EXPERIMENTS WITH THE BAROTROPIC MODEL IN THE AUSTRALIAN REGION

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ABSTRACT

A series of experiments concerned with the variation of parameters, and of the finite difference formulations for the equivalent barotropic model in association with an automatic weather analysis system, have been carried out for the Australia region. These tests precede a more extensive testing of suitable formulations of the model with a view to application in routine forecasting.

The importance of numerical scale matching of both the automatic analysis system and the prognosis system is discussed, and it is shown that in the system presented here some first order approximations to derivatives in the prognosis system can be mis-matched with the analysis system output. The use of higher order finite difference approximations to the Laplacian operator, and of an energy conserving Jacobian operator is investigated in association with a "balanced" equivalent barotropic model and a geostrophic equivalent barotropic model.

The effect of varying amounts of smoothing, neglect of map factor on a Lambert conformal map, variation of Coriolis parameter, varying time step and initial conditions, and the practical stability of the model is discussed.

1. INTRODUCTION

The acquisition of a computer system by the Bureau of Meteorology, has given impetus to the gaining of, and development of, practical experience in numerical methods associated with automatic, operational weather analysis and prognosis. The problems of automatic analysis in the Australian Region and the development of a suitable system have been treated elsewhere (Maine, 1966; Maine and Seaman, 1967) and further work is continuing. This paper deals with the development of an equivalent barotropic model intended for the early stages of operational computer usage in the Bureau. An aim of this study is to obtain suitable interim values of parameters and settling of procedures to enable more detailed determination of model performance to proceed profitably. Although the theoretical model concepts relating to the barotropic model have been treated extensively elsewhere (Charney, 1949; Bolin, 1955, 1956), there is scope for improvement of finite difference formulations relating to the basic differential equations. Further, although the work of developing an Australian region model may parallel that already carried out in the Northern Hemisphere there is a need to test the applicability of assumptions, compare results, and to make theoretical or local empirical adjustments to the model on the basis of these.

2. MATCHING OF ANALYSIS AND PROGNOSIS SYSTEMS

Since the barotropic model is intended for an early stage of operational application in Melbourne, it has been an object of this project to prepare test prognoses from initial data which have been analysed by the automatic multilevel analysis program for the Australian region. Thus the characteristics of the prognosis scheme are considered in relation to those of the analysis scheme. It is intended that this concept of matching prognosis with analysis system will be continued as more refined prognosis and analysis systems are developed.
Fig. 1 The effect of the numerical analysis final filter on harmonic waves of given wavelength $k$.

\[ \frac{\partial f}{\partial x} = \frac{(f_{+1} - f_{-1})}{2d} \]

\[ \frac{\partial^2 f}{\partial x^2} = \frac{(f_{+1} - 2f_0 + f_{-1})}{d^2} \]

\[ \frac{\partial f}{\partial x} = \frac{8(f_{+1} - f_{-1}) - (f_{+2} - f_{-2})}{12d} \]

\[ \frac{\partial^2 f}{\partial x^2} = \frac{(-30f_0 + 16(f_{+1} + f_{-1}) - (f_{+2} + f_{-2})}{12d^2} \]

where $d$ is the grid distance.

Fig. 2 The curves 1, 2, 3, 4 relate to the percentage relative errors of the finite difference approximations for derivatives of harmonic waves of wavelength $k$ grid units as indicated.
In common with experience in the Northern Hemisphere the equivalent barotropic level was taken to be about 500 mb, and the 500 mb analysis from the Australian region multilevel analysis model was consequently used for prognosis purposes. This analysis system employs a numerical filter (see Maine and Seaman, 1967) which restricts the introduction of the small scale modes of variation by observational data. It does not use the barotropic prognoses prepared for 500 mb as a first approximation to the analysis at 500 mb.

Fig. 1 shows the effect of two applications of the numerical analysis filter, on the spectrum expressed in terms of the relative reduction in amplitude of a harmonic mode of wavelength k grid units. It can be seen that all waves with k < 5 are significantly attenuated, and only waves for which k > 6 remain well represented.

