

SIMPLE METHODS OF EVALUATING RICHARDSON'S NUMBER AND SCORER'S PARAMETER

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(Manuscript received September 1966)

ABSTRACT

Richardson's number (Ri)

$$Ri = \frac{g}{\theta} \frac{\frac{\partial \theta}{\partial z}}{\left(\frac{\partial V}{\partial z}\right)^2}$$

was modified to the finite difference form

$$Ri = \frac{R | \Delta \ln p | \Delta (T - T_{\theta})}{| \Delta V |^2}$$

and the numerator of the expression represented as an area on a Herlofson skew T - log p diagram. An overlay was constructed which enables the rapid evaluation of Richardson's number without arithmetical calculations.

A similar overlay was developed to enable the evaluation of Scorer's parameter on the same diagram, based on the modification of the expression

$$l^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z} / U^2$$

to the finite difference form

$$l^2 = \frac{g^2}{2.3 RT^2} \frac{\Delta (T - T_{\theta})}{| \Delta \log p |} / U^2.$$

1. INTRODUCTION

In the Australian phase of U.S. Air Force project Hicat which involved a search for high level clear air turbulence, the need arose for the quick and simple evaluation of Richardson's number and Scorer's parameter. Two overlays which fulfilled this need were developed for use on a Herlofson skew T - log p aerological diagram (Bureau of Meteorology, form F160).

2. RICHARDSON'S NUMBER

The rate of production of turbulent kinetic energy per unit volume in a volume sufficiently large that the work done by pressure forces at the boundaries and the turbulent flux of energy across the boundary can be neglected, can be expressed as

$$\frac{\partial E}{\partial t} \approx \iiint \rho K_M \left(\frac{\partial V}{\partial z}\right)^2 \left(1 - \frac{K_H}{K_M} Ri\right) dz dA \quad \dots(1)$$

$$\text{where } Ri = \frac{g}{\theta} \frac{\partial \theta}{\partial z} / \left(\frac{\partial \underline{V}}{\partial z} \right)^2 \quad \dots (2)$$

the well known gradient form of Richardson's number (Richardson 1920)

and

- E = kinetic energy of eddying
- t = time
- g = acceleration of gravity
- \underline{V} = horizontal wind vector
- z = height
- K_H = coefficient of eddy conductivity of heat
- K_M = coefficient of eddy viscosity
- θ = potential temperature
- ρ = density of air.

If $K_H = K_M$, E decreases or increases according as Ri is greater or less than unity.

From a study of lapse rates and wind shears in the free atmosphere, Petterssen and Swinbank (1947) consider that the ratio K_H/K_M is approximately 2/3 rather than unity. The criteria for $\partial E / \partial t < 0$ would then be $Ri \lesssim 1.5$.

According to Sheppard (1961) "A theory of turbulence or good observation is of course required in order to say what maximum positive value of Ri is consistent with maintenance of turbulence in a stably stratified sheared layer. This question is still being actively pursued but the utility of the number does not wait on such knowledge; it provides the parameter of correlation in observational studies as the Reynold's number or Mach number provides a useful parameter in other flow - field problems".

Richardson's number in the above form is not easy to calculate. The expression can be modified, using the hydrostatic relation $\delta p = - \rho g \delta z$

where p = pressure,

and the potential temperature definition

$$\theta = T \left(\frac{1000}{p} \right)^{\frac{R}{c_p}}$$

where R = gas constant for dry air
 T = absolute temperature
 c_p = specific heat of air at constant pressure,

to the form

$$Ri = \frac{R \left(- \frac{\partial T}{\partial \ln p} + \frac{RT}{c_p} \right)}{\left(\frac{\partial \underline{V}}{\partial \ln p} \right)^2} \quad \dots (3)$$

For dry adiabatic conditions $\theta = \text{constant}$ and

$$\left(\frac{\partial T}{\partial \ln p} \right)_{\theta} = \frac{RT}{c_p}$$

Substitution in (3) yields

$$Ri = R \left(-\frac{\partial T}{\partial \ln p} + \left(\frac{\partial T}{\partial \ln p_{\theta}} \right) \right) / \left(\frac{\partial V}{\partial \ln p} \right)^2$$

