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ACCURACY OF OBSERVATIONS AND COMPUTATIONS OF UPPER WINDS USING PLESSEY RADAR TYPE WF2 AND PILOT BALLOON SLIDE RULE

By P.J.R. Shaw

Central Office, Bureau of Meteorology, Melbourne

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ABSTRACT

The radar observational errors and slide rule computation errors made by observers working under routine conditions are compared with the instrumental errors. Errors in vector winds and computed heights are derived and presented in diagrams and a summarizing table.

1. INTRODUCTION

Routine radar upper wind measurements are made in the Bureau of Meteorology by taking from the radar display readings of the range, azimuth and elevation of an ascending balloon-borne target every minute for the first 12 minutes after release, and every two minutes thereafter. In the time between readings the same observer uses a specially designed slide rule (British Meteorological Office pattern) to compute the wind vector and height, so that the results are available immediately. A buzzer warns the observer before the next reading is due. The balloon ascends at about 5 m sec^{-1}.

Both the radar and the slide rule are subject to instrumental errors, which for practical purposes may be assumed to be the same as the errors which occur when the radar or the rule is used with great care and precision by a skilled observer working as slowly as need be. Both items of equipment are subject to observational errors which depend on the skill of the observer and on the conditions under which he operates, one of the conditions being the speed at which he must work. The object of the present study is to obtain estimates of, (i) the accuracy of the radar and slide rule used firstly under "ideal" conditions and secondly in normal operational routine, and (ii) the resulting errors in the vector wind and computed heights. Since the radar errors are independent of the slide rule errors, each can be treated separately and then compounded to give the vector wind and height errors.

SYMBOLS

A	azimuth of the target
E	elevation of the target
R	range of the target
H	height of the target
t	time between observations
x	a symbol representing A, E, R or H
x'	A, E, R or H adjusted for separation of radar sets
D	distance between radar sets
β	bearing of radar No. 1 from radar No. 2

σ_x	standard deviation of x
$\lvert \sigma_{\underset{\sim}{v}} \rvert$	magnitude of the vector s.d. of the wind vector error
N	number of observations in a flight.

2. RADAR ERRORS

The radar instrumental error as defined above has been determined by Laby and Sparrow (1965), using two WF2 radar sets at Laverton (Victoria), both following the one balloon and assuming:

(i) that all the observations and hence their errors are independent,

(ii) that the two radar sets have equal instrumental errors, so that the error of each can be found by differences in observations made simultaneously by the two radars.

The errors determined in this way are given in row 1 of Table 1.

Table 1. Standard deviations of errors in measurements with WF2 radar

Error	Standard Deviations of errors		
	Range (km)	Azimuth (deg.)	Elevation (deg.)
Instrumental (Laby and Sparrow)	0.035	0.057	0.078
Total instrumental plus observational, under routine operation	0.12	0.29	0.19

Table 2. Analysis of the differences between observations in four flights with two WF2 Radar sets

Flight number	No. of obs.	Range (km)		Azimuth (deg)		Elevation (deg)	
		Av. diff.	s.d. of errors	Av. diff.	s.d. of errors	Av. diff.	s.d. of errors
1	31	+0.05	0.13	+0.14	0.42	+0.09	0.16
2	21	+0.00	0.06	+0.52	0.38	+0.20	0.30
3	39	+0.30	0.09	+0.06	0.20	+0.09	0.16
4	31	+0.02	0.16	+0.16	0.10	+0.09	0.17

The total instrumental plus observational errors in radar observations during routine operations were found in this study using the same method as Laby and Sparrow - two radar sets operating simultaneously. The total errors so found are given in row 2 of Table 1. The four flights studied all reached heights over 27 km, at ranges up to 60 km. They were fairly typical flights with light or moderate winds, maximum speeds in each flight ranging from 18 m $\sec^{-1}$ to 32 m $\sec^{-1}$. All observations with one of the radar sets were made by the one observer for all four flights; three different observers operated the other radar.

Tables 1 and 2 were constructed as follows:

The two WF2 (3 cm) radar sets at Laverton are 168 m apart, No. 1 bearing 074 deg from No. 2. This separation produces slight differences in range, azimuth and elevation measured by the two radars, so approximate formulae were applied to adjust the readings of No. 2.

$$A' = A + \frac{180}{\pi} \cdot \frac{D \sin (A - \beta)}{R \cos E} \qquad \ldots (1)$$

$$\tan E' = \frac{\tan E}{1 - \dfrac{D \cos (A - \beta)}{R \cos E}} \qquad \ldots (2)$$

$$R' = R - D \cos E . \cos (A - \beta) \qquad \ldots (3)$$

In these equations A is the reciprocal bearing of the target, this being the form in which readings are taken from the radar display in order to simplify the slide rule computations of wind.