In finite difference schemes (Berezin and Zhidkov, 1965) first order approximations to first and second derivatives are given by

\[
\frac{\partial f}{\partial x} = \frac{(f_{+1} - f_{-1})}{2d} + f^{iii} d^2 / 6
\]

\[
\frac{\partial^2 f}{\partial x^2} = \frac{(f_{+1} - 2f_0 + f_{-1})}{d^2} + f^{iv} d^2 / 12
\]

while second order approximations are given by

\[
\frac{\partial f}{\partial x} = \frac{[8(f_{+1} - f_{-1}) - (f_{+2} - f_{-2})]}{12d} + f^{v} d^4 / 30
\]

\[
\frac{\partial^2 f}{\partial x^2} = \frac{[30f_0 + 16(f_{+1} + f_{-1}) + (f_{+2} + f_{-2})]}{12d^2} + f^{vi} d^6 / 90
\]

where \(f^{iii}, f^{iv}, f^{v}, f^{vi}\) are the third, fourth, fifth, and sixth derivatives of the function respectively. Here \(d\) is the grid distance and \(f_0\) is the value of the function \(f\) at the grid point where estimates of the derivative are required, and \(f_{+i}, f_{-i}\) the values of \(f\) at \(i\) grid units in the positive and negative \(x\) directions.

It can be shown (Hamming, 1962) that the percentage relative errors \(R_1\) and \(R_2\) to be expected from the first order approximations above, for first and second derivatives are

\[
R_1 = 100 \left[ \frac{1 - \sin \left( \frac{2n}{k} \right)}{\left( \frac{2n}{k} \right)} \right]
\]

\[
R_2 = 100 \left[ \frac{1 - \sin^2 \left( \frac{2n}{k} \right)}{\left( \frac{4n}{k} \right)} \right]
\]

while the corresponding errors \(R'_1\) and \(R'_2\) from the second order approximations are,

\[
R'_1 = 100 \left[ \frac{1 - (8 \sin \left( \frac{2n}{k} \right) - \sin \left( \frac{4n}{k} \right))}{6 \left( \frac{2n}{k} \right)} \right]
\]

\[
R'_2 = 100 \left[ \frac{1 - (15 - 16 \cos \left( \frac{2n}{k} \right) + \cos \left( \frac{4n}{k} \right))}{6 \left( \frac{2n}{k} \right)^2} \right]
\]

Fig. 2 shows the relation between the \(R_1, R_2, R'_1, R'_2\) and wavelength.

From Fig. 2 it is seen that if the relative errors of the first and second derivatives of the waves \(k \geq 6\) are to be kept below five percent then second order approximation should be used for the derivatives in the prognosis schemes. However, if wavelengths in excess of about 15 grid units are to be considered then the first order approximations may be sufficient for estimation of the first and second derivatives.
It should be noted that if the analysis system is constructed so that scale variations are limited by the observation data density, as in the case for upper level analysis in the Australian region at present, the introduction of the shorter waves will be restricted.

3. MODEL DESCRIPTION

Two variations of the equivalent barotropic model have been used in the experiments and these are described below. All parameters now refer to the equivalent barotropic level, assumed to be 500 mb.

Model A

\[
\frac{\partial}{\partial t} \left( (v^2 - q/m^2) \psi \right) = -J \left( m^2 \frac{\partial^2}{\partial x^2} \psi + f, \psi \right) \quad \ldots (1)
\]

\[
g v^2z = -f v^2 \psi + v \psi \cdot v f - 2 \left( \frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y^2} \right) \quad \ldots (2)
\]

with boundary conditions:

\[
\frac{\partial z}{\partial t} = \frac{\partial \psi}{\partial x} , \quad \frac{\partial \psi}{\partial y} = 0 . \quad \ldots (3)
\]

where \( \psi \) is the stream function, \( m \) is the map factor, \( Z \) is geopotential height, \( \bar{f} \) is a mean value of the Coriolis parameter \( f \) around the boundary \( S \) of the grid, \( g \) is gravity, \( q \) is the Cressman long wave parameter, and \( x \) and \( y \) are the grid co-ordinates.

Model A is similar to the barotropic model employed by the U.S.W.B. (Cressman, 1958) except that terrain and friction effects have been omitted. However, the term \( \partial \psi / \partial t \) \((-q^2/m^2)\), used to compensate for the excessive retrogression of long waves, has been included in the form used by Ito and Isobe (1960). The model uses the balance equation as the wind-pressure relationship in the advective term and this assumes that \( \frac{d}{dt} (\text{div} \, \vec{V}) \) and \( (\text{div} \, \vec{V})^2 \) are negligibly small (see Miyakoda, 1960a).