Applied to a layer using finite differences this becomes

$$Ri \approx \frac{R \Delta \ln p \Delta (T_{\theta} - T)}{(\Delta V)^2}$$

as $\Delta \ln p$ is negative upwards

$$Ri = \frac{R |\Delta \ln p| \Delta (T - T_{\theta})}{|\Delta V|^2} \quad \dots (4)$$

where $\Delta (T - T_{\theta})$ is now the difference between the environmental temperature at the top of the layer and the temperature, also at the top of the layer, of a parcel lifted dry adiabatically from the base of the layer. ΔV is the wind shear over the same layer.

(a) Construction Of Overlay

Using the same scale for $\Delta \ln p$ as on the diagram F160 and the scale of temperature along the isobars on F160 for $\Delta (T - T_{\theta})$, isolines of $R \left[|\Delta \ln p| \Delta (T - T_{\theta}) \right]$ were then drawn for constant increments. These are the family of hyperbolas in Fig. 1.

From equation (4)

$$Ri^{\frac{1}{2}} = \frac{R |\Delta \ln p| \Delta (T - T_{\theta})}{|\Delta V|} = \frac{S}{V'}$$

where V' is the magnitude of the wind shear.

A line through the origin in Fig. 1, at 45° to the axis, cuts the hyperbolas at $\ln p$ coordinate values of $K S$, where K is a constant.

Thus S is represented linearly on the $\ln p$ axis.

The wind shear is represented on the V' axis using a scale of 5 kt to $1^{\circ}C$ measured along an isobar on an F160.

Isolines of selected values of Ri number were then drawn. The overlay is shown full scale in Fig. 1.

(b) Instructions For Use Of The Overlay

- (1) Divide the temperature trace into layers terminating at points of discontinuity on the trace.
- (2) Calculate the shear over each layer and plot each at the base of the layer extending from the left hand edge of the F160 (scale $1^{\circ} = 5$ kt).
- (3) Follow the dry adiabat from the temperature at the base of the layer to a point P at the top of the layer.
- (4) Place the overlay so that the T axis lies along the isobar at the base of the layer and the point P lies on the $\ln p$ axis. Mark point Q on the overlay (see Fig. 2a).

