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ESTIMATION OF MAXIMUM WIND SHEAR IN THE BOUNDARY LAYER

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ABSTRACT

A brief description of the instrumentation and some of the results obtained from vertical wind shear measuring equipment installed at Melbourne Airport (Tullamarine) during 1967 is presented. A technique of estimating the maximum value of downwind and crosswind vertical wind shear between 150 ft and 50 ft averaged over various times, is explained. The results in this paper refer to near neutral lapse rates in the boundary layer, although there are indications that similar techniques may be applied to unstable cases as well.

1. INTRODUCTION

The advent of swept wing jet aircraft has produced the necessity to know more about the wind shear in the vertical in the boundary layer. This shear has important implications for the safe landing of these aircraft. At present there is no firm agreement on the magnitude of shear which will cause difficulty in landing or take-off. This will vary considerably from one aircraft type to another. Studies by Friedman (1964) have shown that under some conditions, even if the shear is known exactly, these aircraft will not be able to make a safe landing.

In 1967 the Commonwealth Bureau of Meteorology began to make measurements of wind shear on a 150 ft radio tower at Melbourne Airport (Tullamarine). These measurements have indicated that wind shear in the vertical under many conditions may be described in terms of basic meteorological parameters.

To date evaluation of the data has been mainly confined to near neutral lapse rate cases. The criterion used in deciding if the lapse rate was neutral or not was the C class gustiness classification of Singer and Smith (1953). The turbulence in this class is almost entirely mechanical, with virtually no contribution from convective processes. This is a fairly common condition, occurring during both day and night under heavy cloud, and is seldom associated with light winds. The results presented in this paper refer only to this class.

It has been found that reasonably accurate estimates can be made of the maximum shear averaged over a given time by using relatively few, common meteorological parameters.

2. INSTRUMENTATION

The tower was instrumented at the 50 ft, 100 ft and 150 ft levels with standard U.K. Meteorological Office type cup anemometers and direction vanes. The outputs of these instruments were transmitted to standard recorders. The wind speed output from each head was electronically differenced. This difference was recorded on a Speedomax-G recording voltmeter with a chart speed increased to slightly over one inch per minute (the standard instrument for radiosonde recording in Australia).

Fine scale wind direction differences were also measured. Instrumentation for these measurements was not available for all runs in which wind speed differences were recorded.

The differencing equipment was automatically switched on twice a day at predetermined times and after a run of about one hour was automatically switched off.

3. THE MEAN WIND PROFILE IN THE BOUNDARY LAYER

Many forms of the mean wind profile in the boundary layer have been suggested. The problem of describing this profile is complicated by many influences. In this paper the power law profile due to Frost (1948) is used:

$$\frac{\overline{V}_1}{\overline{V}_2} = \left(\frac{Z_1}{Z_2}\right)^{\alpha} \qquad \ldots (1)$$

where $\overline{V}_1$ and $\overline{V}_2$ are the mean winds at heights Z_1 and Z_2. In class C conditions at Melbourne Airport, for speeds meaned over one hour, α took a mean value of 0.16 with a range of 0.15. As α varies with different localities, at a given airport it would be desirable, under operational conditions, to measure it directly.

An extensive investigation into the dependence of α on meteorological parameters was undertaken by De Marrais (1959) and is a convenient reference on this subject.

4. COMPONENTS OF MEAN SHEAR

A landing aircraft will require the shear in components along the runway (longitudinal) and across the runway (latitudinal), as the effects of these components can be interpreted independently in relation to aircraft control. A more natural system of coordinates to the meteorologist is the downwind and crosswind shear components, and it is these components which are used here. The results obtained using the wind oriented axes would have to be resolved to the runway oriented axes before use by a pilot.

