

MOUNTAIN LEE WAVE ACTIVITY OVER TASMANIA ON 17 APRIL 1966

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ABSTRACT

Mountain lee waves were observed on a satellite photograph over Tasmania on the morning of 17 April 1966.

Estimations of the wavelength and amplitude of these waves is obtained from a study of the Hobart Airport radio-sonde flight which passed through the wave pattern. This is achieved by analysing the sonde recorder trace to obtain the vertical velocity of the sonde during its flight.

A calculation of Scorer's l^2 parameter shows a maximum value in the lower troposphere. This is considered a necessary condition for lee wave activity.

The relationship between the mean tropospheric wind speed and the wavelength of the lee wave activity is studied. On theoretical grounds a linear relationship would be expected and in fact, using a number of actual cases of lee waves, a high correlation is found between these two parameters.

I. INTRODUCTION

Figure 1 is an enlargement of a portion of an APT facsimile photograph received from the United States weather satellite ESSA 2. This photograph was taken on orbit 600 at 0740 EST on 17 April 1966, when the satellite was at a height of approximately 800 miles. The significant feature of this photograph is the approximate north-south banding in the cloud over Tasmania. This was interpreted as due to the presence of lee wave activity, as the banding was at right angles to the direction of the surface wind. (Compare Fig. 1 with Fig. 2, the synoptic situation at 0900 EST for this region.)

To obtain confirmation that lee wave activity did in fact exist over Tasmania on this occasion, and also to obtain a more accurate estimate of the geographical position of the wave system, the Hobart Airport radio-sonde flight for 0900 EST was studied to see if the influence of lee wave activity could be detected in it.

It was assumed that the lee wave pattern would remain constant throughout the period of 80 minutes from the satellite picture till the sonde flight. This assumption was justified after noting the small change in the synoptic pattern and upper winds from 1700 GMT to 2300 GMT.

Earlier studies of lee waves over Tasmania and Southern Victoria include Desmond and Radok (1949) who described lee waves caused by the Tasmanian Central Plateau, as reported by commercial aircraft, Radok (1950) who described lee waves over the the Dandenong Ranges, Victoria, studied by using a specially instrumented aircraft to fly through the wave system, and Reid (1952) who studied lee waves in the same position over Tasmania as in the present study although in his study the wave system appeared to have a much larger amplitude than in the present case.