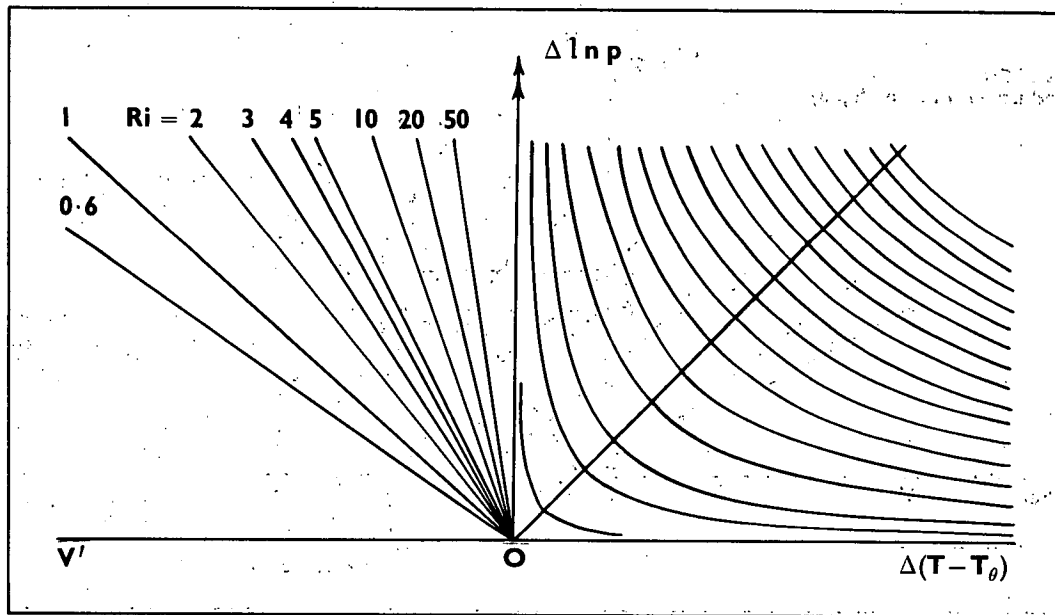
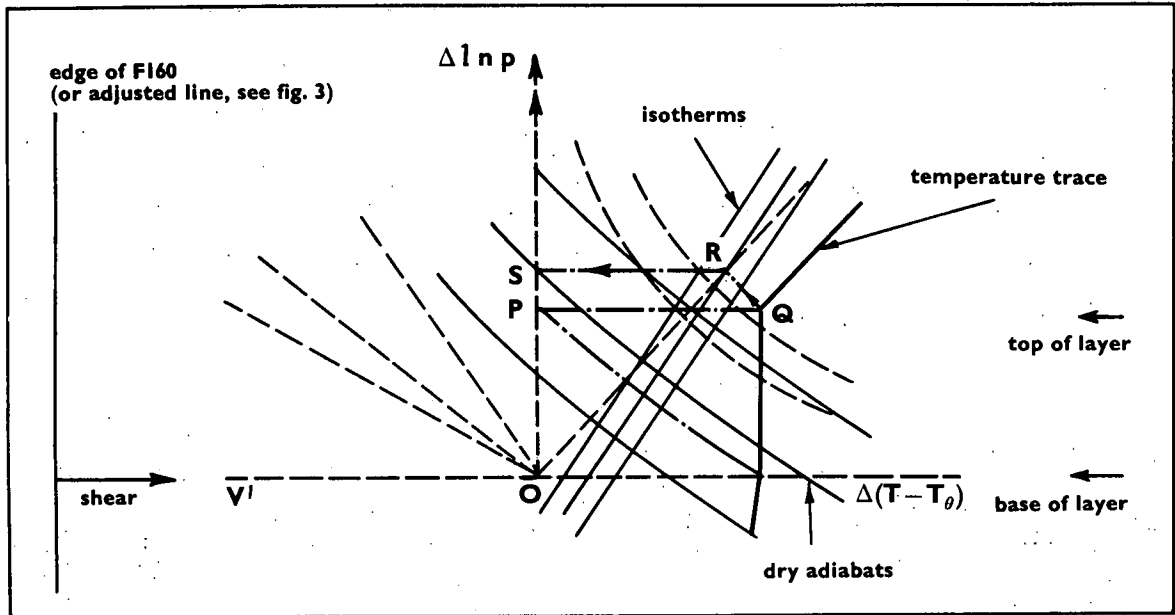


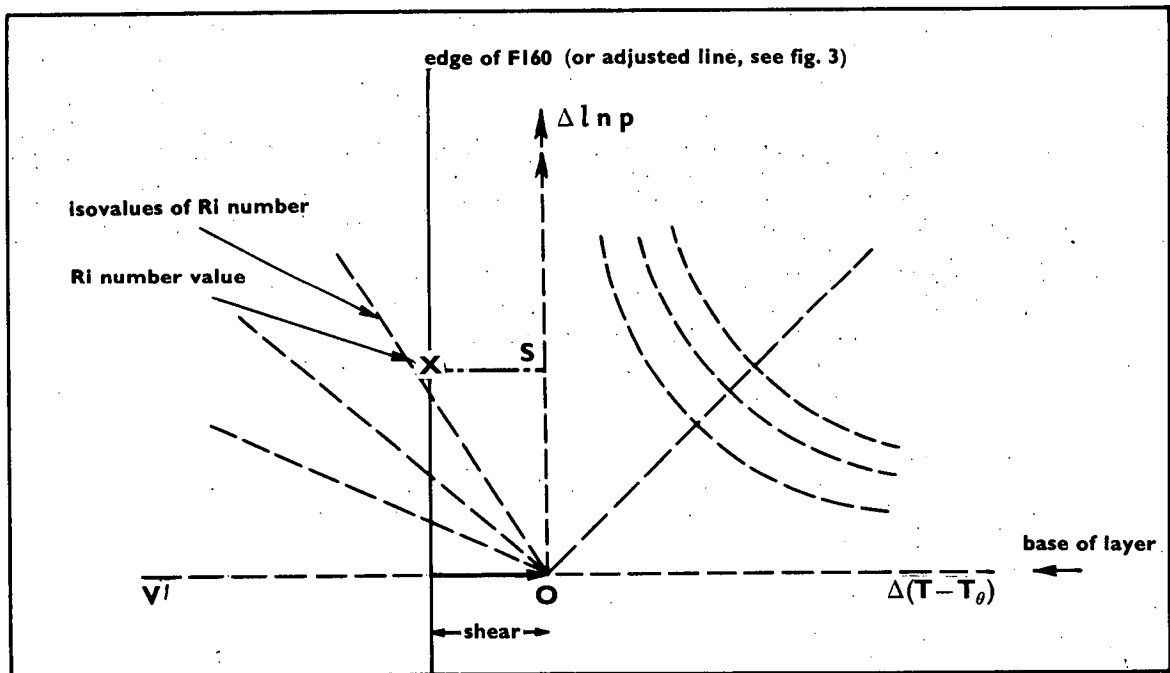
Fig. 1 Overlay for calculating Richardson's number using a skew T-log p diagram (F160).