Model B

\[
\frac{\partial z}{\partial t} = -\frac{\bar{g}}{\bar{f}} \left( m^2 \psi + f, Z \right) \quad \ldots (4)
\]

\[
\zeta = \frac{\bar{g}}{\bar{f}} v^2 Z \quad \ldots (5)
\]

with boundary conditions \( \frac{\partial z}{\partial t} = 0 \)

where \( \zeta \) is the relative vorticity, \( Z \) is the height of the 500 mb surface, \( g \) is gravity and \( \bar{f} \) is the mean value of Coriolis parameter over the grid and \( J \) the Jacobian operator. The Lambert conformal projection was used in the integration of (4) in which case \( m \), the map scale factor, is given by

\[
m = nk \tan \left( \frac{90 - \theta}{2} \right) / \sin (90 - \theta)
\]

where \( \theta \) is latitude and, if \( \theta_1 \) and \( \theta_2 \) are the standard latitudes,

\[
n = \frac{\log \sin \left( 90 - \theta_1 \right) - \log \sin \left( 90 - \theta_2 \right)}{\log \tan \left( \frac{90 - \theta_1}{2} \right) - \log \tan \left( \frac{90 - \theta_2}{2} \right)}
\]

and \( k = \frac{\sin(90 - \theta_1)}{n(tan (90 - \theta_1))^n} \).
Model B is equivalent to an early model reported by Phillips (1960) and has been included as a control for the comparisons which are to be made subsequently. The model is of course a geostrophic system and the absolute vorticity is advected with the "geostrophic" wind. The effect of "spurious anticyclogenesis" (see Shuman 1957) is partly suppressed by putting \( f = \frac{1}{2} \) in the expression for \( \zeta \). This is necessary for consistency with the assumption that the wind field is non-rotational.

4. FINITE DIFFERENCE OPERATIONS

The domain over which the barotropic model has been integrated is given in Fig. 3 and the grid points used for the Australian region analysis model are the same as those for the prognosis model.

The map projection is Lambert conformal with standard latitudes 100S and 400S, and because of this in these first experiments the map magnification factor in the usual finite difference approximations for the prognosis equation has been neglected. This introduces an extreme six percent error in gradients on the southern boundaries, and was considered practically insignificant because of the greater inevitable error in determining the analysis in this area. The total number of mesh points was 864 arranged in a 24 x 36 grid of mesh length 137 n mi (254 km), as shown in Fig. 3.

Time Differencing

The time differencing scheme is the conditionally stable centred leap-frog scheme in which one forward half time step is taken to initialize the values of the vorticity for the subsequent full time steps which are all centred differences. Results based on full time steps of one hour and half an hour were compared. For full time steps (\( \delta t \)) of 1 hour and grid distance (\( \delta x \)) of 137 n mi and allowing speeds (\( V \)) of 100 kt, we have

\[
V \delta t/\delta x = 0.72
\]
satisfying the Von Neumann stability criterion.

For both models the time sequences of prognostic values were established as follows:

\[
\begin{align*}
C_{t+1} &= C_t + \frac{1}{2} \delta C_t & t = 0 \\
C_{t+1} &= C_t + \delta C_{t+\frac{1}{2}} & t = 1 \\
C_{t+\frac{1}{2}} &= C_{t-\frac{1}{2}} + \delta C_t &
\end{align*}
\]

where \( C \) is the parameter which carries the history of the motion. For Model A, \( C \) is the quantity \( (V^2 - q) \psi \) and \( \delta C \) is \(- J(V^2 \psi + f, \psi) \delta t\), the advective change over a full time step. In the case of Model B the parameter \( C_t \) is the vorticity, also \( \delta C_t = \frac{1}{4} \psi J(t + f, Z) \delta t \) the change in \( \zeta \) over a full time step at a point.