Using a set of X and Y axes as shown in Fig. 1, with Y in the direction of $\overline{\underset{\sim}{V}}_{150}$, the mean 150 ft wind over one hour, the resolved components of the wind shear, $\underset{\sim}{V}_s = \underset{\sim}{V}_{150} - \underset{\sim}{V}_{50}$, become

$$\left.\begin{aligned} V_{sx} &= V_{50} \sin \Delta\theta \cos\theta - (V_{150} - V_{50} \cos \Delta\theta) \sin\theta \\ V_{sy} &= -V_{50} \sin \Delta\theta \sin\theta - (V_{150} - V_{50} \cos \Delta\theta) \cos\theta \end{aligned}\right) \qquad \ldots (2)$$

Here V_{150} and V_{50} are the moduli of the 150 ft wind $\underset{\sim}{V}_{150}$ and the 50 ft wind $\underset{\sim}{V}_{50}$, θ is the angle between $\overline{\underset{\sim}{V}}_{150}$ and $\underset{\sim}{V}_{150}$, and $\Delta\theta$ the angle between $\underset{\sim}{V}_{150}$ and $\underset{\sim}{V}_{50}$ (see Fig. 1).

The direction difference record of a single C class run was examined as typical of this class. On the basis of this run it was found that the difference between $V_{150} - V_{50} \cos \Delta\theta$ and $V_{150} - V_{50}$ was negligible. Thus $V_{150} - V_{50} \cos \Delta\theta \simeq V_s$, making Eq. (2)

$$\left.\begin{aligned} V_{sx} &\simeq V_{50} \sin \Delta\theta \cos\theta - V_s \sin\theta \\ V_{sy} &\simeq -V_{50} \sin \Delta\theta \sin\theta - V_s \cos\theta \end{aligned}\right) \qquad \ldots (3)$$

(V_s was measured by the wind speed differencing equipment).