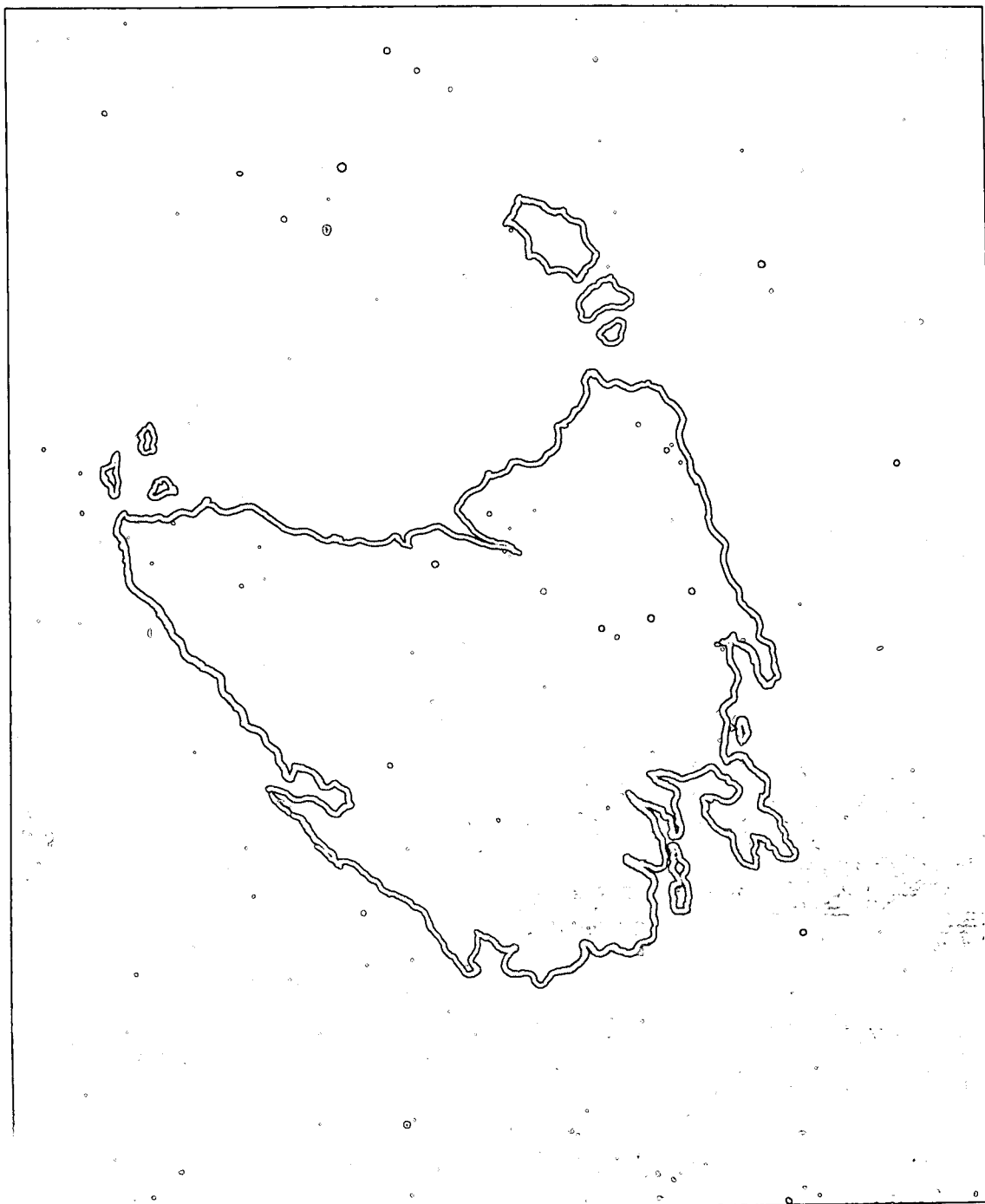


Fig. 1 APT photograph from U.S.A. ESSA 2 satellite on orbit 600, at 0740 EST, 17 April 1966.

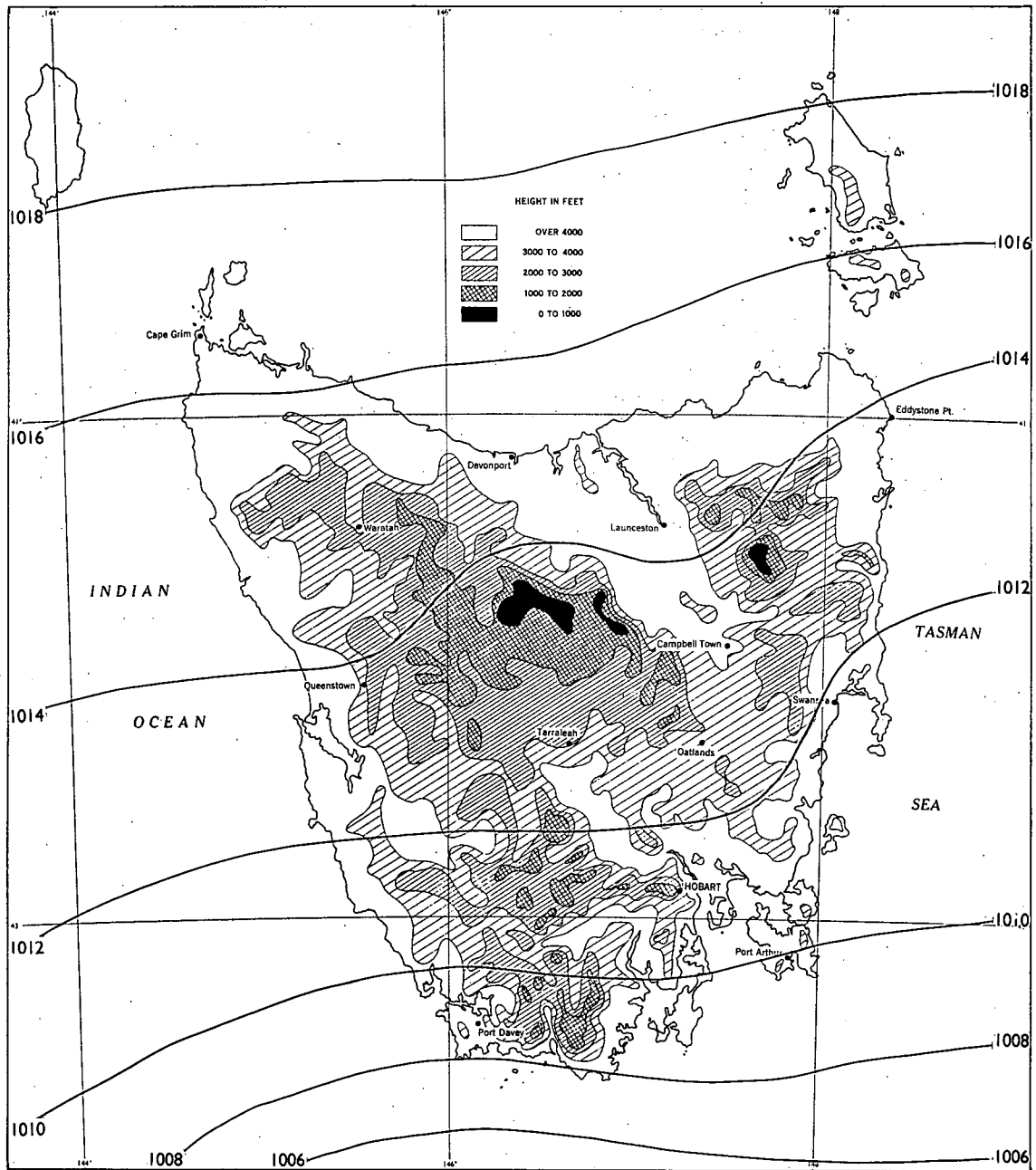


Fig. 2 M.S.L. analysis, 0900 EST, 17 April 1966.

