DEVELOPMENTS FOR AN OPERATIONAL AUTOMATIC WEATHER ANALYSIS SYSTEM IN THE AUSTRALIAN REGION

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ABSTRACT

Automatic numerical weather analysis and prognosis methods will soon be used on a day to day basis in the Australian Bureau of Meteorology. The present paper describes meteorological and other improvements which have been introduced into the basic automatic differential analysis scheme which was developed for the Australian Region.

1. INTRODUCTION

The Bureau of Meteorology commenced an investigation into methods of automating the daily production of weather analyses and prognoses early in 1963. Since that time, methods have been investigated and suitable programs have been written to accomplish these tasks, and from the beginning, the programs were designed with both experimental and operational aspects in mind. Thus, all programs may be run in an acceptable real time and at the same time are very flexible since most activities are parametrically controlled for experimental reasons. Appendix 1 contains a list of the activities which can be controlled both external to the program via control cards and internal to the program via parameters set in data statements.

The basic analysis system has been fully described by Maine (1966), and the additional developments set out in this paper have been programmed for and incorporated into this system. The system has been built up against a background of methods and techniques employed by meteorological organizations in other countries, of which the most relevant are: U.S. National Meteorological Centre, Washington; U.S.N. Fleet Numerical Weather Facility, Monterey; U.S.A.F. Strategic Air Command, Omaha; Deutsche Wetterdienst, Offenbach, Federal Republic of Germany.

In brief, the analysis procedure is based on the Cressman (1959) method and is applied in a framework of a vertical differential analysis with time continuity, maintained by use of simple advection procedures. Owing to the paucity of data in the southern hemisphere, the information which is available must be used to the fullest extent and the analysis of thickness charts using observed shears and thicknesses is an important part of the system.

In experimenting with the initial analysis system on a wider range of meteorological situations that that used by Maine (1966), various shortcomings have appeared. For example, isolach maxima were observed to lack the relatively sharp nose and tail often characteristic of manual analysis of these fields. This was ascribed to excessive smoothing and the use of a circular influence region for analysis and an isotropic correlation function for wind speed. It is common experience that the correlation function is not isotropic; however, when attempts are made to incorporate this feature a difficulty arises in finding sufficient statistical information on the correlation function itself. Consequently, an empirical elliptic shaped model influence area was introduced such that the major axis was directed along the wind and its length was increased in proportion to the wind strength. It was also observed that the frequently used two directional linear interpolation was giving rise to poor contour definition on the analysis charts. Consequently, the interpolation and inverse interpolation schemes throughout the analysis model were changed to two directional Bessel type quadratic formulae.
Fig. 1 Illustrating the elliptical influence area around an observation station 'O' at which the wind speed exceeds a preassigned value.
The scope of the analysis system is not intended at this stage to encompass equatorial regions, although an attempt has been made to improve on the geostrophic approximation by introducing a pseudo gradient relationship, using contour curvature and a standard Coriolis parameter. The results to date have indicated that this approach is useful in latitudes south of 5° South.

All computations have been programmed in Fortran IV and run on the C.S.I.R.O. (C.D.C. 3600) computer located in Canberra, A.C.T. The Bureau of Meteorology has selected and ordered a dual IBM 360/65 computer system, each with 65K of high speed store and several disc pack backing stores, and it is expected that the first half of this system will be installed in 1968. Since Fortran IV is also available on this system little difficulty is expected in converting programs.

2. DATA

The data used in these developments were taken from the 2300Z 14 June 1964 magnetic tapes prepared by Maine (1966), and additional tapes for the hours 2300Z 7 September 1965 to 1100Z 8 September 1965 inclusive. In addition experience with the analysis system has been acquired by working on a smaller grid covering the Australian continent for the dates 7, 8, 9, 10 and 11 September 1965. The observation data and starting analysis fields for these latter cases were obtained from the Weather Records and Statistics Section and the Central Analysis Office of the Bureau of Meteorology, Melbourne.