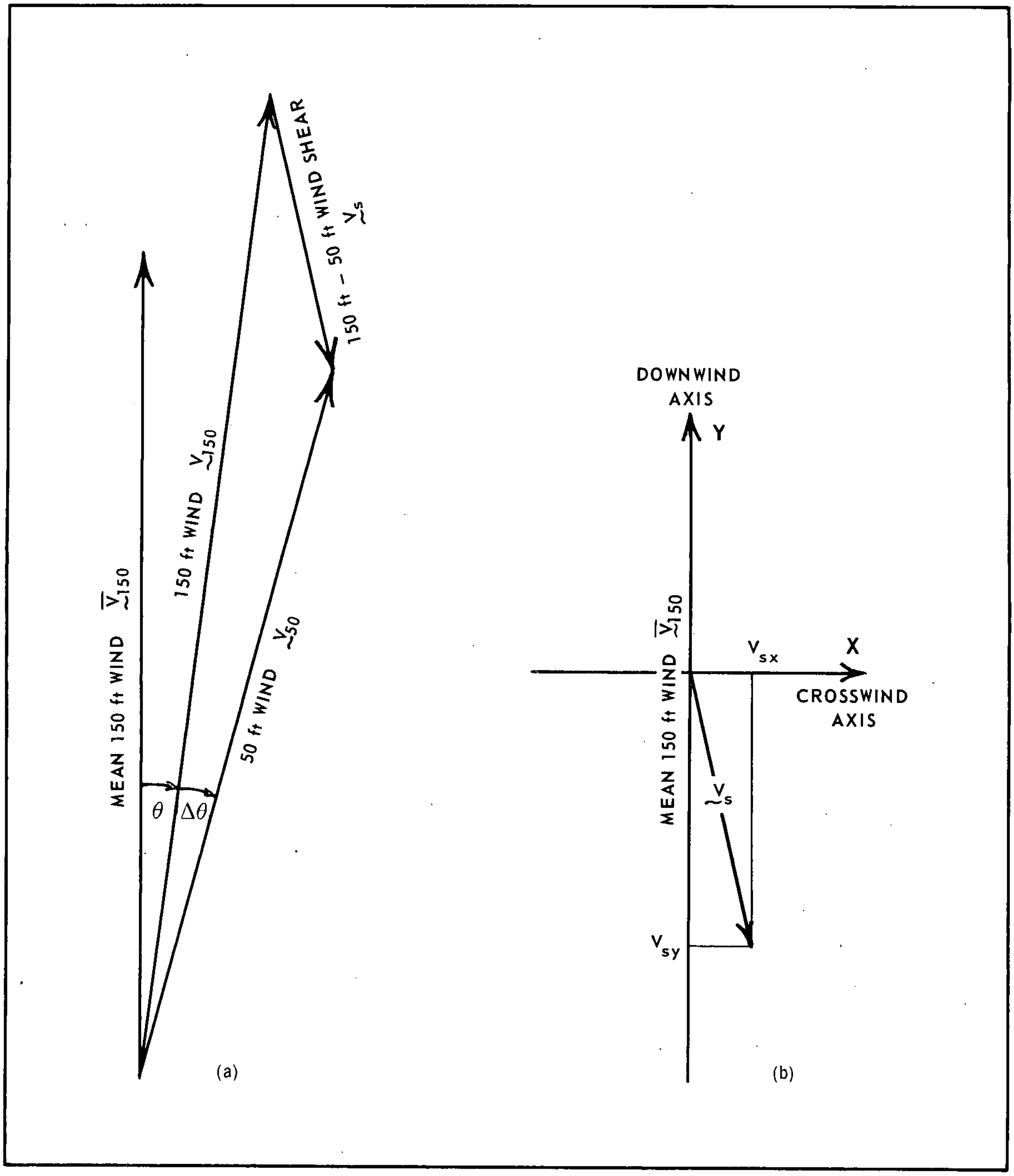

Fig. 1 Vector Diagram (a) shows instantaneous shear, $V_s = V_{150} - V_{50}$, relative to the mean 150 ft wind $\overline{V}_{150}$. Vector Diagram (b) shows the resolved crosswind and downwind components of V_s relative to the mean 150 ft wind direction $\overline{V}_{150}$

An examination of the selected run shows that $\Delta\theta$ and θ may be considered independent. It is assumed that θ, the angle the 150 ft wind makes with the mean 150 ft wind, is independent of the 50 ft wind speed and the magnitude of the shear. Under these conditions the mean values of the shear components are given from Eq. (3) by

$$\left.\begin{aligned} \overline{V}_{sx} &\simeq \overline{V}_{50}\,\overline{\sin\Delta\theta}\;\overline{\cos\theta} - \overline{V}_s\,\overline{\sin\theta} \\ \overline{V}_{sy} &\simeq -\overline{V}_{50}\,\overline{\sin\Delta\theta}\;\overline{\sin\theta} - \overline{V}_s\,\overline{\cos\theta} \end{aligned}\right) \quad \ldots(4)$$

For C class runs θ has a small standard deviation, about 8^{o} in the selected run, with a mean, by its definition, of 0^{o}. Thus $\overline{\sin\theta} = 0$ and $\overline{\cos\theta} \simeq 1$. Then Eqs. (4) become

$$\left.\begin{aligned} \overline{V}_{sx} &\simeq \overline{V}_{50}\,\overline{\sin\Delta\theta} \\ \overline{V}_{sy} &\simeq -\overline{V}_s \end{aligned}\right) \quad \ldots(5)$$

In the selected run $\overline{\sin\Delta\theta} = .024$, $\overline{V}_{50} = 23.8$ kt and $\overline{V}_s = 2.4$ kt (per 100 ft); thus the ratio

$$\left|\frac{\overline{V}_{sy}}{\overline{V}_{sx}}\right| = 4.2 \quad \ldots(6)$$

This is a particular case, but as a general rule for shears between 150 ft and 50 ft in near neutral lapses the mean downwind shear is probably of the order of 4 to 6 times the mean crosswind shear.