2. ACCURACY OF THE APT GRIDGING SYSTEM

As the satellite subpoint on the surface of the earth moves at more than 3 n.mi./sec an error of one second in recording the time the picture is taken can result in an error of more than 3 n.mi. in the position of a point near the centre of the picture. Due to the curvature of the earth the error is greater than this near the edges of the picture.

Intermittent variation in roll, pitch or yaw of the spacecraft cannot be detected by the ground APT receiving station unless geographical features are present in the picture to allow accurate referencing. In the picture under study (Fig. 1) no geographical features were discernible, so an error may be present due to the above causes.

Thirdly, errors may occur due to the fact that distortion in the film on which the grids are printed can occur under certain temperature and humidity conditions.

A combination of these errors can cause an uncertainty of at least 10 n.mi. in the position of any point in the APT photograph. This is comparable in magnitude with the wavelength of the wave system under study. Measurements from Fig. 1 give a wave length of 11 n.mi. for the lee wave system. Another method is therefore required to accurately find the geographical location of the wave system. This is discussed in the following section.

3. A METHOD OF FINDING THE VERTICAL VELOCITY IN LEE WAVES

The method used is similar to the one used by Reid (1952) and Corby (1957a).

The radio-sonde calibration strip and recorder trace for the 9 a.m. flight on 17 April were obtained. The calibration strip shows the pressure at which the sonde baro-switch will switch from one sensor element to another during the flight (e.g., from temperature reading to humidity reading).

The recorder trace is drawn when a pen, which is activated electronically by the sonde sensor elements, is allowed to write on a paper roll which passes under the pen at a constant rate of $\frac{1}{2}$ inch per minute during the sonde flight. As the sonde ascends horizontal lines are drawn on the trace when a change of element occurs. From the calibration strip the pressure at which these lines are drawn can be found, so this trace is in fact a graph of pressure vs. time. Using the hydrostatic equation the recorder trace of pressure vs. time can be converted to a graph of height vs. time.

In theory the rate of ascent of the sonde at any instant could be approximated to by dividing the thickness between changes in sensor element by the time between changes, i.e. by $\Delta Z/\Delta t$. But as the pen line on the recorder trace can only be measured to an accuracy of $1/64$ inch this introduces an uncertainty of 2 seconds in the time of the switch. It is obvious that this could be improved by using an accurate clock at the sonde station and electronically recording the time of the sensor change, but this was not possible in the present case. Another source of error is the fact that the pressure at which the sonde will switch is not known to a greater accuracy than about $\frac{1}{4}$ millibar. So if Δt is taken as about 20 seconds and Δp as 4 mb, which is representative of the pressure difference between elements, an error of up to 30 per cent in the estimated rate of ascent can occur.

To overcome this difficulty the time and height of each change of element were plotted on a graph and a smooth curve fitted to these scattered points. By measuring the tangent along this smooth curve the rate of ascent at each instant of time was found. By taking the time the sonde took to reach 24,000 ft the mean rate of ascent of the sonde for this layer was found. This was subtracted from the instantaneous rate of ascent to give the vertical velocity of the air. It has been assumed that a sonde balloon in a region of zero vertical wind will ascend at a constant rate. This is not so, as effects of balloon distortion, icing, etc. can affect the rate of ascent. But as no estimate of these effects in the present study was possible they have been ignored. It was assumed they would be of a smaller magnitude and also not of a sinusoidal type as are the variations due to lee waves.

Fig. 3 is basically a graph of vertical velocity vs. time. By using the height vs. time graph previously obtained, vertical velocity vs. height axes were drawn on the graph. Finally, by using the horizontal winds obtained from the radar (Fig. 4) and noting the fact that the winds are almost zonal, axes for vertical velocity vs. horizontal distance were drawn on Fig. 3.

4. WAVELENGTH AND AMPLITUDE OF THE LEE WAVES

Assuming that the lee wave system is vertical near the level of maximum amplitude, a measure of the distance (Fig. 3) from maximum vertical velocity to the next maximum or from minimum to minimum is a measure of the wavelength of the system. An average value of 11 n. mi. was obtained. This is in excellent agreement with the value obtained from the satellite photograph so the assumption of quasi-vertical wave fronts seems valid.

To obtain an estimate of the amplitude of the wave system the following assumptions were made:-

- (i) Vertical wave fronts in the region of maximum vertical velocity,
- (ii) Simple sinusoidal wave form,
- (iii) The sonde passed through the region of maximum vertical velocity.

Assume a wave of the form

$$y = a \sin 2\pi x/L \quad \dots(1)$$

where a is the semi-amplitude
 L is the wave length
 y is the vertical displacement of a stream line
 x is the distance from the origin.

Differentiating (1), $\frac{dy}{dx} = a \frac{2\pi}{L} \cdot \cos 2\pi x/L$.

For $x = 0$, $\frac{dy}{dx}$ is a maximum and equals $a \frac{2\pi}{L}$.

Also
$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta t} \cdot \frac{\Delta t}{\Delta x} = \frac{w}{U}$$

where w is the vertical velocity
 U is the horizontal wind speed, and
 t is time.