Using observations adjusted as above, the mean differences $(\Sigma \Delta x)/N$ were calculated for each flight and are tabulated in Table 2. Standard deviations of Δx were then calculated for each flight. On the assumption that the two radars with their observers contribute equally and randomly to the differences, the standard deviation of a radar error is given by:

$$\sigma_x = \frac{\text{s.d. of } \Delta x}{\sqrt{2}} \qquad \ldots (4)$$

and these are tabulated in Table 2 for each flight, and in Table 1 row 2 for all flights combined (by weighted variances).

The errors in the observed range, azimuth and elevation produce errors in the computed vector wind and in the computed height to which it is assigned. The formulae for errors in computed winds and heights, in terms of errors in the observations are:

$$\left| \underset{\sim}{\sigma_V} \right| = \frac{(2)^{\frac{1}{2}}}{t} \left\{ (\sigma_A R \cos E)^2 + (\sigma_E R \sin E)^2 + (\sigma_R \cos E)^2 \right\}^{\frac{1}{2}} \qquad \ldots (5)$$

$$\sigma_H = \left\{ (\sigma_R \sin E)^2 + (\sigma_E R \cos E)^2 \right\}^{\frac{1}{2}} \qquad \ldots (6)$$

In deriving equations (5) and (6) it is assumed that the error in a balloon displacement which is obtained by differencing two observations of position, has a standard deviation given by $2^{\frac{1}{2}}$ times the s.d. of the error in one observation. This is true only if the errors are random, which appears to be the case.

3. SLIDE RULE ERRORS

The slide rule "instrumental" errors were found by computing winds and heights from four WF2 flights using the slide rule with painstaking care, keeping as many significant figures as the slide rule would permit,and checking the results with a digital computer. The computer program allowed for the approximation of 6000 ft to 1 nautical mile made in the slide rule computations, computed wind components and heights from the raw data, and printed out differences between its own computations and the observer's. All four flights used reached heights of 26 km or more.

The slide rule operational errors for wind were found in the same way, using 56 flights comprised of 1311 separate wind computations made by 31 separate observers working under the usual procedures. For height, 39 flights by 29 observers were treated in a similar manner.

4. DISCUSSION

Figures 1 to 4 show the results, for observations after the twelfth minute, made every second minute. The "instrumental" and "observational" errors shown in the figures are as defined in the Introduction. The "total" error is obtained by combining the variances of the instrumental and observational errors.

Ground range (R cos E) was used as the abscissa in Figs. 1 and 2 because it was found that the errors are closely fitted by single curves. In the case of slide rule errors it can be shown that the vector wind error is proportional to R cos E (Spillane, 1968) for purely instrumental errors, i.e. not including "mistakes" in setting or in reading the slide rule, or in the associated mental arithmetic. Height errors for the slide rule can be treated in a similar manner and are found to be a function of height (R sin E) only. The radar errors for height cannot be shown as a function of a single variable as can be done for wind vector, and a family of curves is needed (Figs. 3 and 4).

Although only 60 km range was the maximum reached in the flights used for determining observational radar errors, the curves of Figs. 1 to 4 have been extrapolated to greater ranges, the justification being that the observational errors were found to be substantially independent of range and elevation.

The instrumental radar and slide rule errors are compared and combined in Figures 1 and 3. In the flights studied the standard deviation of the wind vector error remains below 2 m sec^{-1} except at extreme ranges, i.e. even at 100 km ground range only 5 percent of winds will have errors greater than 4 m sec^{-1}. At moderate ranges (up to 60 km) the standard deviation remains below 1 m sec^{-1}. The slide rule makes only a relatively small contribution to the height error as only one slide rule operation is required to compute height. The height error due to radar increases with range, and arises mainly from elevation errors at great range. The standard deviation of about 140 m at 100 km range could be objectionable where there is a strong vertical shear.

In the routine use of the slide rule, about 10 percent of computed winds had large errors (more than 5 m sec^{-1}) which were identifiable as mistakes. The mean absolute value of the mistakes was 15 m sec^{-1} and their contribution to the overall standard deviation was about 5 m sec^{-1}, as shown in Fig. 2. If mistakes are excluded, the errors in routine slide rule computation compare favourably with the instrumental errors of Fig. 1. Mistakes in the computation of heights were very rare, as only one simple operation is required,and the observer can easily detect them during the flight.

It may be inferred that the large radar observational errors also include mistakes in reading the display dials and recording the readings.

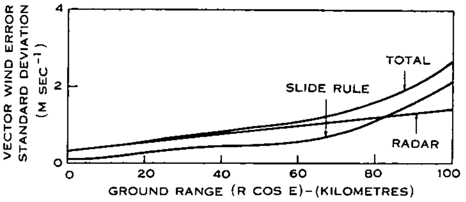

Fig. 1 Vector wind error due to instrumental error, for observations made every two minutes.