3. DEVELOPMENTS TO BASIC PROCEDURE

(a) Analysis in the Vicinity of Jet Streams

A more satisfactory representation of contours and isotachs at levels above 500 millibars was obtained by modifying the shape of the area of influence of an observation in the vicinity of jet streams. Instead of a circle of radius \( R \), an ellipse with the major axis oriented in the direction of the wind was chosen if the reported wind speed, \( V \), exceeded a pre-assigned value, \( P \). The lengths \( A \) and \( B \) of the major and minor axes (see Fig. 1) were determined by:

\[
A = R \left( 1 + \frac{K (V - P)}{P} \right), \quad \text{and} \quad B = R
\]

where \( K \) is an empirically determined constant.

The weight, \( W \), for a correction to a preliminary field value at a grid point from an observation was still given by the usual formula:

\[
W = \frac{R_a^2 - d^2}{R_a^2 + d^2}.
\]

Referring to Fig. 1, the line \( 0G \), from the observation point \( 0 \) to the grid point \( G \), produced intersects the ellipse at \( D \), \( R_a \) is the length of \( 0D \), and \( d \) the length of \( 0G \). From geometrical considerations,

\[
R_a^2 = \frac{1}{\cos^2 \phi + \frac{\sin^2 \phi}{A^2 + \frac{B^2}{A^2}}}
\]

where \( \phi \) is the angle between the wind direction at \( 0 \) and the line \( 0G \).
Fig. 2 Automatic contour/isotach analyses for 200 mb 11002 8 September 1965. The upper chart (a) employed circular influence area for all observations; the lower chart (b) employed elliptical influence area for wind speeds in excess of 80 knots. Relevant wind observations are plotted.
80 knots has proved a satisfactory value for the parameter $P$, and introduction of the foregoing procedure has resulted in isotach fields being more continuous and less cellular, with maxima exhibiting fairly sharp curvature in the entrance and exit regions of the jet stream. Fig. 2 illustrates the effect of using an elliptical influence area for wind observations in excess of 80 knots, in a particular situation.

(b) Use of the "Gradient" Wind Equation during Analysis

If it is assumed that observed winds obey the gradient wind equation more closely than the geostrophic, then the geostrophic equation

$$\left| \frac{\partial z}{\partial n} \right| = \frac{f V}{g} \tag{1}$$

where $z$ is geopotential height, $f$ the Coriolis parameter, $V$ wind speed, and $\partial / \partial n$, the differential operator normal to a contour of $z$, should be replaced by

$$\left| \frac{\partial z}{\partial n} \right| = \frac{f V}{g} + \frac{V^2}{g R_T} \tag{2}$$

where $R_T$ is the instantaneous radius of curvature of the trajectory of a fluid particle at the point of observation.

If $R_T$ is approximated by the contour radius of curvature $R$, equation (2) becomes

$$\left| \frac{\partial z}{\partial n} \right| = \frac{f}{g} \left( V + \frac{V^2}{fR} \right)$$

Details of the finite difference scheme used to compute $R$ are given in Appendix 2.

Comparing the preceding equation with (1), the height gradient $\left| \frac{\partial z}{\partial n} \right|$, under the gradient wind assumption, may be found by using an "equivalent geostrophic" wind speed $V_{EG} = V + \frac{V^2}{fR}$ instead of the observed wind speed $V$ in the geostrophic equation (1).

This procedure has been introduced into the analysis at MSL and constant pressure levels, using the value of $R$ computed from the preliminary field before each pass, except the first, subject to the following restrictions -

(i) The height gradient is restricted so that the observed wind $V$ lies between 0.5 and 1.5 times the geostrophic, i.e.,

$$0.5 \frac{f}{V} \left| \frac{\partial z}{\partial n} \right| \leq V \leq 1.5 \frac{f}{V} \left| \frac{\partial z}{\partial n} \right|$$

(ii) Computed radii of curvature less than two grid units were not admitted due to limitations in the accuracy of the finite difference method used to compute $R$. A value of $\pm 2$ grid units was used for $R$ in these cases. This was of little practical significance, since restriction (i) was usually effective first.