The Laplacian Operator

In both models the \( V^2 \) operator was replaced with the following finite difference approximations:

\[
V_1^2 F = (F_1 + F_2 + F_3 + F_4 - 4F_0) / \delta^2 = \psi_1^2 F/\delta^2 \\
V_2^2 F = -(F_9 + F_{10} + F_{11} + F_{12}) / 4 + 4(F_1 + F_2 + F_3 + F_4 - 15F_0) / 3\delta^2 = \psi_2^2 F/\delta^2
\]

where the function values \( F \) are defined for the given points in Fig. 4. The estimates \( V_1^2 F \) and \( V_2^2 F \) contain error terms \( O(\delta^2) \) and \( O(\delta^4) \) respectively, since the approximations \( \psi_1^2 \) and \( \psi_2^2 \) above contain the first and second order approximations to the second order derivatives discussed previously.
The Jacobian Operator

Initially both models A and B were run using the simplest expression for $J(n, \psi)$; reference Fig. 4,

$$J_1(n, \psi) = \left( \psi_2 - \psi_3 \right)^2 + \left( \psi_2 - \psi_4 \right)^2 / 4d^2 \quad \ldots(6)$$

Later, use was made of the mathematically and physically more consistent operator due to Arakawa (1966):

$$J_2(n, \psi) = \frac{1}{3} (J^{++} + J^{+x} + J^{x+}) / 4d^2 \quad \ldots(7)$$

where

$$J^{++} = (\psi_1 - \psi_3) (\psi_2 - \psi_4) - (\psi_2 - \psi_4) (\psi_1 - \psi_3)$$

$$J^{+x} = \psi_1 (\psi_5 - \psi_8) - \psi_3 (\psi_6 - \psi_7) - \psi_2 (\psi_5 - \psi_6) + \psi_4 (\psi_6 - \psi_7)$$

$$J^{x+} = \psi_5 (\psi_2 - \psi_1) - \psi_7 (\psi_3 - \psi_4) - \psi_6 (\psi_2 - \psi_3) + \psi_8 (\psi_1 - \psi_4)$$

This Jacobian operator possesses the property

$$J_2(n, \psi) = -J_2(\psi, n)$$

and also $J_2(n, \psi)$ preserves the transfer of the mean square kinetic energy and vorticity from one grid point to another, without false changes, a property which the $J_1(n, \psi)$ operator does not possess (Arakawa 1966). The operator is applicable to quasi-non-divergent flows.

### 5. SOLUTION OF EQUATIONS

#### The Poisson and Helmholtz Equations

A Poisson equation is solved for the cyclic solution of the balance equation and the inversion of the balance equation to obtain $Z$ from $\psi$, in the prognostic procedure. The procedure used was the Sheldon (1962) speeded Liebmman procedure, in which relaxation is carried out first on the odd numbered mesh points and secondly on the even numbered mesh points during each scan. This type of speeding was also applied successfully to the solution of the Helmholtz equation of Model A. The convergence criterion was set in the range 1.0 - 0.5 ft for inversion of the balance equation and $10^8 - 10^7$ CGS units for stream function values. An 'a priori' estimate of the over-relaxation co-efficient for both Poisson and Helmholtz equations in the case of the $\psi_1$ operator was obtained using the formulae after Miyakoda (1960, b):

$$\lambda = e - (e^2 - 1)^{1/2}$$

$$e = \left( \frac{4 + q/m^2}{2t} \right)^2 - 1$$

$$t = (\cos \pi/P + \cos \pi/Q) / 2$$

where $\lambda$ is the over-relaxation coefficient and $P$ and $Q$ are the grid dimensions. This was very closely verified from 'a posteriori' results of calculations with various values of $\lambda$ for the given grid (see Fig. 5 for the convergence rate of the Poisson equation). In the case of the $\psi_2$ operator, 'a posteriori' estimates of Straede (1967) were used to obtain a suitable value for the relaxation coefficients for the Poisson equations.

#### The Balance Equation

The integration of Eq. (3) was accomplished using finite difference "Method C" of Miyakoda (1960a) and the equation was solved cyclically in the form:

$$\psi^2 = -f \pm \left[ \psi^2 \psi^2 + \psi^2 \psi^2 - \psi^2 \psi^2 - \psi^2 \psi^2 \right] \quad \ldots(8)$$
Fig. 4 Reference grid for definition of grid point values used in finite difference expressions.

Fig. 5 Optimal selection of over-relaxation coefficient. Continuous curve is a-posteriori result, and theoretical (Miyakoda 1960) predicted value is indicated.
with boundary condition \( \psi = g \frac{Z}{f} \) and the restrictions:

\[
2g \frac{\psi Z}{f} + f^2 - 2 \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) > 0
\]

and that the term under the square root of Eq. (8) remains positive. When a negative condition of the discriminant in (8) arose at a point, the forcing function was replaced with \(-f\) and the field smoothed with the filter whose characteristic appears in Fig. 1.