5. VARIANCE OF WIND SHEAR

Estimates of σ_s^2 (τ, t), the variance of the wind speed difference between 150 ft and 50 ft averaged over non-overlapping intervals of t seconds in runs of τ seconds duration, were obtained for 10 independent runs. τ = 1 hour in these runs. The simple relation

$$\sigma_s^2(\tau, t) = h^2(\tau, t)\,\overline{V}_{150}^2 \quad \ldots(7)$$

was found to hold for smaller values of t. However, as t was increased the ratio of the range of the estimates of h (τ, t) to the mean value of h (τ, t) increased. This is shown in Table 1.

The estimates of σ_s (τ, t) and $\overline{V}_{150}$ will each be subject to two sources of uncertainty, sampling variance and error variance. The expected variance in h (τ, t), Var (h(τ, t)) is given by

$$\text{Var}\left\{h(\tau, t)\right\} = \overline{V}_{150}^{-2}\left\{\text{Var}(\sigma_s) + h^2(\tau, t)\,\text{Var}(\overline{V}_{150})\right\}$$

where Var (σ_s) and Var ($\overline{V}_{150}$) are the variances of σ_s (τ, t) and $\overline{V}_{150}$. The sampling variance of σ_s is given by $\sigma_s^2(\tau, t)/2N$, where N is the sample size used to estimate σ_s^2 (τ, t), (N = τ/t), while the instrumental error variance is .0133 kt^2. The sampling variance in estimating $\overline{V}_{150}$ is found to be negligible, while an error variance of $(0.7)^2$ $(kt)^2$ has been used.

Table 1. The values of h (τ, t) meaned over 10 runs for τ = 1 hour and various t. The ratio of the range of estimate of h (τ, t) to the mean of h (τ, t) is also shown.

t (sec)	9	13	17	20	25	30	35	40	60	120	180	240	300
h (1 hour, t)	.100	.089	.081	.074	.069	.065	.060	.056	.049	.037	.032	.029	.028
Ratio of range to h (τ, t)	.30	.24	.23	.24	.20	.26	.30	.25	.29	.46	.72	.70	.89

Table 2. The observed values of the maximum downwind shear (between 150 ft and 50 ft averaged over 9 seconds) for the values of α and $\overline{V}_{150}$ shown, compared with the most likely values

Mean 150 ft wind (kt)	α	Max 9 sec wind shear (kt/100 ft)	
		Observed	Not exceeded on 50% of runs
22.9	.095	10.2	9.6
15.8	.178	7.1	7.9
14.8	.246	7.1	8.2
19.9	.218	10.6	10.6
17.8	.251	10.2	10.0
13.8	.224	7.4	7.3
15.7	.084	6.6	6.4

Table 3. The ratio of the maximum cross wind shear averaged over t seconds during one hour to the mean wind speed at 150 feet.

t (sec)	9	13	17	20	25	30	35	40	60	120	180	240	300
$\frac{V_{sx}(\tau, t)_{max}}{\overline{V}_{150}}$	.26	.21	.19	.16	.15	.14	.12	.12	.09	.06	.05	.04	.04

Assuming the estimates of $h(\tau, t)$ are normally distributed about the mean value of $h(\tau, t)$ and that the true mean values are given in Table 1, it was found that all of the 130 individual estimates of $h(\tau, t)$ lay within the 99% confidence limits calculated on the above assumptions. As a result it was reasonable to accept the values of $h(\tau, t)$ for the larger values of t.

In all individual runs it has been found that the frequency distribution of the meaned windspeed differences are very well described by a normal distribution.

The value of mean wind speed at 150 ft was selected for use in expression (7) rather than that at a level nearer anemometer height because this higher level is more characteristic of the turbulence in the boundary layer. The dependence of Richardson's number, and thus turbulence in the lowest few hundred metres of the atmosphere, and the wind speed at the top of the layer is discussed by Lyons, Panofsky and Wollaston (1964).