- (5) Follow the hyperbola through Q to the straight line normal to all hyperbolas (at the point R), then move to R and marks R's value on the $\ln p$ axis (point S).
- (6) Shift the overlay horizontally so that the origin \bar{O} is at the end of the line representing the shear. The Ri number is now found at a distance $\bar{O} S$ from the V' axis along the edge of the F160 (see Fig. 2b), or, in the case when the shear is interpolated, along an adjusted line obtained as in Fig. 3.

In many cases, winds are not available at the top and bottom of the layer. In these cases the shear is linearly interpolated as shown in Fig. 3. Then, placing \bar{O} at the end point of the interpolated shear, the Richardson number is read off at a distance $\bar{O} S$ from the V' axis along the adjusted line.



(a) Steps 1-5



(b) Step 6

Fig. 2 Sequence of steps (dash-dot lines) required to find Richardson's number, overlay lines dashed. Diagrams not to scale.

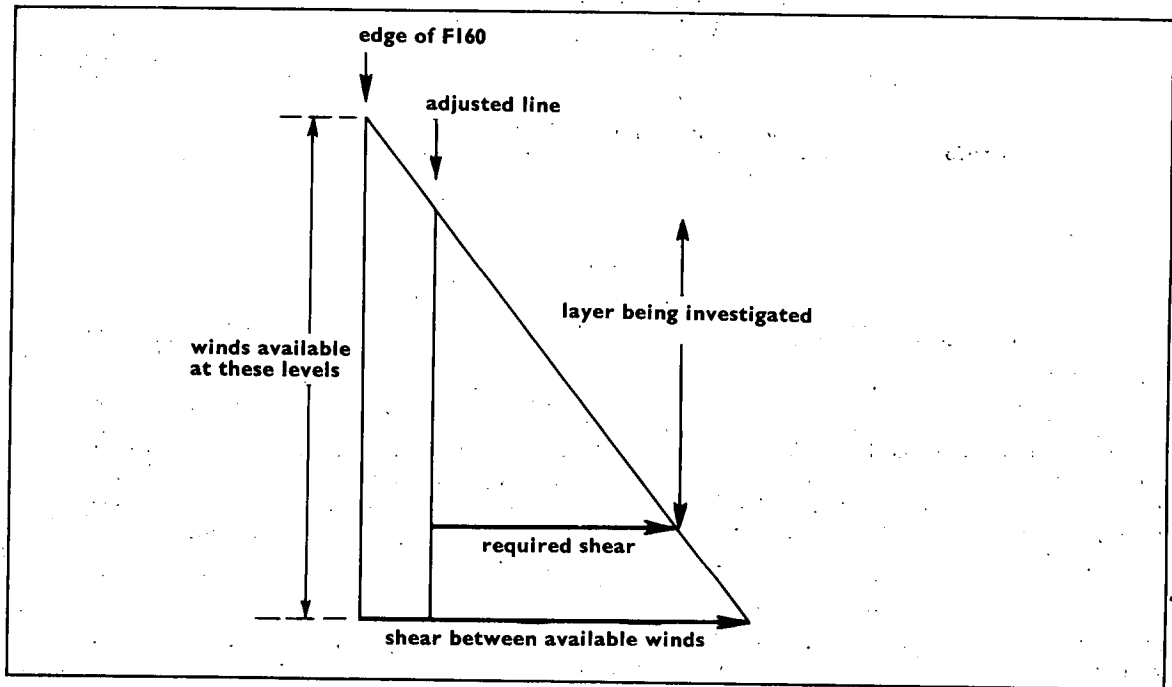


Fig. 3 Vector wind shear determined by a linear interpolation method.