(iii) The geostrophic approximation was found to produce a more realistic analysis than the "gradient" in the vicinity of jet streams. The foregoing procedure was therefore not applied to observed winds in excess of a pre-assigned limiting value $V_L$ (60 knots has proved satisfactory), and the value of $\left| \frac{\partial z}{\partial n} \right|$ in this circumstance was determined from equation (1).
(c) Use of the Gradient Wind Equation in Preparation of the Isotach Preliminary Field

Using grid point values of contour height from a previously analysed constant pressure chart, preliminary field values of wind speed for the corresponding isotach analysis were obtained, subject to the restrictions enumerated below, by solving for $V$ in the gradient wind equation (2). The trajectory curvature $R_T$ was approximated as in Section 3(b) above and computed as in Appendix 2.

Restrictions on the above procedure were;

(i) The wind speed $V$ was limited so that

$$0.5 \frac{f}{\beta} \frac{\partial z}{\partial n} \leq V \leq 1.5 \frac{f}{\beta} \frac{\partial z}{\partial n}.$$  

This criterion is equivalent to

$$-4.5 \frac{f}{\beta^2} \frac{\partial z}{\partial n} \leq R \leq 0.5 \frac{f}{\beta^2} \frac{\partial z}{\partial n}.$$  

(ii) The geostrophic rather than the "gradient" wind equation was used for grid points on the boundary, since $R$ could not be satisfactorily determined in this case.

(iii) As in Section 3(b), the geostrophic relationship was preferred to the "gradient" in the vicinity of jet streams, and was used when $\frac{f}{\beta} \frac{\partial z}{\partial n} > V_L$, where $V_L$ was a pre-assigned limiting velocity.

(d) Use of Wind Observations from Tropical Stations

Although observed winds in tropical latitudes in general conform less closely to the geostrophic (or "gradient") relationship than do winds at higher latitudes, it was considered desirable to allow tropical wind observations some control over the contour directions in the analysis. Tropical wind observations were therefore used in a similar way to extra tropical observations, but with the Coriolis parameter increased in the ratio $\sin \Phi_S / \sin \Phi$ where $\Phi_S$ was a pre-assigned standard latitude. Obviously this amounts to application of the geostrophic (or gradient) relationship as if the latitude of the observation station were $\Phi_S$.

Useful results have been obtained with a value of 15° for the parameter $\Phi_S$. However, the limitations of this approach are realized, and it is intended eventually to develop a stream function approach to analysis in these regions.

(e) Refinement of the Procedure for Interpolation in a Field of Grid Point Values

Given values $\beta_P$, $\beta_Q$, $\beta_R$ and $\beta_S$ of a scalar variable $\beta$ at four equally spaced collinear points $P$, $Q$, $R$ and $S$ (see Fig. 3), it is assumed a quadratic relationship exists between $\beta$ and $x$, viz.,

$$\beta = ax^2 + bx + c$$

where $x$ is the distance from an arbitrary origin.

The constants $a$, $b$, and $c$ are determined using the criteria that the curve defined by this equation must fit the values $\beta_P$, $\beta_Q$, $\beta_R$ and $\beta_S$ exactly, and the values $\beta_P$ and $\beta_S$ in a "least squares" sense. It can then be shown that the value of $\beta$ interpolated at $X$ is given by

$$\beta_X = \beta_Q + \frac{d}{h} (\beta_R - \beta_Q) + \frac{1}{4} \left( \frac{d^2}{h^2} - \frac{d}{h} \right) (\beta_P - \beta_Q + \beta_R + \beta_S)$$

where $d$ and $h$ are defined in Fig. 3. This is the familiar Bessel type interpolation formula.
Fig 3  Illustrating the distances d, h and x for quadratic interpolation.