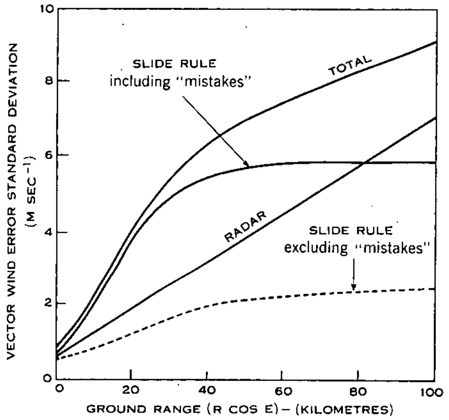

Fig. 2 Vector wind error due to instrumental plus observational errors, for observations made every two minutes.

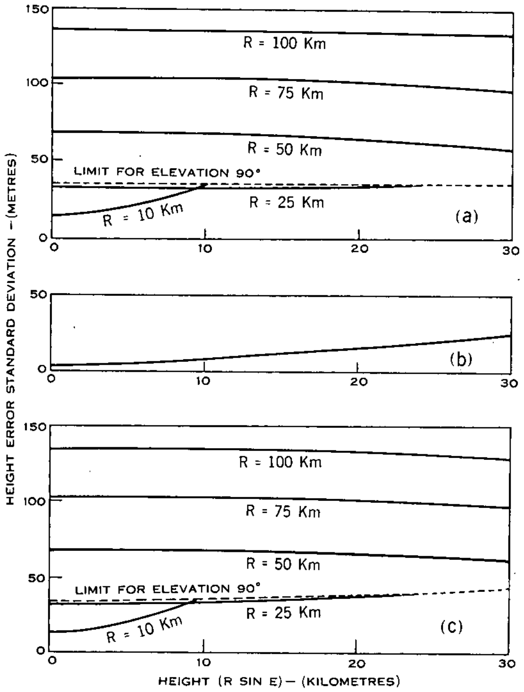

Fig. 3 Height error due to instrumental errors,
(a) radar only
(b) slide rule only
(c) radar and slide rule combined.

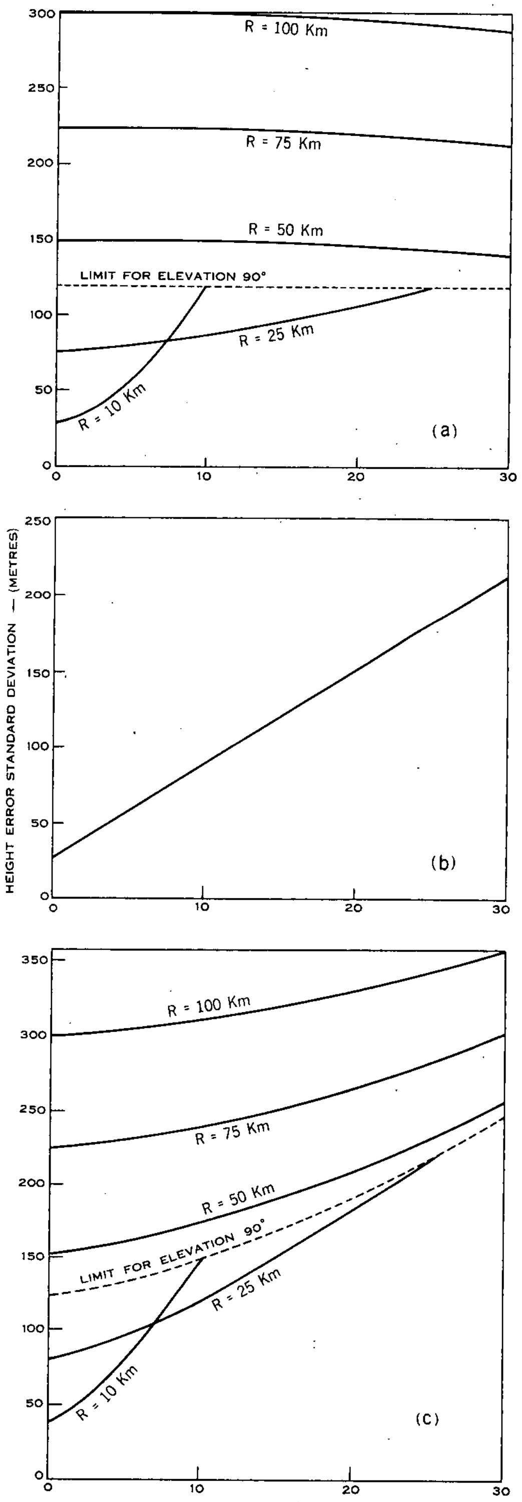

Fig. 4 Height error due to instrumental plus observational errors,
(a) radar only
(b) slide rule only
(c) radar and slide rule combined.