Therefore
$$a = \frac{L}{2\pi} \frac{w_{\max}}{U} \quad \dots(2)$$

From Fig. 3 $w_{\max} = 5$ kt and, from the vertical profile presented in Fig. 5, U at the level of w_{\max} is equal to 65 kt.; $L = 11$ n.mi.

Using these values in (2), $a = 800$ ft is obtained.

Therefore the maximum displacement of a particle passing through this mountain lee wave system is 1600 ft.

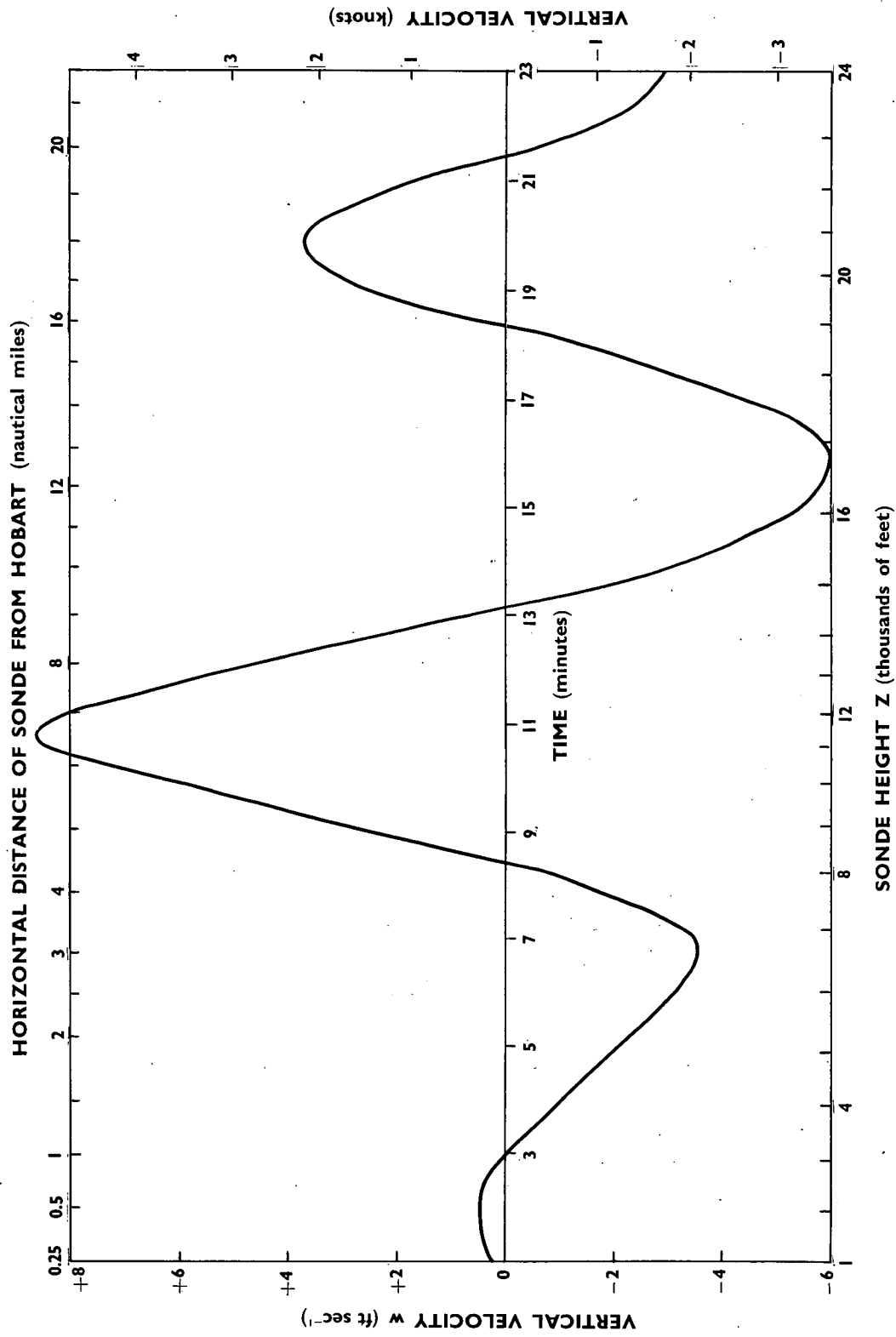


Fig. 3 Vertical velocity estimated from Hobart radiosonde at 2300 GMT on 16 April 1966.