The difference operator for \( \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right) + 4 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right) \) was given by

\[
((v_1^2)\psi) - (\psi_1 - \psi_2 - \psi_3 - \psi_4)^2 + (\psi_5 - \psi_7 - 2\psi_0)(\psi_6 - \psi_8 - 2\psi_0) / d^4
\]

and the term \( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \) by

\[
( (f_1 - f_0)(\psi_1 - \psi_0) + (f_2 - f_0)(\psi_2 - \psi_0) + (f_3 - f_0)(\psi_3 - \psi_0) + (f_4 - f_0)(\psi_4 - \psi_0) ) / d^2
\]

The initial approximation to the \( \psi \) field was taken as \( \frac{\partial Z}{\partial t} \) and convergence was terminated when the difference between the stream function means for two successive cycles did not differ by more than about 1.5 x 10^7 CGS units at every grid point.

Some computational economies were made in the cycle scan technique by flagging of points where the solution had converged sufficiently for that cycle (convergence criterion decreases with cycle) and avoiding the computation of the forcing function at that point for the duration of another cycle. However, care was needed for when the quantity under the square root sign in (8) became negative it was necessary to disregard flagging of converged points in the neighbourhood and to apply a local elementary filter to remove sharp changes in the forcing function resulting from replacing the force by \(-f\).

Smoothing Filter

During the testing of variants to the basic models it was necessary to incorporate a numerical filter capable of suppressing effectively short wavelength fluctuations without altering the larger waves.

A method due to Wallington (1962) has been used to compute the co-efficients of the two-dimensional smoothing operator whose approximate response curve is shown in Fig. 1. The details relating to this are described by Maine and Seaman (1967).

In each model the value of the parameter carrying the history of the motion \( (\tau, \text{ for Model } B) \) was retained for the half time step and the full time step. When smoothing was required both of these fields were smoothed before continuing the prognosis.

6. EFFECTS OF VARYING PROCEDURES

The Operators

Numerical instability was encountered during the early testing stages in association with the use of the simple finite differencing scheme \( \nabla^2 \), \( J_k (n, Z) \), but this was found to be due mostly to the loss of quality in the gradients from the analysis model near the boundaries of the analysis region. The analysis model was subsequently improved, giving more reliable and smoother contour values, and also some smoothing carried out about every thirty hours on the vorticity fields of Model B with resultant suppression of any instabilities in the cases tested for a period of eight days, at which time calculations were terminated.

In the case of Model A, smoothing was introduced into the \( (\nabla^2 + q) \psi \) field for the variants which included the \( J_k (n, \psi) \) operator, in order to suppress eventual instabilities arising after about one day of integration with half-hourly time steps. It was found that with the introduction of Arakawa \( J_k (n, \psi) \) operator no smoothing was required for integration periods of about two days, an effect ascribed to the energy conserving properties of this Jacobian.
Two successive runs of Model B were made to establish some effects of the use of a higher order finite difference approximation to the Laplacian. The model characteristics were:

(a) Arakawa type Jacobian $J_2$

(b) Fully variable Coriolis parameter in the evaluation of vorticity and wind.

(c) Magnification reduction filtering every 12 hours.

(d) One hourly time steps.

(e) Cut off criterion in Sheldon speeded Liebmann Poisson relaxation scheme $\delta = 1.0$ ft.

In one case both the initial value of geostrophic vorticity and the $\nabla^2$ operator in the Poisson equation solution were evaluated using the $\nabla^2_2$ operator. An over-relaxation coefficient of about 0.40 was used and this resulted in about 10 iterations for convergence with each time step. In the other case the usual operator $\nabla^2_2$ was used throughout with an over-relaxation coefficient of about 0.79 which resulted in about 15 iterations per step for convergence.

With the cut off criteria set in each case at 1.0 ft only, negligible differences were observed between the forecast charts at + 48 hr. The same result was observed at + 96 hr. Consequently the Liebmann relaxation procedure "cut off" criterion for the model variant which employs the $\nabla^2_2$ operator was lowered to 0.5 ft; this still resulted in only negligible differences from both the previous run and the run employing the lower order $\nabla^2_1$ operator.

The initial 500 mb chart was considered a typical case and it was concluded that there were insufficient significant small scale features present in the initial analysis to highlight the effect of the higher accuracy of the $\nabla^2_2$ operator in the final forecast.

Model B was run with the following characteristics:

(a) $J_2$ Jacobian operator.