Fig 4  Identification of grid points used in the bi-directional quadratic interpolation at point I.
In order to interpolate the value $\beta_1$ at a point I given a two dimensional network of grid values of $\beta$ (see Fig. 4), the value $\beta_A$ is interpolated as in the preceding paragraph from $\beta_{A_1}$, $\beta_{A_2}$, $\beta_{A_3}$, $\beta_{A_4}$ and the values $\beta_B$, $\beta_C$, and $\beta_D$ are interpolated in a similar way. Then $\beta_1$ is interpolated from $\beta_A$, $\beta_B$, $\beta_C$, and $\beta_D$.

This bi-directional quadratic interpolation procedure was used in the analysis system on all applicable occasions, namely:

(i) Interpolation of the scalar preliminary field value at each observation point, during each analysis pass,

(ii) Interpolation of the scalar preliminary field values of pressure or height at four points one grid distance from each wind observation point, in order to determine the gradient in the preliminary field.

(iii) Interpolation of the scalar preliminary field values of pressure or height at four points one grid distance from each wind observation point, on the later analysis passes, in order to determine the curvature at the observation point, and interpolation of pressure or height at a network of points surrounding each grid point in the final analysis field, for the same purpose.

The original bi-directional linear interpolation procedure was retained for points within one grid unit of the boundary of the analysis area. The introduction of the foregoing procedures has resulted in a more satisfactory delineation of analysis in regions of greatest contour curvature.

(f) Refinement of the Chart Drawing Procedure

Given values $\beta_P$, $\beta_Q$, $\beta_R$, and $\beta_S$ of a scalar variable at four equally spaced collinear points $P$, $Q$, $R$ and $S$ (see Fig. 3), the problems of estimating the distance from $Q$ to a point $X$ where the scalar field value is $\beta_I$ is the inverse of the problem discussed in the first two paragraphs of Section 3(e). The solutions are required of the quadratic equation in $d$:

$$\frac{\beta_I - \beta_B}{\beta_C - \beta_B} + \frac{1}{4} \left\{ \frac{\beta_C - \beta_B}{\beta_C - \beta_D} \right\} \left\{ \beta_A - \beta_B - \beta_C + \beta_D \right\} = \frac{\beta_I - \beta_B}{\beta_C - \beta_B} + \frac{1}{4} \left\{ \frac{\beta_C - \beta_B}{\beta_C - \beta_D} \right\} \left\{ \beta_A - \beta_B - \beta_C + \beta_D \right\}$$  \hspace{1cm} (3)

Referring now to Fig. 4, values of $\beta$ are known at analysis grid points such as $A_1$, $A_2$, $A_3$, $A_4$, etc. For line-printer display, or for incremental plotting procedures, it is necessary to consider a "fine grid". The "fine grid" interval in the line printer display is determined by an integral number of printer characters and printer lines to the large analysis grid unit, and in the incremental plotter display usually by a multiple of the smallest available plotter increment. The problem reduces to determining at which points of the "fine grid" the value of $\beta$ is closest to a contour value. Values $\beta_A$, $\beta_B$, $\beta_C$, and $\beta_D$ may be found by "bi-directional quadratic interpolation" as described in Section 3(e). The points $A$, $B$, $C$ and $D$ are now analogous to the points $P$, $Q$, $R$ and $S$ in Fig. 3, and the distance $d$ measured from point $B$ to the contour may be found from Eqn. 3. The "fine grid" point between $B$ and $C$ closest to the distance $d$ from $B$ is considered to be a point on the required contour.

The foregoing procedures were used on both the line printer display, and in the incremental plotting procedure for continuous chart display, with the following restrictions:

(i) Near the boundary of the analysis grid, a linear instead of a quadratic technique was used.

(ii) It can be shown that, referring to Fig. 3, when

$$\frac{(\beta_R - \beta_X)(\beta_P - \beta_Q - \beta_R + \beta_S)}{4(\beta_R - \beta_Q)^2} \leq \frac{p}{100}$$

then the distance $d$ found by solving Eqn. (3) will differ from that found by simple linear interpolation between $Q$ and $R$ by less than $p$ percent.
This test provides an option for using linear instead of quadratic interpolation where the two methods give similar results.