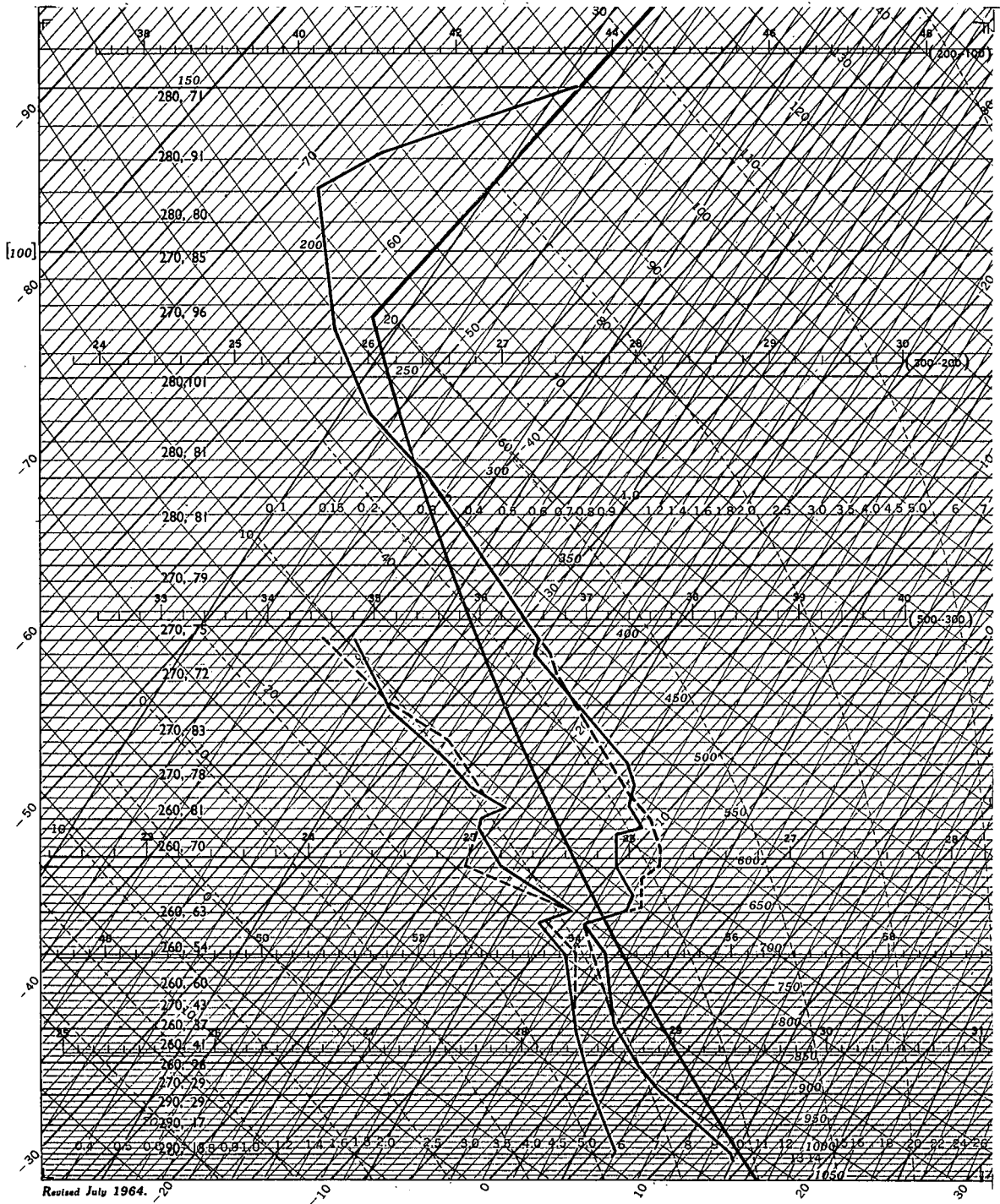


Fig. 4 Hobart Airport, upper-air sounding 2300 GMT 16 April 1966. Broken line is the modified trace which is representative of the undisturbed air mass. Upper winds (degrees, knots) are shown on the left.

