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THE GENERATION OF AVAILABLE POTENTIAL ENERGY BY SENSIBLE HEATING IN
SOUTHERN OCEAN CYCLONES

by B. R. Bullock

Dr. Ben Bullock is the last of a series of research fellows from the Department of Meteorology, University of Wisconsin, who have come to spend a year working at the International Antarctic Meteorological Research Centre. The Research Centre which had initially been set up in 1959 as the International Antarctic Analysis Centre, was closed as from 12 June 1969.

Dr. Bullock commenced his discussion by noting that the migratory cyclones of middle and high latitudes are the principal atmospheric sites for the conversion of potential to kinetic energy. Questions which are not yet fully answered are:

- (i) why is the available potential energy concentrated in these cyclones?
- (ii) what are the details of the conversion mechanism?

Diabatic processes are of particular interest in this context.

The concept of available potential energy (A.P.E.) was originated by Margules about 1900 when he was investigating the source of kinetic energy (K.E.) in cyclones. Following Margules' pioneering studies, there was no significant progress for 50 years until Lorenz applied the concept of A.P.E. in his work on the energy budget of the general circulation. Other workers - notably Dutton and Johnson - have refined Lorenz's equations, and produced "exact" expressions for the A.P.E. budget of a local volume which may be considered as translating with a cyclone.

Available potential energy may be defined by the equation

$$A = \pi - \pi_r \quad \dots(1)$$

where π = the total potential energy in the actual atmosphere, and π_r = the equivalent energy in a "reference" atmosphere resulting from a redistribution to a horizontally homogeneous, statically stable, density stratification. (This definition is essentially that given by Margules.) Integrating over a volume defined by the area σ and the bounding isentropic surfaces $\theta = \theta_0$ and $\theta = \theta_T$

$$\pi = \int_{\sigma} \int_{\theta_0}^{\theta_T} c_p T \rho \frac{\partial z}{\partial \theta} d\theta d\sigma \quad \dots(2)$$

(where c_p = specific heat of dry air at constant pressure and ρ = density).

Dr. Bullock showed how an equivalent expression for π_r could be obtained by substituting for T in terms of θ and replacing the pressure p by the reference pressure p_r . (In the reference state, p is a function of θ only.) Also, applying conservation of mass principles, the reference pressure p_r was found to be equal to \bar{p}_θ (the area-averaged value of p for the isentropic surface). Combining these results with equations (1) and (2)

$$\bar{A} = \frac{c_p}{p_{00} K} \int_{\theta_0}^{\theta_T} \theta (\bar{p}^K - \bar{p}^K) \rho \frac{\partial z}{\partial \theta} d\theta \quad \dots(3)$$

where \bar{A} = the area-averaged value of the A.P.E., p_{00} = the standard pressure (1000 mb), and $K = R/c_p = 0.286$, (where R is the gas constant). Using the definition $p = \bar{p} + p'$, Dr. Bullock derived the approximate expression

$$\bar{A} = \frac{c_p K(K-1)}{2 p_{00} K} \int_{\theta_0}^{\theta_T} \bar{p}^K \frac{(p')^2}{\bar{p}} \rho \frac{\partial z}{\partial \theta} d\theta \quad \dots(4)$$

Equation (4) implies that the A.P.E. is proportional to the variance of pressure on isentropic surfaces, i.e. increasing baroclinicity will be associated with increasing A.P.E.

Considering the energy budget of a translating atmospheric volume, the rate of change of \bar{A} may be written

$$\frac{d\bar{A}}{dt} = \frac{d\pi}{dt} - \frac{d\pi_r}{dt} = \bar{G} + \bar{B}_A - \bar{C}(A, K) \quad \dots(5)$$

where \bar{G} = the (area-averaged) generation by diabatic processes, \bar{B}_A = the boundary flux of A.P.E. (the advection of A.P.E. through the boundaries of the volume) and $\bar{C}(A, K)$ = the conversion of A.P.E. to K.E. Similarly, the rate of change of the area-averaged K.E. may be written

$$\frac{d\bar{K}}{dt} = \bar{C}(A, K) + \bar{B}_K - \bar{D} \quad \dots(6)$$

where \bar{D} = the frictional dissipation of K.E.

Following a procedure analogous to that used in deriving equation (4), the rate of generation of A.P.E. by diabatic processes may be written

$$\bar{G} = \frac{1}{\sigma} \int_0^{\theta_T} \int_{\theta_0}^{\theta_T} \epsilon Q \rho \frac{\partial z}{\partial \theta} d\theta d\sigma \quad \dots(7)$$

where Q = the local rate of diabatic heating, and the efficiency factor ϵ is defined by

$$\epsilon = 1 - \left(\frac{p_r}{p}\right)^K \quad \dots(8)$$

From equations (7) and (8) it follows that $\bar{G} > 0$ - i. e. A. P. E. will be generated - if diabatic heating occurs where $p > p_r$, and diabatic cooling where $p < p_r$. For a cyclone, this means that maximum generation of A. P. E. takes place if there is heating at low levels in the warm air, and cooling at high levels in the cold air.

In his current work Dr. Bullock has been investigating the generation of A. P. E. due to the turbulent transfer of sensible heat from the ocean to the atmosphere in a Southern Ocean cyclone. A volume with a horizontal radius of 2000 km (measured from the storm centre) was studied. Owing to the absence of conventional meteorological data, a model of the storm structure had to be used. The basic assumptions involved in this model are:

- (i) an adiabatic lapse-rate prevails throughout the mixed layer (the layer of sensible heat addition) - the value of θ at any point within the layer was taken to be equal to that of θ_s at the sea surface vertically beneath that point - which was further assumed to be constant;
- (ii) the horizontal divergence of the heat flux is negligible;
- (iii) $v_n = v_{gn}$, i. e. the wind component perpendicular to the isentropes on an isobaric surface is equal to the geostrophic wind component in the same direction. v_{gn} is independent of pressure, since the thermal wind will be parallel to the isentropes.

