_1 - z_0)$$

and from the equation of state $e = \rho R_w T$

$$\frac{R_w}{2\,D\,f(r)}\left[r_1^2 - r_0^2\right] = \frac{1}{\bar{w}}\left(\frac{e_\infty}{T_\infty} - \frac{e_r}{T_r}\right)\left(z_1 - z_0\right) \qquad (3)$$

where T_∞ is the temperature of the ambient air and e_∞ is the vapour pressure of the ambient air and is written now as e_d the saturation vapour pressure at dewpoint T_d.

According to Kinzer and Gunn (1951) the surface temperature of a falling raindrop assumes very closely the temperature of a well ventilated wet bulb in the same environment. Consequently T_r is approximately the wet bulb temperature of the ambient air (T_w) and e_r is equivalent to e_w the saturation vapour pressure at T_w.

Thus equation (3) may be written

$$\left[r_1^2 - r_0^2\right] = \frac{2\,D\,f(r)}{R_w\,\bar{w}}\left[\overline{\frac{e_d}{T} - \frac{e_w}{T_w}}\right]\left(z_1 - z_0\right) \qquad (4)$$

The term $\left[\overline{\frac{e_d}{T} - \frac{e_w}{T_w}}\right]$ depends on the distribution of temperature, dewpoint, and wet bulb temperature in the layer and the effect on drop radius of the whole sub-cloud layer may be obtained by numerically evaluating

$$r^2_{\text{initial}} - r^2_{\text{final}} = \frac{2}{R_w}\sum_{i=1}^{n}\left\{\frac{D_i\,f(r_i)}{\bar{w}_i}\left(\overline{\frac{e_w}{T_w} - \frac{e_d}{T}}\right)_i\left(\Delta z\right)_i\right\} \qquad (5)$$

Such computations have been performed for various isothermal atmospheres of constant relative humidity with the aid of the Smithsonian Tables 1954 and are shown in Fig. 1. The increment for computation was, drop size diameter change in a layer is .1 mm ($\Delta r = .1$ mm), and results are presented as "Length of Fall Path" to shrink to a final diameter of .2 mm, commencing with initial drop diameter in the range .2 mm to 2.2 mm.

The term $\left(\overline{\frac{e_d}{T} - \frac{e_w}{T_w}}\right)$ of equation (4), shows the influence of the dryness of ambient air and is, by virtue of Clapeyron's Equation, which shows saturation vapour pressure to be a function of temperature, a function of e_d, e_w and T. Invoking then the so-called "psychrometric formula" at a particular pressure, T_w and thus e_w is a function of dry bulb temperature T, and actual vapour pressure e_d.

Therefore at a particular pressure

$$\left(\overline{\frac{e_d}{T} - \frac{e_w}{T_w}}\right) \quad \text{for a comparatively thin layer}$$

is a function of e_s and e_d, i.e. $= \overline{B\,(e_s,\ e_d)}$ say, where e_s is saturation vapour pressure at T. While the function $\overline{B\,(e_s,\ e_d)}$ exists, it is far from a simple algebraic form, and the simplest form that recommends itself for an empirical parameter is the saturation vapour deficit which has been used extensively in empirical formulae to estimate evaporation from open water surfaces;

i.e. $\overline{B\,(e_s,\ e_d)}$ is regarded proportional to $(\overline{e_s - e_d})$.

The Dryness Index is thus defined as,

$$E_I = (\overline{e_s - e_d})_1\,\Delta z_1 + (\overline{e_s - e_d})_2\,\Delta z_2 + \cdots \qquad (6)$$

where $(\overline{e_s - e_d})$ is the average saturation vapour deficit in the layer Δz between significant levels as reported by the radiosonde. Eqn. (6) thus ignores the influence of pressure that was introduced