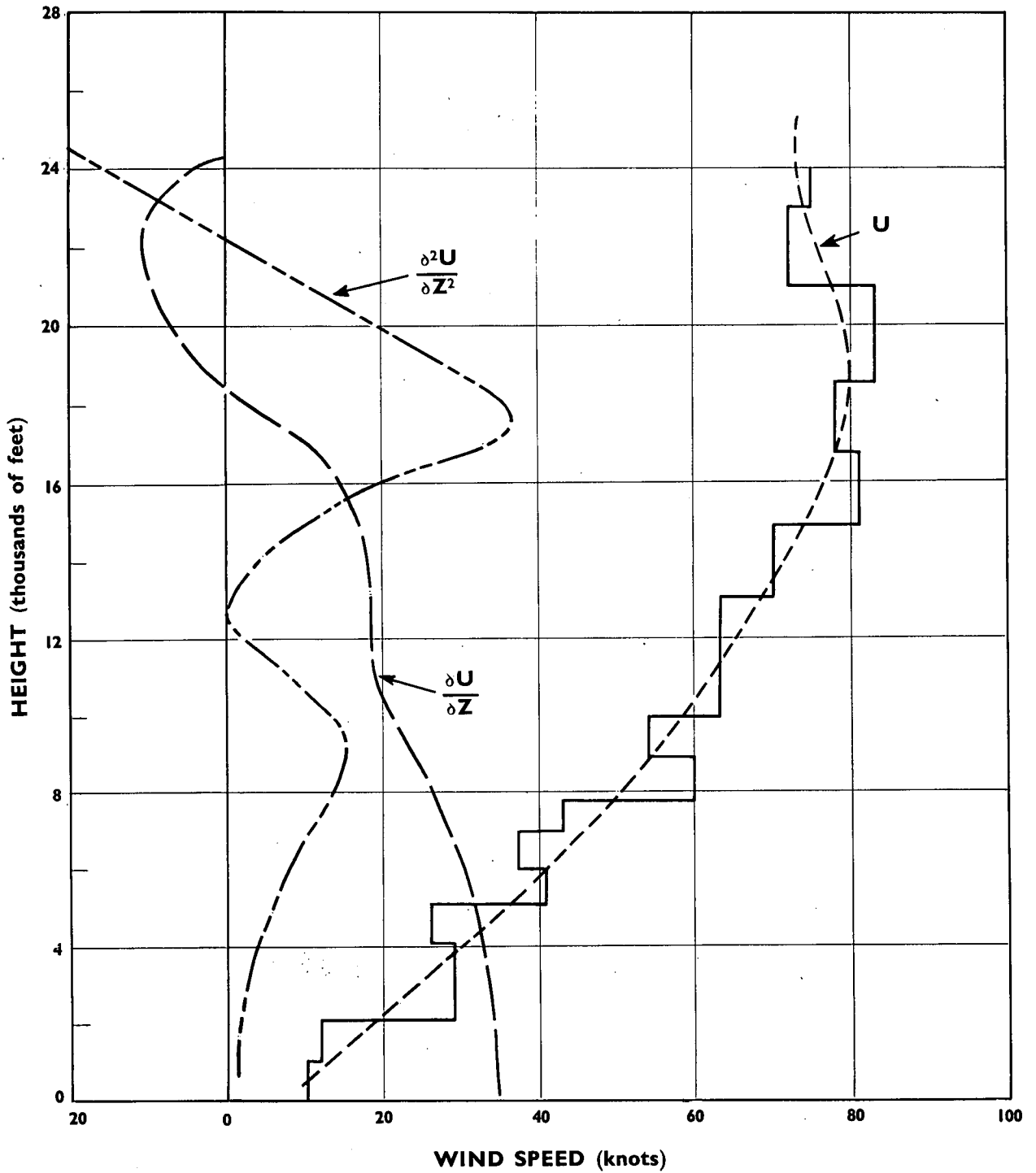


Fig. 5 Vertical profiles of U (kt), $\Delta U/\Delta Z$ (kt/n.mi) and $\Delta^2 U/\Delta Z^2$ (minus kt/n.mi²).

5. THE l^2 PARAMETER AS AN INDICATOR OF LEE WAVE ACTIVITY

Scorer (1949) found the relevant parameter for mountain lee wave occurrence was

$$l^2 = \frac{g\beta}{U^2} - \frac{1}{U} \cdot \frac{\partial^2 U}{\partial z^2} \quad \dots(3)$$

where $\beta = \frac{1}{\theta} \cdot \frac{\partial \theta}{\partial z}$,

U is the horizontal wind speed,
z is height and
 θ is the potential temperature.

The wave length λ was given by

$$\frac{2\pi}{l_{\min}} > \lambda > \frac{2\pi}{l_{\max}} \quad \dots(4)$$

To compute the l^2 profile for this case it was first necessary to amend the Hobart Airport temperature trace so as to be representative of the undisturbed air stream. Using the value of semi-amplitude found in the previous section the displacement of each parcel of air from its equilibrium position was calculated. By moving each part of the trace back adiabatically to its equilibrium position an amended temperature trace was obtained. This is given as the broken line in Fig. 4. The solid line is the original temperature trace.

To calculate the first term on the right hand side of equation (3) an overlay devised by Spillane and Colquhoun (1966) for Bureau of Meteorology aerological diagram (form F160) was used.

The second term in equation (3) is usually ignored in routine evaluation of l^2 . This term is difficult to evaluate as it involves the rate of change of wind shear with height. An attempt was made to evaluate this term in this study. The stepped continuous line in Fig. 5 is the actual reported upper wind profile. As the flow was almost zonal, changes in wind direction with height have been ignored. The smooth broken line in Fig. 5 was drawn as a line of best fit to the observed winds. $\partial U / \partial z$ was obtained by measuring the tangent to this curve and $\partial^2 U / \partial z^2$ was similarly obtained. The accuracy of $\partial^2 U / \partial z^2$ estimated in this manner is not very good because:

- (i) The U profile is interpolated between spot winds at two minute intervals.
- (ii) A small change in the curvature of the U profile causes a correspondingly large change in $\partial^2 U / \partial z^2$. Due to the possible errors in each term of equation (3) and the fact that l^2 is the algebraic difference of these terms, the resultant l^2 profile (Fig. 6) can only be used as a guide to the true l^2 profile. Any attempt to estimate the wave length using this profile and the inequality (4) cannot be expected to give conclusive results.

Comparison of Fig. 6 with a number of similar l^2 profiles found by Corby (1957b) indicate that Fig. 6 is of the type of profile usually found when mountain lee wave activity occurs. The main feature of these profiles is a maximum value of l^2 in the lower troposphere with a decreasing value above this. The meteorological elements required to give this type of profile are:- (i) increasing wind speed with height, (ii) a level of marked stability in the lower troposphere.