(b) $\nabla^2_2$ Laplacian operator.

(c) Smoothing of vorticity fields every 24 hours.

(d) Hourly time steps.

In one run a mean Coriolis parameter $\bar{F}$ was used in the evaluation of vorticity and wind speed and in the other $F$ was allowed to be variable. No drastic difference was observed out to + 48 hours, but as was suspected slightly more development of high pressures occurred in the run with variable $F$. Speeds of advection remained about the same. It was concluded that the reason for the partial suppression of the spurious building of the high pressure centre was due to the energy and vorticity conserving properties of the $J_2$ operator. On the other hand it was demonstrated that when the $J_1$ operator was used in conjunction with a variable $F$, substantial erroneous building of the high pressure centres occurred.

Comparison of $J_1$ and $J_2$ Adveotive Operators

Successive runs of Model B were made with the $J_1$ and $J_2$ operator formulations, and also with the following characteristics:

(a) $\nabla^2_2$ Laplacian operator.

(b) Use of $\bar{F}$ in evaluation of vorticity.

(c) One hour time steps.

(d) Smoothing every 24 hours with magnification reduction filter.
Appreciable differences between the two runs could be observed at +24 hours with all waves being advected at a slower rate by the $J_2$ operator. Over a 48 hour run there was about a 25 percent difference in speeds of observed waves advected by the $J_1$ and $J_2$ operators. In addition the $J_2$ operator was able to preserve in a realistic fashion more of the original detail of the chart, whereas the prognosis made by the $J_1$ operator became smoother. The same test was tried with Model A and gave similar results.

Time Step

In order to test the truncation error associated with the time step, successive runs of Model B were made with values of $\delta t = 0.5$ hr and $\delta t = 1.0$ hr. The model characteristics were:

(a) $J_2$ Jacobian operator for advection of vorticity.
(b) $V_2^2$ Laplacian operator for Poisson solution and vorticity evaluation.
(c) Constant average Coriolis parameter in the evaluation of vorticity and wind speed.
(d) Smoothing every 48 hours.

Although the actual numbers resulting from calculations at grid points were not identical at +24 hours and +48 hours there were only negligible observed variations in the contour display. Consequently it was tentatively decided that the time step of one hour did not cause any serious discrepancies in the integration. Similar runs for Model A gave the same result.

Smoothing

In order to compare the effects of a simple nine point filter and the magnification reduction filter, two comparative hindcast runs were made with Model B employing:

(a) Simple Jacobian $J_1$.
(b) Simple Laplacian $V_1^2$.
(c) $\nabla$ approximately $\nabla$ at lat. 40°S for vorticity and wind speed evaluation.
(d) Time step $-0.5$ hr.

The initial chart was 500 mb 0500 GMT 8 September 1965 and at -30 hours both hindcasts gave insufficient westward movement of a major low pressure system. The centre of the low in the nine point smoothed hindcast was broader than the centre of the low for the magnification reduction filter version. This situation could be explained by the flatter frequency response of the nine point filter. At -156 hours the 9 point filter version had developed a second high pressure system over Australia, and only a strong ridge developed from the magnification reduction filter version. On this basis and because of the better theoretical response curve the magnification reduction filter was preferred to the 9 point filter.

Two comparison runs were made with Model B employing smoothing of the vorticity fields, firstly at six hourly intervals and secondly at 12 hourly intervals. The model had the following characteristics:

(a) Arakawa Jacobian $J_2$.
(b) Second order Laplacian $V_2^2$.
(c) Fully variable Coriolis parameter in evaluation of vorticity.
(d) One hour time steps.
(e) Magnification reduction filter.

The initial analysis used was 0500 GMT 8 September 1965 and the +18 hr (2300 GMT 8 September 1965) and +66 (2300 GMT 10 September 1965) hr prognoses were selected for comparison.