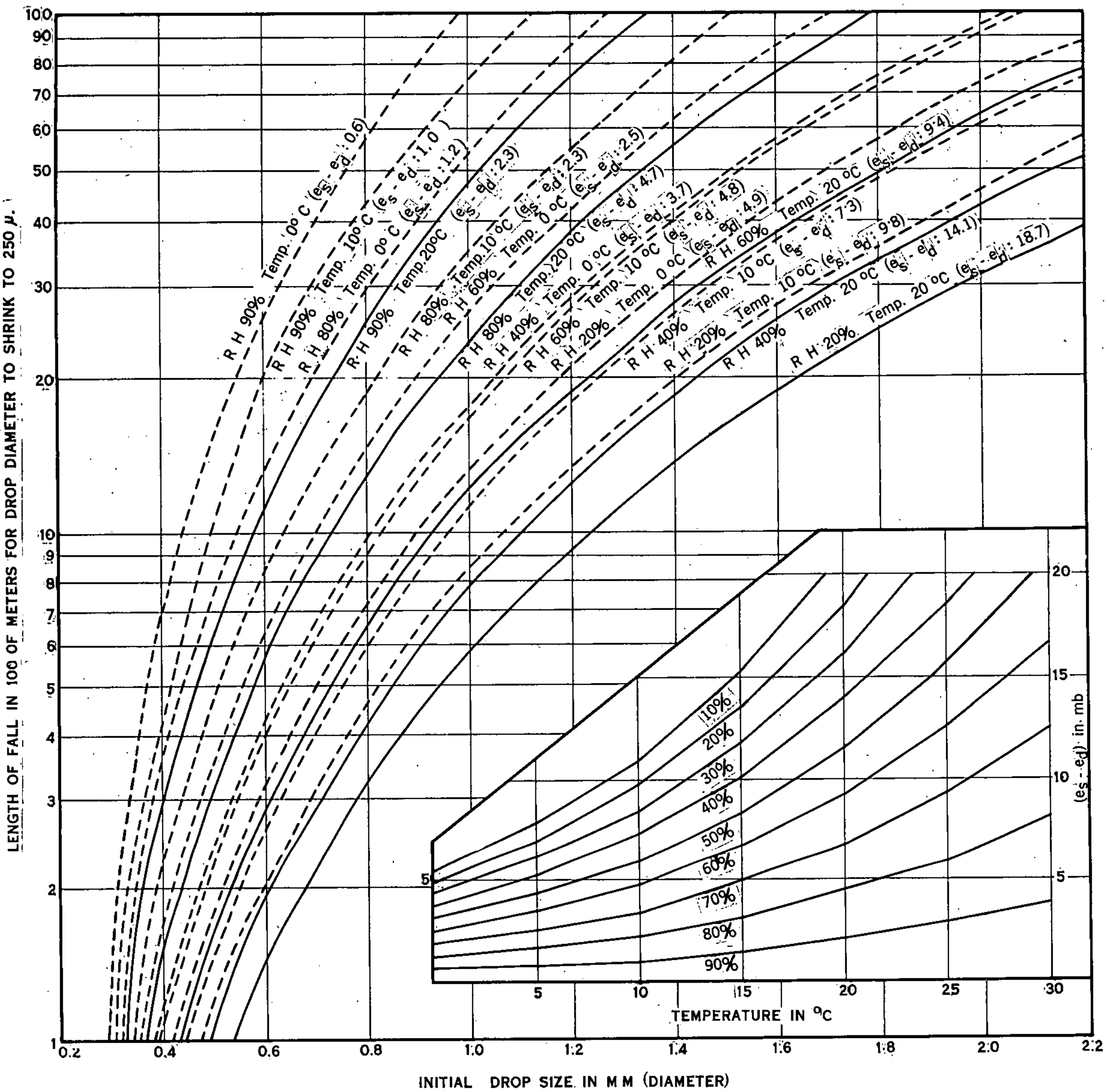

FIG. 1 Nomogram giving length of fall necessary to evaporate drops of certain diameters to a diameter of 250 μ in various environments (based on results of Kinzer and Gunn,1951) .

Relationship between temperature,humidity and saturation vapour deficit shown in lower right of diagram.

by the psychrometric formula, but is suggested as a useful parameter from equation (4), in which $\frac{D_i\, f(r)_i}{\overline{w}_i}$ has a range of about 1/3 of its maximum value for drop diameters ranging from 2.0 mmto 0.5 mm.

It is considered the dryness index will be subject to much greater changes and as a measure of evaporation effects will be related to rainfall intensity for various initial values of maximum drop size leaving the cloud.