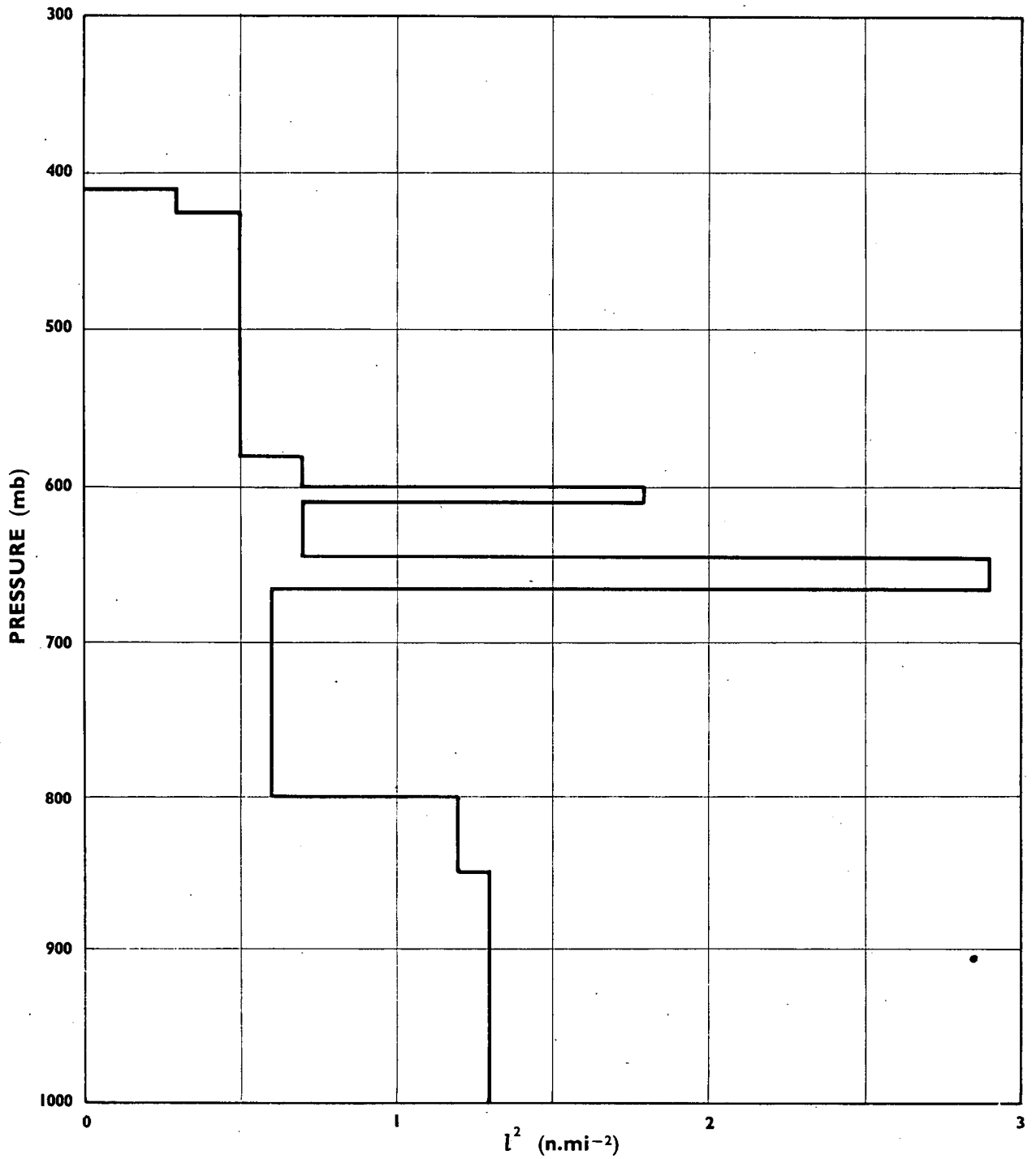


Fig. 6 Profile of Scorer's l^2 parameter derived from Hobart's upper air sounding at 2300 GMT, 16 April 1966.

6. CORRELATION BETWEEN WAVELENGTH AND MEAN TROPOSPHERIC WIND SPEED

Assuming that the wind shear is constant throughout the troposphere, equation (3) reduces to

$$l^2 = \frac{g\beta}{U^2} \quad \dots (5)$$

If l^2 is a constant, the inequality (4) reduces to

$$\lambda = \frac{2\pi}{l} \quad \dots (6)$$

Combining Eqs. (6) and (5) when $U = \bar{U}$, the mean tropospheric wind speed,

$$\lambda = \frac{2\pi}{\sqrt{g\beta}} \cdot \bar{U} \quad \dots (7)$$

In most cases the actual mean tropospheric lapse rate is approximately half the dry adiabatic lapse rate. Using this value and the usual value for g , equation (7) becomes

$$\lambda = 0.18 \bar{U} \quad \dots (8)$$

where λ is in nautical miles and \bar{U} in knots.

Actual observations of mean tropospheric wind speed and lee wave length were plotted in Fig. 7. The dots in Fig. 7 refer to data from Corby (1957a) in which the wavelength was determined by the anomalous rate of ascent of sondes. The triangles refer to data of Fritz published by Hanson (1963). These data were obtained from satellite photographs. In this case \bar{U} is the mean wind from 850 mb to 200 mb. Using these observations the correlation coefficient between lee wave length and mean tropospheric wind speed was found to be 0.86.