The result of the introduction of the quadratic procedure has been better contour delineation in areas of greatest curvature. However, over large areas of the chart little change in the contour positions estimated by linear means was observed. Since computing time was considerably increased by the quadratic methods the need was felt to employ economies using the option just presented for line printer display purposes. This procedure becomes pointless in the case of the incremental plotter since steps are made at much higher resolution.

(g) Generation of Pseudo-Observations from Observations Outside the Boundary of the Analysis Grid

In order to allow observations from outside the boundary of the analysis area to exert a greater influence on the analysis near the boundary, pseudo-observations of pressure or height were generated at the nearest boundary point to the outside observation. This was done after the first analysis pass, and in this way all outside observations within a distance \( r \) (the largest pass radius) of the boundary, retained some influence on the analysis near the boundary on all subsequent passes.

The pseudo-observations were generated in the following way:

(i) Observations of pressure or height only:

\[
\beta_{XB} = \beta_{IB} + \frac{R_1^2 - d_B^2}{R_1^2 + d_B^2} \left( \beta_0 - \beta_E \right)
\]

where \( \beta_{XB} \) = pseudo-observation of the pressure (height) at point B, the nearest point on the boundary to 0, the observation point.

\( \beta_{IB} \) = the interpolated current preliminary field value of \( \beta \) at point B.

\( \beta_0 \) = observed value of \( \beta \) at 0.

\( \beta_E \) = the extrapolated value of \( \beta \) at 0, using the current preliminary field.

\( d_B \) = the distance 0B.

\( R_1 \) = the pass radius for the first analysis pass.

(ii) For an outside observation of wind only:

\[
\beta_{XB} = \beta_{IB} + \frac{R_1^2 - d_B^2}{R_1^2 + d_B^2} \left\{ \beta_E - \beta_{IB} + \frac{1}{2} \left( \delta \beta_{BO} + \delta \beta_V \right) \right\}
\]

where \( \delta \beta_{BO} \) is the change in \( \beta \) from boundary to observation point determined by extrapolation from the preliminary field, and

\( \delta \beta_V \) is the change in \( \beta \) determined by geostrophic application of the observed wind at 0.

(iii) For an outside observation of both wind and pressure or wind and height:

\[
\beta_{XB} = \beta_{IB} + \frac{R_1^2 - d_B^2}{R_1^2 + d_B^2} \left\{ \beta_0 - \beta_{IB} + \frac{1}{2} \left( \delta \beta_{BO} + \delta \beta_V \right) \right\}
\]
(h) Frictional Correction to Observed Surface Winds

Frictional forces at the earth's surface result in observed low level wind speeds less than the geostrophic (or gradient) speed and observed surface wind direction turned slightly clockwise from the geostrophic in the southern hemisphere. A facility has been introduced whereby observed surface winds from ships and selected islands and well exposed coastal stations may be increased in speed and backed in direction by specified amounts before the geostrophic or "gradient" equation is applied to determine the pressure gradient. A 25% increase in speed and a 20 degree backing in direction have been tentatively adopted.

(i) Mapping Facility on Line Printer Display Charts

To enable interpretation of line-printer output without using an overlay, an option was made available to produce the outline of Australia and New Zealand in addition to the contours on any line-printer chart.

For specified map projection, scale and grid parameters, a subsidiary program generates the co-ordinates of points around the coastline of the Australian continent and Tasmania in units of line-printer character width and line spacing.

(j) Modification of Pass Radii According to Information Density

So that the scales of motion analysed remain consistent with spatial density of information, it is desirable to reduce the pass radii more slowly in sparse data regions than elsewhere. To enable this an "information density" analysis is performed as follows, in conjunction with each meteorological field. At the end of each analysis pass, the sum $S_{ij}$ at each grid point, of the "weights" associated with each observation, is expressed as a fraction of $S$ which is the average value of $S_{ij}$ over a specified sub-grid selected from the densest data region. The resulting quantity $I_{ij} = \frac{S_{ij}}{S}$ is defined as the "information density" at the grid point $(i, j)$. Interpolation in the grid point field of information density will then yield the information density $I_s$ at the location of any station.