(b) The Role of Coalescence

In the case of rain originating as ice crystals and then falling through a cloud layer warmer than 0°C, the raindrops will grow primarily by collision with cloud droplets. In the layer -5°C to -15°C where supercooled drops exist, ice crystals smaller than .2 mmgrow both by accretion and sublimation (Houghton 1950). The equation of growth of a precipitation element of graupel, or ice crystal, is then,

$$\frac{dM}{dt} = 4\pi r D(\rho_\infty - \rho_r) + \int_0^{r^1_{max}} Q(r, r^1)\, V(r, r^1)\, m(r^1)\, dr^1 \qquad -(7)$$

where M is the total mass of a drop of radius r,
D is coefficient of water vapour diffusion, ρ_∞ and ρ_r water vapour density of ambient cloud air and at the surface of the drop respectively,
$Q(r, r^1)$ a coalescence function relating the cross-sections of the falling drop, and cloud droplets of radius r and r^1 respectively,
$V(r, r^1)$ the relative terminal velocity of drops and droplets, and
$m(r^1)\, dr^1$ is the mass per unit volume of droplets in the radius range r^1 to $r^1 + dr^1$.

In the supercooled part of the cloud the first term on the right hand side of equation (7) cannot be ignored, but is negligibly small in the layer warmer than 0°C (Houghton 1950, Ludlam 1951) and in this layer eqn. (7) can be solved numerically using available experimental or theoretical values for $Q(r, r^1)$, Kinzer and Gunn's empirical values of $V(r, r^1)$ and observed droplet spectra.

For the purpose of this paper, equation (7) can be simply written

$$4\pi r^2 \rho_w\, dr = \pi r^2 E w V\, dt \qquad -(8)$$

where ρ_w, the density of water in drop, is taken = 1, w is mean liquid water content of cloud, and E Langmuir's collection coefficient, and V is relative fall speed of the raindrop.

Therefore $\frac{dr}{dt} = w\frac{E}{4}V$

If $\frac{dz}{dt} = \mu - V$, where μ is updraft speed.

$$\frac{dr}{dz} = w\frac{E}{4}\cdot\frac{V}{\mu - V}$$

and $$4\int_{r_0}^{r^1} \frac{\mu - V}{E\,.\,V}\, dr = \int_{z_0}^{z_1} w\,.\,dz \qquad -(9)$$

In the deep stratified cloud systems studied, it is assumed that the updraft current is uniform and constant, and the liquid water content and droplet spectrum is constant.

For raindrops in such an environment E is constant and from equation (9)

$f(r_0, r_1) \propto (z_1 - z_0)$, i.e. for a drop of initial radius r_0 falling through the 0°C level, the final size is simply related to the depth of the warm part of the cloud.

The initial size of ice crystal or graupel falling through $0^{\circ}C$ may vary from 100 μ to 1000 μ in diameter but no measurements of this nature or of the actual liquid water content of the warm part of the cloud are available.

From the similar physical structure of the clouds studied it is thought that the spectrum of precipitation elements reaching $0^{\circ}C$ would be rather similar and maximum drop size leaving the base of the cloud would be determined largely by the coalescence process in the warm part of the cloud.

(c) Deduced Assumptions

The assumptions set up then are,

(i) Evaporation of the largest raindrops depends primarily on the dryness and depth of the sub-cloud layer as indicated by $(\overline{e_s - e_d})$ x (Height of Cloud Base).

(ii) The size of the largest drops leaving a deep stratiform cloud depends largely on coalescence in the warm part of the cloud.

3. RESULTS

Data was selected from cases of "ice crystal origin" rain during 1957, 1958 and 1959 at Adelaide, South Australia, according to the following conditions,

(i) Cloud thickness exceeding 20,000 feet, stratified and with cloud-top temperatures colder than $-15^{\circ}C$,

(ii) Rain occurred in the six hour period centred on time of release of radiosonde.

The dryness index was evaluated for each case according to equation (6) by using the radiosonde data plotted on a skew T - log P diagram. The thickness of cloud layer warmer than $0^{\circ}C$ was obtained by use of radiosonde data and surface observations of base heights. Average rain intensity during the period was calculated from pluviograph traces.