With more frequent smoothing of the vorticity fields a sharp trough pattern was broadened and gradients were weaker than with the less frequently smoothed series. Further runs were made with less frequent smoothing and it was found that two sweeps of the magnification reduction filter every twelve time steps (12 hr) was excessive, causing a sudden change in the orientation of the trough axis after smoothing. These effects were ascribed to the effect of the filter orientational response. On the other hand the prognosis pattern broke up into smaller scale features after about 72 hourly time steps in the absence of explicit smoothing. This also occurred with Model B, but at a slightly later time. A more satisfactory amount of smoothing was found to be one sweep of the filter every 36 hours.
Neglect of Map Factor

In order to assess the effect of the neglect of map factor, a synthetic zonal distribution of height values was chosen to represent the 500 mb surface. The geostrophic vorticity pattern of zonal isopleths in this case is itself zonal, therefore the distortion effects of neglect of map factor should be evident after an extended integration of, say, Model B. This was tested by advecting the initial zonal vorticity pattern for 24 hours neglecting map factor. The final vorticity pattern was relaxed and the resultant height values were found to agree very closely with the original values. The effect of a map factor was therefore not considered to be appreciable in the integrations carried out here on the Lambert conformal projection map base.

The Parameter q

This has been examined in some detail by Bolin (1956) and Cressman (1958), and it is known that a suitable value of q can allow a more correct interaction of wind and pressure fields for the largest scales of motion. However, due to the restricted area of integration used here, it was considered that a detailed examination of the effects of q was not very relevant. Consequently with a view to a later extension of the model to hemispheric use the retrogression of the long waves was only corrected for in the more empirical manner of Ito and Isobe (1960), by setting q = constant. Values ranging from 0.1 to 0.5 CGS were experimented with in various runs of Model A and a value of 0.25 was selected as this gave most realistic results. It was found that an increase in the value of q also resulted in an increase in the meridional extension of the pressure systems.

Initial Condition

It was found in one case tested that the presence of a small scale trough just to the west of a low pressure centre, which was actually "cutting off", resulted in a prognosis for steady southeastward movement instead of slow northeastward movement as observed. On comparing the automatic initial analysis with the corresponding manual analysis it was concluded that the secondary trough appearing in the automatic chart was erroneous. The source of this trough was traced partly to the fact that the preliminary starting values for the analysis south of the Great Australian Bight, which was west of the "cutting off" low, did not suggest an intense ridge. This was partly because the only information used to modify the analysis fields was a ship's observation at the surface, some hours off the synoptic time of the chart. This observation was used directly, giving a weak trough south of the Bight. The trough was then carried vertically upward by the automatic differential analysis procedure and appeared at 500 mb in the absence of any other information at this level. When this spurious trough was eliminated from the automatic analysis by inserting more of pseudo observations into the analysis, it was found that the prognosis did in fact give a very slow eastward movement of the "cutting off" low, followed later by a southeastward drift, more in accordance with the observed results.

7. EXAMPLES OF PRONOSES

Two meteorological situations were chosen for special examination from a series of analyses and prognoses occurring during a simulated operational trial of an Australian region automatic weather analysis and prognosis system described elsewhere (Maine, 1967).

The first situation treated (30 July 1967) was associated with a quasi-stationary cold "cut-off" low development. Both Models A and B used the $J_2$ Jacobian and the $v_2$ Laplacian operators with smoothing applied every 36 hours and time step one hour. In Model B the mean Coriolis parameter was used in the evaluation of vorticity and wind speed.

(a) 500 mb 2300 GMT 30 July 1967

Fig. 6 contrasts the CAO manual analysis (upper half) and the computed version at this time, the lower computer analysis having been built up hydrostatically from the surface but using only limited observations arising during the simulated operational trial referred to previously. Although less data were used for the computer version than for the manual analysis, the similarity of the two charts seems sufficient to allow prognosis comparisons to be made with later verifying manual analysis. The biggest disparity in the initial analysis occurs south-west of the Australian continent where the computed analysis places a trough and ridge slightly further west than in the manual version.

(i) Model A comparisons

Fig. 7 shows Model A prognosis valid for 2300 GMT 31 July 1967. The main trough is located over Tasmania and has apparently moved much too slowly, for the actual trough is
Fig. 6 500 mb C.A.O. manual analysis for 2300Z 30 July 1967 (upper) and computed analysis (lower).
Fig. 7 500 mb C.A.O. verifying manual analysis for 2300Z 31 July 1967 (upper) and +24 hours model A prognosis (lower).
situated over the West Tasman Sea. The pattern of the northern parts of the trough over Queensland corresponds fairly well with the observed and would give a similar isochromatic structure, although out of phase. This phase difference between forecast and observed appears for almost all trough and ridge features over the chart at this time, and in fact the +50 hours prognosis corresponded more closely with the observed than the +24 hours period forecast.