Line (1) in Fig. 7 is the regression of wavelength on mean wind, while line (2) is the regression of mean wind on wavelength. The broken line in Fig. 7 is a line with the theoretical slope of 0.18 (see equation (8)). The equation to this line is $\lambda = 0.18 \bar{U} - 2.2$.

From the Hobart Airport RAWIN flight of 2300 GMT, the mean wind from 850 mb to the tropopause (180 mb) was found to be 74 knots. Using this speed and a wavelength of 11 n. mi., the point marked by a cross on Fig. 7 was plotted. This point is in good agreement with the regression lines previously calculated.

7. CONCLUSION

Provided suitable humidity conditions are present, satellite photographs are a valuable indicator of mountain lee wave activity.

In areas of sparse data the wavelength of lee waves as measured from satellite photographs may be used to give an estimate of the mean tropospheric wind speed and surface wind direction in that area.

Estimates of the amplitude of mountain waves can be obtained from suitably located stations by checking the radio-sonde flight to find any sinusoidal anomalous rate of ascent of the radio-sonde balloon.

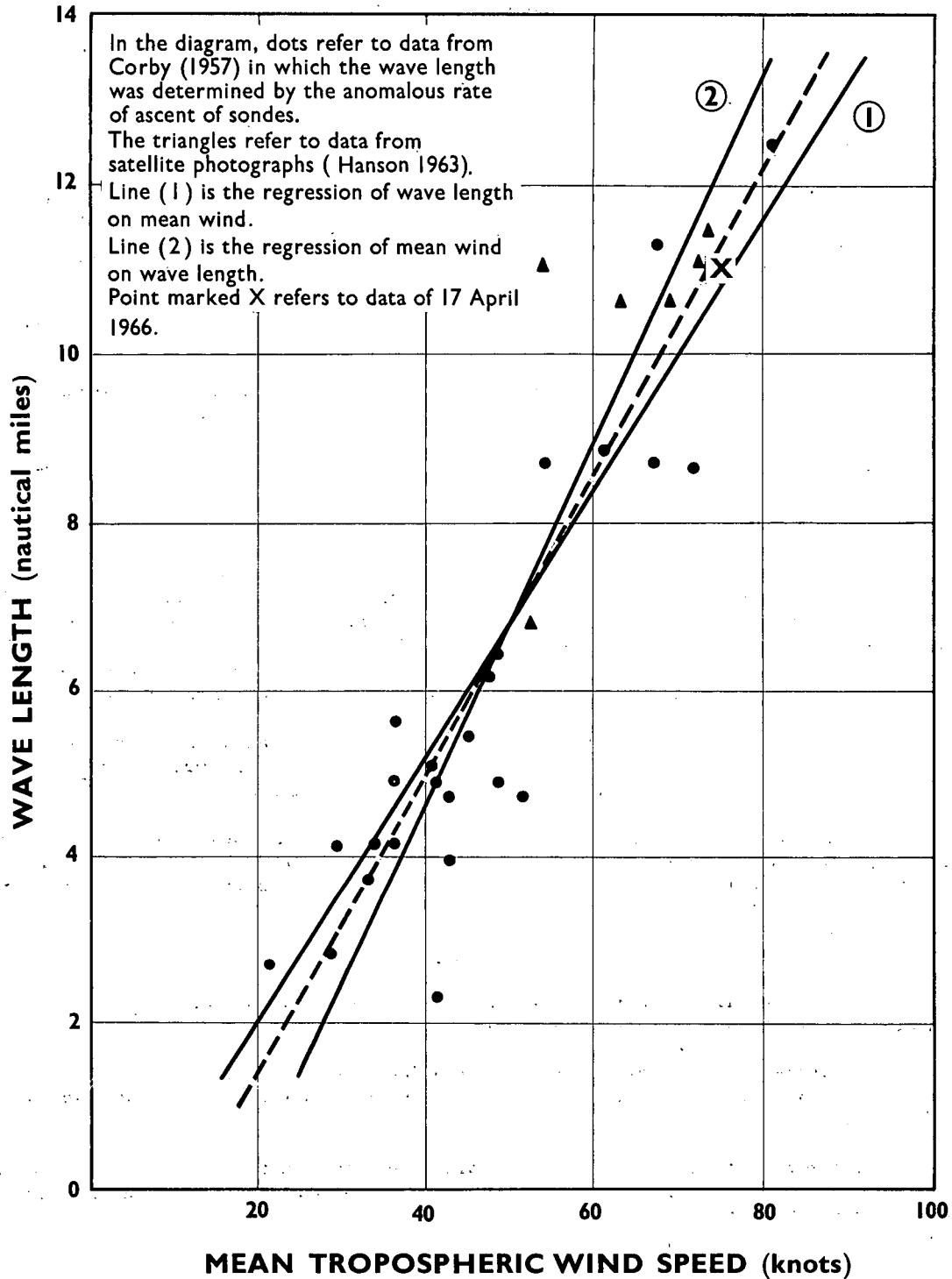


Fig. 7 Relation between mean tropospheric wind speed and wave length of lee waves.

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