The results are shown in Figure 2. The rainfall intensity in points per hour, or millimetres per hour, is described as a function of the depth of cloud warmer than $0^{\circ}C$ and the dryness index of the sub-cloud layer. The dryness index has been grouped into divisions $\leqslant 1.5$, 1.6 - 5.0, 5.1 - 9.5 and $\geqslant 9.6$, so that the trajectories shown in Fig. 1, for the different atmospheres indicated thereon, of one kilometre depth, are divided into four distinct classes without overlapping.

The intensity of rain versus maximum drop diameter, after Blanchard and Spencer, is shown in Figure 3.

From Figure 2, rain intensity increases with increasing depth of the warm part of cloud, and in the range 4 to 16 points per hour is classified clearly by two ranges of dryness index of averages 1.0 and 2.3.

According to Blanchard and Spencer (Figure 3) the maximum diameter of raindrops corresponding to this rain intensity range is approximately 0.7 to 1.1 mm.

From Figure 1 the initial size of such drops leaving the cloud and falling through sub-cloud layers of dryness indices 1.0 and 2.3 can be estimated as these indices correspond to a fall of 1 kilometre in sub-cloud layers of, say, $+ 10^{\circ}C$ and 90% relative humidity, and $+ 10^{\circ}C$ and 80% relative humidity, respectively.

For an index of 1.0 a drop of approximately .78 mm and for an index of 2.3 a drop of approximately .86 mm will shrink to 0.7 mm diameter, while initial drops of 1.15 mm and 1.20 mm respectively for the same indices will shrink to 1.1 mm.