The model prognosis did not indicate the preliminary stages of cutting off suggested by the observed trough in Fig. 7, except in so far as the prognosis axis changed orientation to N/S whereas the observed axis rotated to NNE/SSW. This is perhaps not surprising as it is probable that this is a dynamic effect occurring in association with latent heat released from the production of rain and the changing heat flux across the lower boundary of the atmosphere, all processes which receive no consideration here.

(ii) Model B comparisons

A prognosis was carried out using Model B and at +12 hours from the initial chart (in comparisons not shown here) the observed trough axis had moved about 100 n mi in advance of the computed trough. The computed ridge in the Great Australian Bight was practically coincident with the observed analysis. Suggestions of excessive high pressure building over the north of the continent were appearing.

Fig. 8 shows the validating manual analysis and the Model B prognosis for the 24 hour period ending at 2300 GMT 31 July 1967. The trough position was well predicted about the latitude of Tasmania, but the axis lagged in the rear of the observed position over New South Wales and Queensland. The verifying analysis showed a deeper trough with a NNE/SSW axis orientation suggestive of an incipient "cut-off" low development. The prognosis trough was not deep enough and the axis only changed its orientation from NWW/SSE at the initial time to N/S at +24 hours. At this time a rather excessive ridging over northern Australia is also evident and the flow over northwestern Australia has turned northwesterly instead of westerly.

(b) 500 mb 2300 GMT 25 July 1967

The initial chart comparison is made in Fig. 9 and the mean differences are again in the western half of the chart. Here, due to misplacing of pseudo observations (bogus data) used to help depict the trough pattern over the Indian Ocean, a small erroneous high pressure cell and a double trough pattern were induced.

The small high cell did not have a great effect on the subsequent prognosis from either model and the stream function derived from the balance equation in Model A smoothed this feature considerably. The double trough feature, however, remained in the later prognoses from both models.

(i) Model A comparisons

Fig. 10 shows the observed prognostic charts for 2300 GMT 26 July 1967. This prognosis sequence showed a slow northeastward movement of the low over eastern Australia associated with complete "cutting off" of this feature from the stronger flow pattern further south. The observed system moved further to the northeast in 24 hours and appeared to become attached to the advancing trough from the west. The predicted trough in the western chart area did not move as rapidly eastward as the observed feature. The growth of a ridge from the northeast over New Zealand shown by the observed chart was not apparent in the 24 hour forecast pattern.

(ii) Model B comparisons

At +12 hours the centre of the observed "cut off" low system had moved to the northeast and so also had the prognostic low. Both prognostic and observed patterns had developed a major ridge southeastward over Tasmania, then toward New Zealand. The trough over Western Australia was showing less complexity and was placed in about the same longitude as the verifying trough. At +24 hours (see Fig.11) in both prognostic and analysis the ridge through Tasmania had weakened with the approach of the trough from the west and the ridge pattern over New Zealand became more pronounced. The "cut-off" low centre in the analysis moved slowly northeast whereas the computed low moved northeast at about half the speed.
Fig. 8  500 mb C.A.O. verifying manual analysis for 2300Z 31 July 1967 (upper) and + 24 hours model B prognosis (lower).
Fig. 9 500 mb C.A.O. manual analysis for 2300Z 25 July 1967 (upper) and computed analysis (lower).
Fig. 10 500 mb C.A.C. venting manual analysis for 2300Z 26 Jul 1967 (upper) and + 24 hours model+ orocinesis (lower)
Fig. 11. 500 mb C.A.O. verifying manual analysis for 2300Z 26 July 1967 (upper) and +24 hours model B prognosis (lower).
8. SUMMARY OF TESTS PERFORMED

The tests carried out on Models A and B referred to above have been performed on several cases and have given similar results; however, further trials are planned before introduction of routine running of, say, Model A on the Bureau's own computer system. Further experience in use of the finite differencing scheme and the integration methods should be obtained by them and in addition some history of geographical error distributions and correlations with initial conditions will become available.

The main characteristics of Models A and B for use in further evaluation studies should be as follows:

(a) $J_2$ Jacobian.
(b) $V_2$ Laplacian.
(c) Magnification reduction type filtering at 36 hour intervals.
(d) No map factor variation in Lambert conformal applications.

In the case of Model A the usual "cycle scan" balance equation technique could be employed with the variations mentioned. In the case of Model B the mean Coriolis parameter $\beta$ should still be employed even though the $J_2$ operator reduces excessive anticyclogenesis.

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