6. ESTIMATION OF THE MAXIMUM DOWNWIND SHEAR

We define $V_{sy}(\tau, t)_{max}$ as that downwind shear, averaged over a time t, which has a 50% chance of occurring once during τ seconds. As the shears averaged over time t are normally distributed,

$$\frac{t}{2\tau} = \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_{o}^{\xi} \exp(-x^2) \, . \, dx \qquad \ldots(8)$$

where

$$V_{sy}(\tau, t)_{max} = \overline{V}_{sy} + \sqrt{2} \; \sigma_s(\tau, t) \; \xi \, . \qquad \ldots(9)$$

From the relations given in Eqs. (1), (7) and (9), the ratio

$$\frac{V_{sy}(\tau, t)_{max}}{\overline{V}_{150}} = 1 - (\tfrac{1}{3})^{\alpha} + h(\tau, t)\sqrt{2} \; \xi \qquad \ldots(10)$$

can be formed. This ratio has been calculated for α ranging from 0.1 to 0.3, for τ = 1 hour and for t from 9 seconds to 300 seconds, using the values of $h(\tau, t)$ given in Table 1. The values of the ratio, for various values of t and τ = 1 hour, not likely to be exceeded on 50% of occasions are shown in Fig. 2. By knowing the mean speed at 150 ft and the value of α, the maximum downwind shear averaged over a given time within a time interval of one hour may be calculated from these figures, for near neutral lapse rates at Melbourne Airport.

Table 2 presents some observed and estimated maximum shears for τ = 1 hour and t = 9 seconds.

7. ESTIMATION OF THE MAXIMUM CROSSWIND SHEAR

Using the technique of shear measurement employed at Melbourne Airport, the crosswind component determination depends on the measurement of $\Delta\theta$. The results that follow have been obtained from the single case previously mentioned. Although not absolutely reliable, they may be taken as a guide to the estimation of the maximum crosswind shear component under C class conditions until more observations are available.

In the selected run, as was the case with downwind shears, the crosswind shears averaged over a time t were found to be normally distributed. However, their standard deviation was about 80% of the downwind standard deviation.