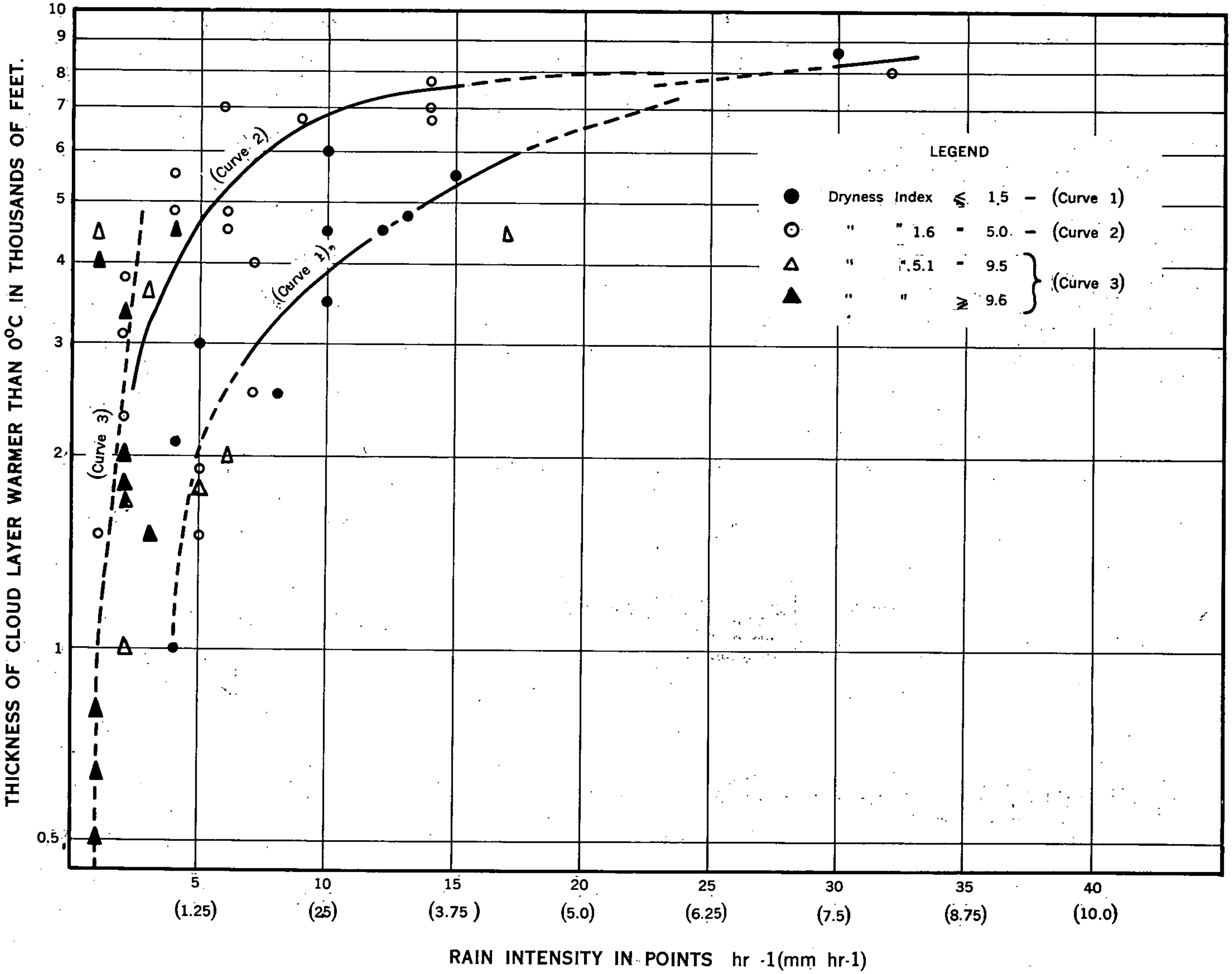

FIG.2 RAIN INTENSITY AS FUNCTION OF COALESCENCE FACTOR AND DRYNESS INDEX

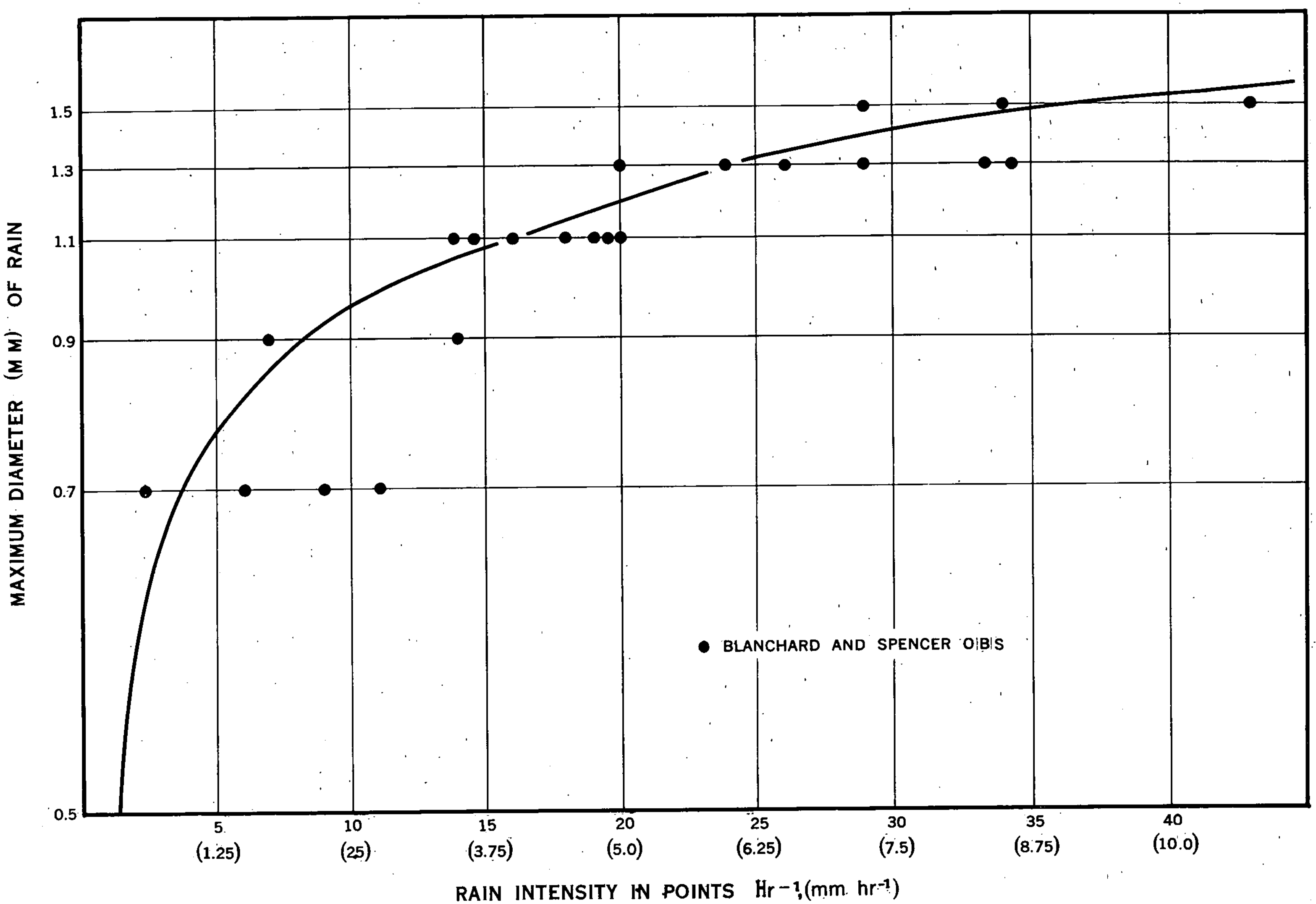

FIG.3 INTENSITY VERSUS MAXIMUM DROP SIZE (AFTER BLANCHARD AND SPENCER)

The required thicknesses of the warm part of the cloud, for these different dryness indices to result in the same intensity of rain, can be examined further in terms of drop size required at cloud base, to see if coalescence in the warm part of the cloud can, as assumed, account for drop size and drop variation.

If the liquid water content of the cloud layer warmer than $0^{\circ}C$ is 0.75 grams per cubic metre and updraft velocity 1 metre per second, a melting ice crystal of 0.25 mm falling through the $0^{\circ}C$ level will grow to 0.8 mm diameter by coalescence after falling through 3,000 feet of cloud, to 0.9 mm diameter after falling 4,000 feet, to 1.1 mm diameter after falling 5,200 feet, and to 1.25 mm after falling 6,500 feet in cloud (Figure 4 after Bowen 1950).

This model of drop growth compares favourably with the results of the investigation for rain between 4 and 16 points per hour as shown on Figure 2.

The fact that for these intensities a reasonable model of droplet growth by coalescence yields a drop size dependence on thickness of the warm part of the cloud, consistent with observations and deductions based on Kinzer and Gunn's evaporation studies and Blanchard and Spencer's observations, indicate that these parameters as suggested in sections 2 and 3 are important.

From Fig. 2, however, it is seen that rainfall intensity less than 1.0 mm per hour depends not so much on coalescence as on evaporation. Apparently the raindrops are greatly reduced in size by evaporation in the sub-cloud layer when the dryness index exceeds 5.1 and the rain intensity is weakened considerably even when the warm part of the cloud is rather thick. It is seen from Fig. 2 that on