3. SCORER'S PARAMETER

Scorer's parameter l^2 is defined as

$$l^2 = \frac{g\beta}{U^2} - \frac{U''}{U} \quad (\text{Scorer, 1958})$$

where $\beta = \text{stability} = \frac{1}{\theta} \frac{\partial \theta}{\partial z}$
 $U = \text{wind speed normal to a barrier}$
 $U'' = \frac{\partial^2 U}{\partial z^2}$

The term $\frac{U''}{U}$ is small and can be neglected.

Corby (1957) demonstrates the use of a nomogram for use with a Tee-Phi diagram on which the term $g\beta$ can be evaluated. The overlay developed here enables l^2 to be evaluated simply from a Herlofson skew T-log p diagram.

Expressing l^2 as

$$l^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z} / U^2$$

$$= \frac{g^2}{RT^2} \left[\left(\frac{\partial T}{\partial \ln p} \right) \frac{1}{\theta} - \frac{\partial T}{\partial \ln p} \right] / U^2$$

$$l^2 \approx \frac{g^2}{RT^2} \frac{\Delta(T_\theta - T)}{\Delta \ln p} / U^2 .$$

Therefore

$$2.3 l^2 = \frac{g^2}{RT^2} \frac{\Delta(T - T_\theta)}{|\Delta \log p|} / U^2$$

$$= \alpha / U^2 \text{ (say)} \quad \dots (5)$$

(a) Construction Of Overlay

Putting $g^2 / (RT^2 |\Delta \log p|) = 10^{-4} \text{ hr}^{-2} \text{ deg}^{-1} \quad \dots (6)$

values of $\Delta \log p$ were found for T values at 20° intervals and ranging from 203°K to 303°K. Then since $\log 1000 - \log 100 = 1$, taking the distance on F160 between the 1000 mb and 100 mb pressure lines as unity, values of a $\Delta \log p$ from equation (6) corresponding to the various temperatures were scaled off from O along OY' in Fig. 4. From these scaled points, lines were drawn perpendicular of OY' and labelled with the appropriate centigrade value.

Values of $\alpha \times 10^4$ ranging from 0 to 18 were then plotted along the OX axis, with the same scale as temperature on an F160 (measured along an isobar). A linear scale was constructed on the OY axis to give the value $\alpha^{1/2} \times 10^2$ by travelling linearly from the OX value to its square root on the OY axis.