Comparisons of Figs. 2 and 4 with Figs. 1 and 3 show that:

(i) the total error with present procedures is larger than the instrumental error by a factor of about three for height, and a factor of about seven for vector wind at moderate ranges (60 km),

(ii) of the four sources of error discussed here (i.e. instrumental and observational errors for both radar and slide rule) the radar observational error is the biggest in its effect on height, but at short to moderate ranges the slide rule observational error, with gross mistakes included, is the biggest in effect on wind.

Any effort to reduce the present errors in computed winds should be directed to reducing both the radar observational error and the computational error. Avenues which might be explored are:

(i) to allow more time for the observations,

(ii) to improve the slide rule, or employ different computational aids, particularly those which allow more time for manipulation of the radar controls.

The following are some additional points in explanation or of interest. Absolute values of differences between the observations of range, azimuth and elevation from the two radars were plotted against range, azimuth, elevation and time, and were found to be substantially independent of them all. There is no reason to expect any dependence, except probably when the signal is weak (i.e. at very great range), or when the elevation is near 90°, or when the elevation is very low.

Table 2 shows that accuracy varied between flights. The variations in accuracy of azimuth and elevation observations, and most of the variations in range, are ascribed to variations of skill or care between the observers, although this was not evident at the time. All the observers were experienced and were judged to be competent, and there is no reason to doubt that their performance was typical.

In flights 3 and 4 there was an abrupt change in ΔR during the soundings. In flight 3 the mean value of ΔR changed from 0.16 km to a new value of 0.51 km, and in flight 4 the change was from 0.05 to 0.19 km. The change must be ascribed to an unidentified minor fault in the equipment, which is designed specifically to make very accurate measurements of range increments rather than absolute range. The scatter of ΔR about the separate means remained approximately the same in each flight, and since only increments of R affect the wind computations to any extent the two parts of each flight have been combined for this investigation.

It might be expected that the observational errors in range, azimuth and elevation would increase with their respective rates of change, because (i) errors in timing the observation would become more serious, and (ii) the observer may be forced to accept less precise settings of the controls in order to minimize the timing error. However, no apparent dependence was indicated when differences between the two radars were plotted against rates of change (up to 1.8 km min^{-1} and 4.2 deg min^{-1}).

Timing errors were not measured separately. They have been included in the standard deviations for range, azimuth and elevation.

The slide rule vector wind errors shown in Figs. 3 and 4 do not include a small additional error (+ 1.3%), which is due to the use of the approximation of 6000 ft for 1 nautical mile (6080 ft) in the slide rule computations where windspeed is in knots.

The "observational" error in slide rule operation includes a rounding-off error, which occurs when speed and direction are entered on the computation form to the nearest m sec^{-1} (or knot) and degree. The vector wind standard deviation of the rounding-off error ranges from about 0.3 m sec^{-1} at zero wind speed to about 0.6 m sec^{-1} at 100 m sec^{-1}.

There are various other sources of error outside the scope of this study; for example,

(i) Errors in height due to radar refraction:

In routine observations, a correction for refraction is made from tables based on an average profile of refractive index, but no allowance is made for day to day variations.

(ii) Errors due to averaging:

The procedures described provide winds averaged over layers about 600 metres thick, so the wind at a specified level can only be obtained approximately, with an accuracy depending on the vertical shear.

5. SUMMARY OF WF2 ACCURACY

In the operational use of upper wind reports, the errors in particular soundings cannot be found immediately from Figs. 2 and 4 because the range and ground range are not usually available. Table 3 has therefore been constructed to provide a rough indication of likely errors in individual flights, in British units. The figures in the table have been taken from Figs. 2 and 4 and rounded off, assuming normal balloon ascent rates.

Table 3. Standard deviation of errors in vector wind and height for particular wind soundings made with WF2 radar under routine observational procedures

Pressure P (mb)	Mean wind in layer (1000-P)mb (kt)	Standard deviation of error at level P	
		Vector wind (kt)	Height (ft)
700	20	3	60
500	50	8	300
200	20	8	300
200	50	14	600
10	20	14	600
200	100	18	1000
10	40	18	1000

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REFERENCES

Laby, Jean E. and Sparrow, J.G.	1965	"Wind studies using level balloons". J. Appl. Met., 4, pp. 585-589.
Spillane, K.T.	1968	"Reliability of Wind Data Determined by 277 Radar in Australia". Met. Study, Bureau of Meteorology, Aust. (To be published).