only one occasion did rain intensity exceed 1.5 mm/hour (6 points per hour) when the dryness index was greater than 5.0. The model of droplet growth applied in the region 1 to 4 mm/hour does not apply to the full range of dryness indices observed with rainfall intensities less than 1 mm per hour.

Due to the shortage of data the relation between rainfall intensity and dryness index when intensity is greater than 5 mm/hour is not clearly shown. However, when the dryness index is less than 5.0, the evaporation effects are apparently small enough to permit moderate and heavy rain provided the warm part of the cloud is sufficiently thick. Moreover, as the terminal velocity of large drops is high, these drops shrink relatively less in the sub-cloud layer and the clearly separated curves in the case of weaker rainfall for different dryness indices converge with increasing intensity. However, according to Fig. 3 (Blanchard and Spencer) the intensity of heavy rain is less dependent on maximum drop size and this convergence or independence of dryness index for large drops as indicated by a thick warm layer of cloud is expected.

From the data it appears that very large drops are not produced from this type of cloud in South Australia and rainfall intensities greater than 5 points per hour occur only with dryness index less than 5.0.

4. CONCLUSIONS

No direct measurements of raindrop size, water content, or updraft speed were available for this study; nevertheless, the result suggests that rainfall intensity is strongly dependent on evaporation in the sub-cloud layer and on coalescence processes in the cloud layer warmer than $0^{\circ}C$, and these parameters can be easily applied to construct objective forecasting aids. It is considered that further studies of this nature should be encouraged with a view to applying the results of studies in the field of cloud physics to routine forecasting of cloud and rain.