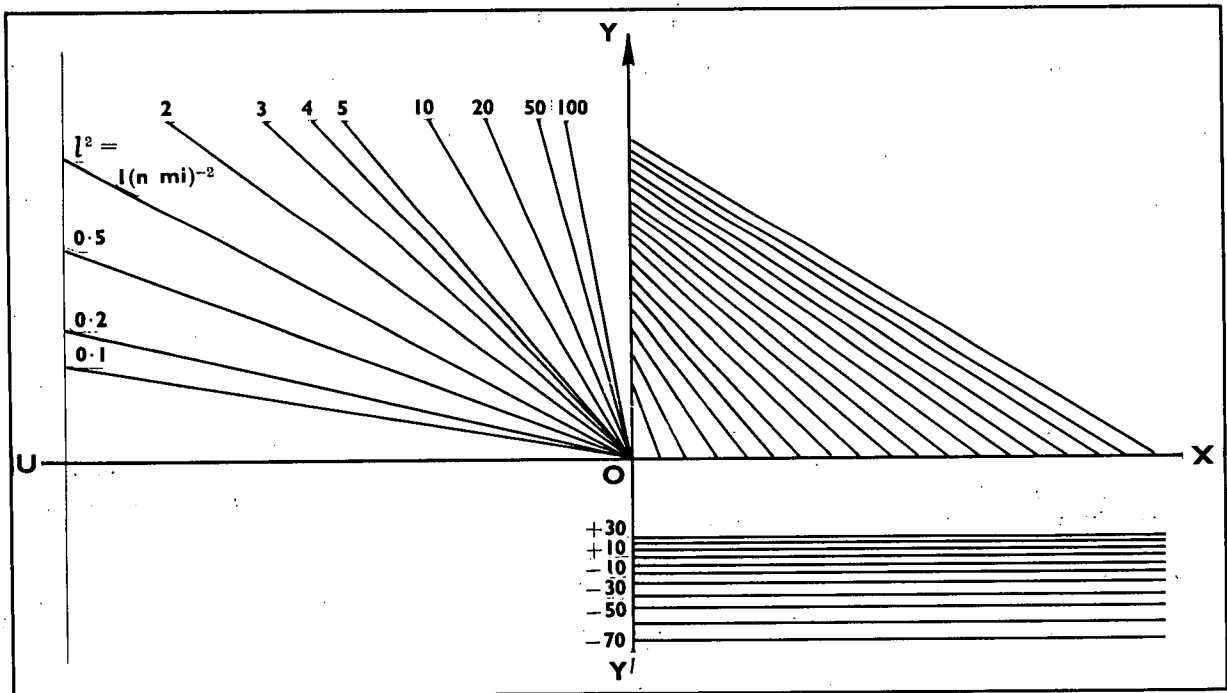
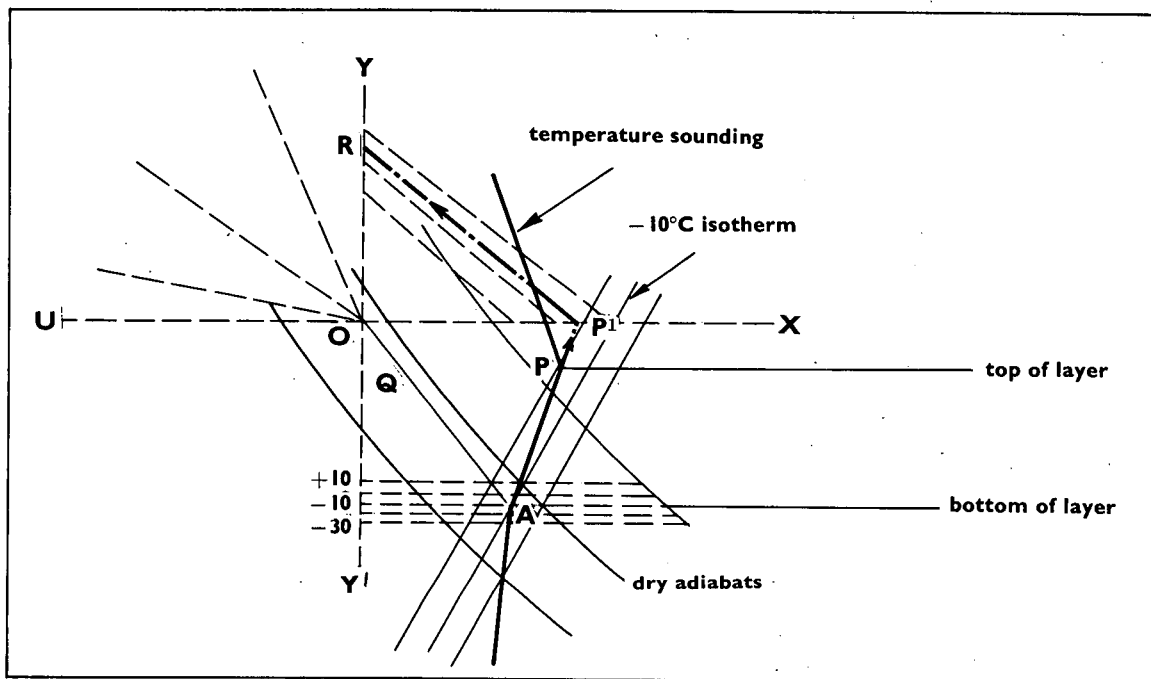
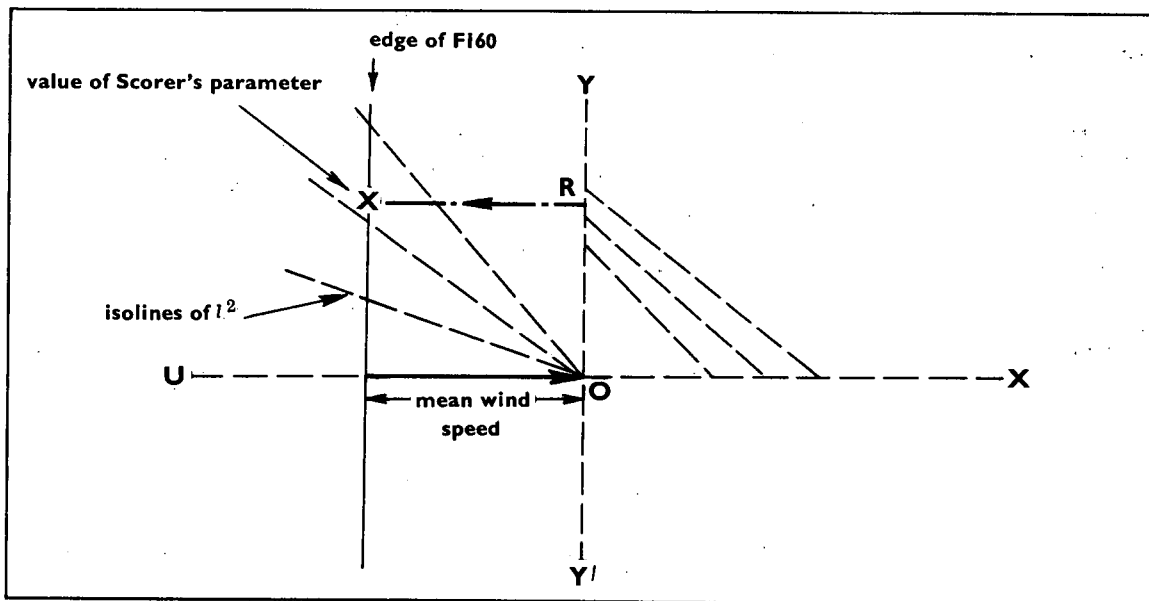