On a subsequent analysis pass $n$, the pass radius $R_s$ about a particular station is given by

$$\begin{align*}
R_s & = R_1 - I_{ij} (R_1 - R_n) \quad \text{when } I_s < 1 \\
R_s & = R_n \quad \text{when } I_s \geq 1
\end{align*}$$

where $R_1$ = pass radius on pass 1, and $R_n$ = nominal pass radius on pass $n$.

(k) Modification of Rejection Criteria According to Information Density

In sparse data areas, where the preliminary fields are usually less reliable than elsewhere, it is also desirable that rejection criteria be less stringent. The information density field previously described is therefore used to modify the tolerances on the deviations of observations from the current preliminary field, in the following way:

$$\begin{align*}
T & = T_n (K_{\text{TOL}} - I_s) \quad \text{when } I_s < 1 \\
T & = T_n \quad \text{when } I_s \geq 1
\end{align*}$$

where $T$ = actual tolerance $T_n$ = nominal tolerance for pass $n$ $K_{\text{TOL}}$ = an empirical constant $> 1$. 

The actual tolerance on an observation will always lie in the range \( T_n \leq T < K_{\text{tol}} T_n \).

The modifications in this and the preceding sub-section have been independently tested by Maine and Gauntlett (1967) with encouraging results.

1. **An Improved Smoothing Filter**

The smoothing process originally employed consisted of applying, in both co-ordinate directions, the one-dimensional filter

\[
Z_0 = 0.5 \, Z_0 + 0.25 \left( Z_{-1} + Z_{+1} \right)
\]

where the suffixes 0, +1, -1 indicate the values of the scalar \( Z \) at a grid point and at displacements of +1 and -1 grid units from this original point in the co-ordinate direction under consideration. The response of this one-dimensional filter is shown by the continuous line in Fig. 5. Although strongly attenuating the amplitude of short wavelength disturbances, this filter also produced a smaller but significant attenuation of longer wavelengths. Consequently, synoptic features already in or advected into areas of no data became oversmooth, with maxima decreased and minima increased. Also, the effect of the elliptic influence-function in producing sharper nosed isolach patterns was counteracted.

Wallington (1962) described a technique by which a one-dimensional filter of the desired response could be constructed by successive application of two five-point filters. The dashed line in Fig. 5 shows the response curve resulting from one application of each of the following filters in succession:

\[
\begin{align*}
Z_0 &= 4.916 \, Z_0 - 2.457 \left( Z_{-1} + Z_{+1} \right) + 0.499 \left( Z_{-2} + Z_{+2} \right) \\
Z_0 &= 0.333 \, Z_0 + 0.250 \left( Z_{-1} + Z_{+1} \right) + 0.083 \left( Z_{-2} + Z_{+2} \right)
\end{align*}
\]

(W)  

Wavelengths of 2 and 3 grid units are completely removed by this filter (W) whilst amplitudes of above 5 grid units are attenuated by less than 2 percent.

At present, the smoothing process employed on the two-dimensional grid point analysis consists of a double application of filter (A) in both co-ordinate directions, after selected analysis passes. Figure 6 illustrates the different character of the analyses produced by application in both coordinate directions of filter (A) and the less discriminating three-point filter (Chart (b)). In the no-data areas at the south and west of the chart, and over New Zealand (data from New Zealand was not used), the central values of the synoptic features have been maintained. Smaller-scale features to the south of Western Australia, introduced by ship reports at an earlier time, have similarly been maintained by filter A. In dense-data areas the differences are less pronounced, since the attenuation of the three-point filter at longer wavelengths is counteracted on all but the final analysis pass by application of the observational data again on the next analysis pass. Nevertheless, the trough over Queensland was considerably degraded even on this one pass.