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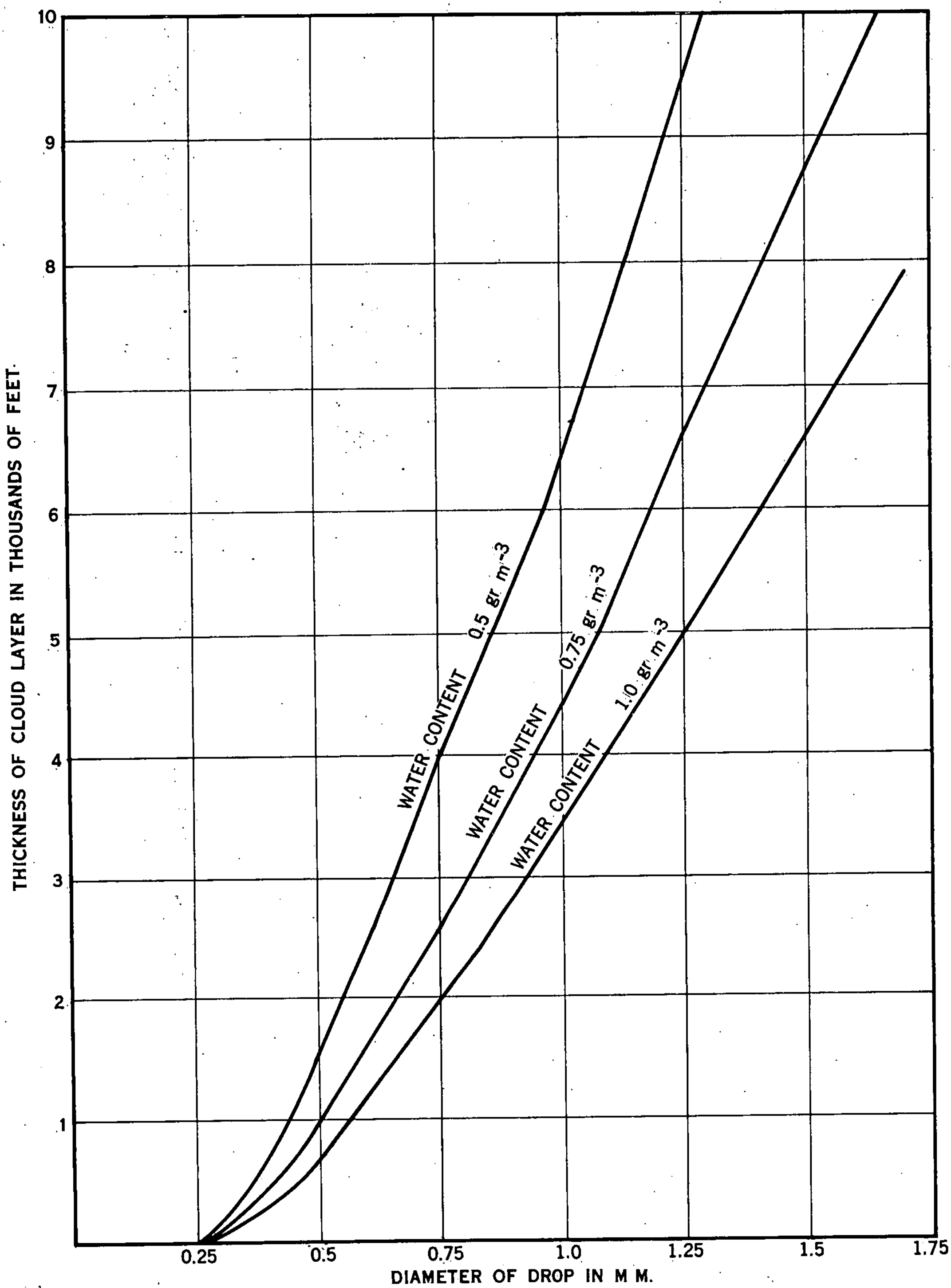

FIG.4 GROWTH OF A DROP 250 μ DIAMETER AT INITIAL STAGE, WITH THICKNESS OF CLOUD LAYER IN WHICH THE CONSTANT SPEED OF UPDRAFT IS 100 cm sec $^{-1}$, FOR THE DIFFERENT CLOUD WATER CONTENT. (AFTER BOWEN)

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