Fig. 4 Overlay for calculating Scorer's parameter on a skew T-log p diagram.



(a) Steps 1-3



(b) Step 4

Fig. 5 Sequence of steps (dash-dot lines) required to find Scorer's parameter, overlay lines dashed. Diagrams not to scale.

The OU axis represents the wind speed normal to the barrier over which it flows (scale $1^{\circ}\text{C} = 5\text{ kt}$). The selected isolines of l^2 in units of $(\text{n. mi.})^{-2}$ were then drawn. The overlay is shown full scale in Fig. 4, except that in practice the U axis and isolines of l^2 less than and equal to unity were extended to the left to cope with strong winds.

(b) Instructions For Use Of The Overlay

- (1) From the environmental temperature at the bottom of the layer, (point A in Fig. 5a) a dry adiabat is followed on form F160 to the point Q at the pressure at the top of the layer and the chord through A and Q drawn.
- (2) The overlay is positioned so that point A lies on the appropriate temperature line of the overlay and OQ, or OQ produced, passes through its origin O. AP, or AP produced, then cuts OX at P'. The line QP must be parallel to the isobars on the F160. (The error in using the temperature at the base rather than the middle of the layer is negligible.)
- (3) From P' move along the family of straight lines to R.
- (4) The mean wind speed normal to the barrier for the layer, U (kt), is drawn as a straight line extending from the lefthand edge of the diagram and along the isobar at the base of the layer being studied (scale $1^{\circ}\text{C} = 5\text{ kt}$). The l^2 value in units of $(\text{n. mi.})^{-2}$ can then be found as shown in Fig. 5b.

When the layer under consideration is cloudy, the wet bulb temperature should be substituted for the environmental temperature and the moist adiabatic lapse rate substituted for the dry.

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