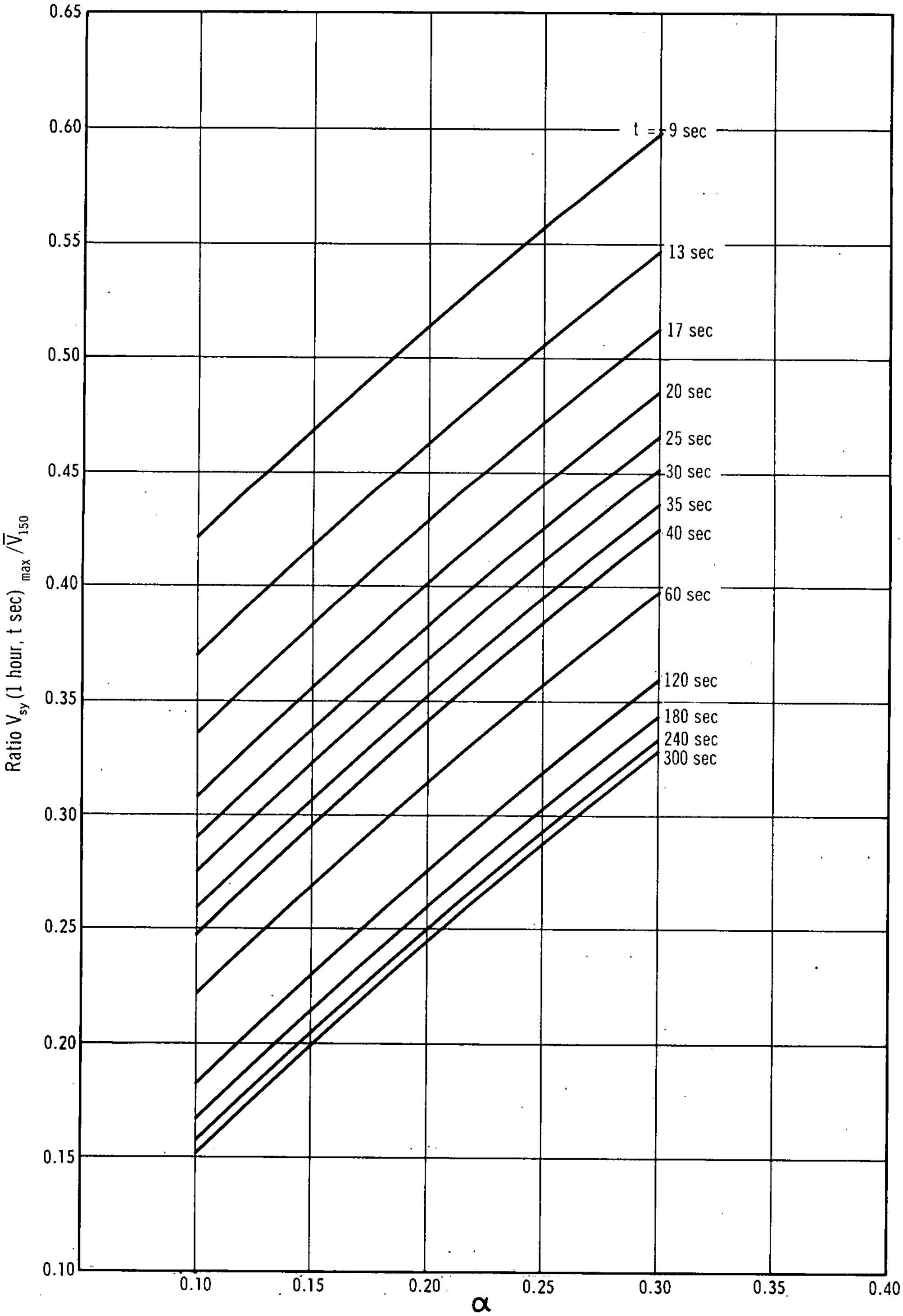

Fig. 2 Ratio of the maximum downwind shear between 150 ft and 50 ft to the mean 150 ft wind not likely to be exceeded on 50% of occasions, for various values of α and t, when τ =1 hour.

As was noted in Eq. (6) the mean crosswind shear was relatively small, and by ignoring it the ratio

$$\frac{V_{sx}(\tau, t)_{max}}{\overline{V}_{150}} = .8\sqrt{2}\ h\ (\tau; t)\ \xi \qquad \ldots(11)$$

may be formed similar to Eq. (10).

This assumes that the ratio of the crosswind and downwind standard deviations remains constant over a range of wind speeds. This constancy has not been verified, and so at present these values can only be used as a guide.

The values of the ratio are presented in Table 3 for τ = 1 hour and various values of t. The ratios are tabulated as the most likely estimate (i.e. the 50% level). It may be desired to add to the value of maximum shear calculated from Table 3 a correction for the mean shear, say 20 percent of the mean absolute shear. This correction will probably only be significant for larger values of t, and in any case in most instances will not exceed 1 kt per 100 ft.

8. CONCLUSIONS

It is considered that for near neutral lapse rates, using only the mean 150 ft wind speed and α, a reasonably accurate estimate can be made of the maximum downwind vertical wind shear between 150 ft and 50 ft. The maximum crosswind shear is probably also reasonably estimated by the techniques described. At present the estimates are confined to near neutral lapse rates, but preliminary investigations into unstable situations indicate that these techniques will be able to be extended to cover the unstable classes.

The cases studied have had mean windspeeds up to 27 knots, but the author is confident that these techniques apply to higher speeds as well. It is likely that at higher wind speeds airport operations for swept wing aircraft would be restricted in any case due to strong wind gusts.

The values of h (τ, t) will be influenced by terrain, but for sparsely wooded, lightly undulating grassland as at Melbourne Airport, the values quoted here would probably be valid.

The shears measured on a tower will not be the shears that are encountered by an aircraft landing or taking off. The inter-relation between tower measured shears and aircraft encountered shears will be the subject of further investigation.

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