In a two-dimensional field a wave of wavelength \( \lambda \) oriented at an arbitrary angle to a grid co-ordinate axis will undergo a magnification

\[
M (\lambda, \theta) = m (\lambda / \cos \theta) \, m (\lambda / \sin \theta)
\]

where \( m (\lambda / \cos \theta) \) and \( m (\lambda / \sin \theta) \) are the magnifications corresponding to wavelengths of \( \lambda / \cos \theta \) and \( \lambda / \sin \theta \) on the theoretical frequency response curves of the one-dimensional analogue. The value of \( M (\lambda, \theta) \) will not in general be the same as \( m (\lambda) \), except in the degenerate cases of \( \theta = 0 \) or \( \pi/2 \). For response curves of the type shown in Fig. 5, curves oriented along a co-ordinate direction will be magnified less (or attenuated more) than curves not so oriented.
Fig. 5 The continuous line illustrates response of the three-point one dimensional filter described in Section (i). The dashed line illustrates response of the filter consisting of single applications, in succession, of the two five-point filters described in Section (i).
Fig. 6 Automatic mean sea level analyses for 1100Z 8 September 1965. The upper chart (a) employed the two five-point filters in both coordinate directions. The lower chart (b) employed the three-point one-dimensional filter in both coordinate directions, for the same number of passes. Selected relevant observations are shown.
the difference being greatest over that wavelength band in which the magnification changes most rapidly in the one-dimensional analogue.

The left hand side of Figure 7 shows the magnification \( M \) of the two-dimension filter in current use, as a function of orientation and wavelength.

The variation of magnification with orientation may be reduced by the following process. After applying the filter \((A)\) once in each of the co-ordinate directions \(OX\) and \(OY\), another filter \((B)\) with a similar response to \((A)\) is applied once in each of the directions \(OX'\) and \(OY'\), oriented at \(45^\circ\) to the directions \(OX\) and \(OY\). The grid points used in applying filter \(B\) will be those lying along the diagonals of the array, with a spacing of approximately 1.414 times that when applying filter \((A)\). A different set of coefficients will therefore be necessary to produce a similar response to filter \((A)\).

This "quasi-isotropic" two-dimensional smoothing filter has been tested successfully and it is planned to introduce it into the analysis program. The right hand side of Fig. 7 shows its magnification as a function of wave orientation and wavelength.

\[ (m) \] Premature Termination of Analyses.

The analysis of a particular field is terminated at less than the specified maximum number of analysis passes when the scalar and vector deviations of each observation from the current field are less than specified "cut-off" values, which are set parametrically within the program. This procedure is slightly different from the original one when "cut-off" criteria were based on the overall scalar and vector statistics. This latter procedure, however, occasionally allowed an analysis to terminate with one or two observations deviating considerably from the analysis, although the overall statistics were under the specified limits.

4. FURTHER MODIFICATIONS PLANNED

Because the Lambert conformal chart has been used for the Australian region, it was felt unnecessary to introduce the effect of the map magnification factor over the grid during early trials. Neglect of this effect introduces an approximately six percent error in pressure gradients derived near the southern boundary of the chart and less than this elsewhere. The system will be refined further by introducing the map magnification factor into computations of gradients.

As the order of the interpolation scheme has been increased so also will the order of the finite difference approximations to the first and second space derivatives be raised to second order. It is likely that in estimating isostach starting fields the "Balance Equation" (Bolin (1956)) will be more reliable than the "gradient" type approximation already introduced and this will be tested at selected levels.

The complete lack of data from parts of the Indian Ocean indicates that a great deal of reliance must be placed on the use of surface temperature and cloud to infer the vertical structure of the atmosphere. The system is intended to generate automatically estimates of geopotential for the various standard pressure surfaces up to 500 mb on the basis of climatological relationships between cloud base, cloud type, surface temperature and pressure.

The existence of deep layers in the atmosphere having super-adiabatic lapse rates for long periods of time is physically unreal, especially in the sparse data regions, and at the present time this is not explicitly prevented, although starting fields for temperature analyses are computed by assuming a realistic lapse rate from the middle of a standard layer to its upper surface. The successful application of multilayer non-geostrophic numerical prediction models using these analyses as initial data will require explicit prevention of non-negative static stability throughout the differential analysis.

It has been pointed out by Bedient (1964) that the Cressman type analysis scheme is not efficient in its use of large quantities of wind reports. This situation often occurs in the stratosphere and in tropical regions, both regions in which the Australian meteorologist is
Fig. 7 Polar diagrams illustrating the relationship between wavelength magnification factor and wave orientation for (a) two passes of filter A (see Section (1)) in both coordinate directions and (b) one pass of filter A in both coordinate directions and one pass of filter B in both diagonal directions. Radial distance from pole represents magnification (0 – 100%). Azimuth represents wave orientation relative to grid coordinate axes. Full lines are isopleths of wavelength (grid units).
intensely interested. Bedient has suggested that a stream function defined from separate analyses of wind components be incorporated into the system. The method consists of first finding the stream function for the entire area, then inverting this field (through use of the "balance equation") back to an equivalent height field. A weighted mean is then formed between any observed heights and these newly found height values. Both this method and a method similar to this indicated by Maine (1966) will be tried in developing improved tropical and stratospheric analyses.

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APPENDIX 1

PRINCIPAL PARAMETRIC CONTROLS

(1) External

Origin and grid length of analysis grid.
Basic analysis projection parameters (usually Lambert conformal).
Highest standard level analyzed.
Analysis pass radii at each level.
Scalar and vector validation tolerances of each analysis pass.
Relative "weights" given to scalar and vector observations at each analysis pass.
Option to receive additional information in the form of pseudo-observations at a level while the program is in execution.
Option to suppress line printer output at each level.

(2) Internal

Type of analysis projection.
Levels used for analysis.
Option to use geostrophic or gradient wind equation.
Ellipticity of elliptic influence function.
Functional relationship between vector tolerance and preliminary field wind speed.
Smoothing parameters controlling response of smoothing filter.
Smoothing option at each analysis pass.
Functional relationship between analysis pass radius and information density.
Functional relationship between tolerances and information density.
"Cut-off" tolerances for each level.
Fig 8  Illustrating identification of points used in determining curvature from finite - differences.
APPENDIX 2

FINITE-DIFFERENCE METHOD OF COMPUTING CURVATURE (SEE FIG. 8)

The curvature $K_P$ at a point $P$ on the segment of a curve $AB$ of length $S$ is defined as
\[ \lim_{\delta S \to 0} \frac{\delta \psi}{\delta S}, \]
where $\delta \psi$ is the angle between the tangents at A and B. If the curve shown in Fig. 8 is considered as a contour $\beta_1$ of an analytic field $\beta = f(x,y)$, where $x$ and $y$ are distances measured relative to arbitrary $x$ and $y$ cartesian co-ordinate axes, it can be shown that
\[ K_P = \frac{\frac{d^2y}{dx^2}}{\left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^{3/2}} \]

Referring to Fig. 8, finite differences approximation to $dy/dx$ and $d^2y/dx^2$ at point 0 are
\[ \frac{dy}{dx} = \frac{\beta_1 - \beta_3}{\beta_2 - \beta_4} \]
\[ \frac{d^2y}{dx^2} = \frac{2(\beta_1 - 2\beta_0 + \beta_3)}{S(\beta_2 - \beta_4)} + \left( \frac{\beta_1 - \beta_3}{2S} \right) \left( \frac{1}{\beta_5 - \beta_8} - \frac{1}{\beta_6 - \beta_7} \right) \]

where $x$ and $y$ are measured relative to the axes $0X$ and $0Y$.

By always choosing the nine point grid oriented such that the line through $1$ and $3$ is tangential to the contour at $0$, the denominators in the finite-difference approximations will never become zero.

Applying the foregoing argument to a meteorological field, where the point $0$ is a point in the analysis grid, the value of $\beta$ may be found by the interpolation processes described in Section 3(e), and thus the curvature at point $0$ estimated. The accuracy of the method may be improved by interpolating a 25 point set around $0$ and raising the order of accuracy of the derivatives.