Making these assumptions, Dr. Bullock showed that the net heat flux into the mixed layer, between heights $z = 0$ and $z = H$, is given by

$$F_o - F_H = \frac{c_p}{g(1+K)p_{oo}} v_{gn} \frac{\partial \theta_s}{\partial n} (p_o^{1+K} - p_H^{1+K}) \quad \dots(9)$$

In applying equation (9) the values of θ_s were obtained from sea-surface temperatures, which in turn were obtained from climatological values, supplemented by point measurements where available. The values v_{gn} were obtained from carefully re-analysed MSL analyses. For the particular storm chosen for the study, two special radiosoundings made by the "Eltanin" were available - these indicated that the pressure at the top of the mixed layer (p_H) was about 850 mb. Since satellite photographs of the area indicated no significant variation in the meso-scale structure of the convective clouds, it was assumed that $p_H = 850$ mb throughout the area of cold-air advection.

Dr. Bullock presented a series of slides showing the results of his analyses for two different stages in the life-history of the storm - the developing stage and the occluding stage. The slides showed the MSL pressure patterns, the distribution of heating (with a definite maximum coinciding with the strongest southwesterly gradient, just northwest of the cyclone centre) and the distribution of ϵ at the surface and at 850 mb. The frontal zone appeared at the 850 mb level as a zone of strong isobaric

gradient of ϵ . It was noticeable (particularly in the analyses of the occluding stage) that the maximum heating occurred where the value of ϵ was relatively small - the heating that made the major contribution to \overline{G} took place further north, where ϵ was larger.

Values of \overline{G} were computed using three different assumptions about the distribution of diabatic heating in the vertical. These were

- (a) a uniform distribution of heating with height, i. e. $\frac{\partial Q}{\partial z} = 0$;
- (b) $Q = 0$ at $z = H$, and $\frac{\partial Q}{\partial z} = \text{a constant}$; and
- (c) all the heating concentrated at the surface.

The third of these assumptions is obviously unrealistic, but gives an idea of the maximum possible value of \overline{G} .

The estimated values of \overline{G} were of the order of 1 watt m^{-2} , with the values being a little larger in the occluding than in the developing stage. This indicates that the low-level diabatic heating associated with the cyclone did result in a positive contribution to the generation of A.P.E. and was thus a factor assisting in the development of the storm. (This result contrasts with those obtained in earlier studies, using formulae due to Lorenz.)

To gain an idea of the relative importance of this effect in the energy budget of the storm, values of \overline{D} were also estimated, using a method devised by Lettau. The ratio $\overline{G}/\overline{D}$ was found to be approximately $1/3$ in the developing stage, and $1/5$ in the occluding stage. Finally, Dr. Bullock stated that the computed values of \overline{G} seemed to be significantly smaller than those obtained in another study of a developing cyclone off the north-eastern coast of the United States. It would seem probable that this effect should be more important in the case of a storm in which the cold air is moving from a continental source-region out over the ocean than in the Southern Ocean case, where the cold air-mass is essentially maritime in character.

A lively discussion followed. Mr. K. Morley asked whether the effect considered might be more significant over a continent, where the effects due to latent heat release would be relatively less important. In addition, considerably more surface and upper-air data would be available, leading to improved analyses. Dr. Bullock said that he could not agree that the latent heat effect would be relatively less important, but he did agree that improved θ analyses would be an advantage. He had not included the effect of any form of latent heat conversion in the present study.

Mr. R.H. Clarke then asked what the magnitude of the contribution to \overline{G} due to latent heat release might be. Dr. Bullock in reply quoted values of 2, 8 and 4.5 watt m^{-2} , respectively for the developing, mature and occluding stages of a storm, and added that the first of these values was probably an underestimate.

Dr. G.B. Tucker commented that Dr. Bullock's address had been interesting from two viewpoints - both in view of the methods used and of the results obtained. He felt that as only one cyclone from each hemisphere had been studied in detail, the contrast in the $\overline{G}/\overline{D}$ values that had been observed must be regarded as a tentative result only. Dr. Tucker then pointed out that the assumption that $\frac{\partial \theta}{\partial t} = 0$ implies

that the sensible heating contribution must result in an increase in H, and yet the further assumption had been made that H was constant. Dr. Bullock said that since he had had no way of observing or determining the change in H, he had used a constant value as a matter of convenience. He had run his computations using different values for this constant, and had found that as H was increased \bar{G} decreased.

Dr. F.A. Berson asked whether there was no simpler method of computing the net heat flux. Dr. Bullock replied that the absence of surface data had forced him to use equation (9) in estimating the heat flux. Mr. P. Price enquired whether the results were highly sensitive to variations in the MSL analysis. Dr. Bullock said that Mr. H.R. Phillpot had prepared independent MSL analyses, which had then been used to re-compute the \bar{G} values, and there had been very little change in the results. Mr. R.H. Clarke commented that 850 mb seemed a rather low level to take as the top of the heating layer - in some respects it may be better to take H as the height of the tropopause. Dr. Bullock agreed with this comment, but nevertheless felt that his conclusions were not really affected. In reply to a further comment by Mr. Clarke, Dr. Bullock said that he regarded the effect of the cooling of the warm air-mass to be insignificant, owing to the high stability of the air-mass at low levels. Mrs. J. Hopwood asked whether the results were dependent to a large degree on the assumptions that had been made - by changing these assumptions, might it be possible to arrive at a small negative value for \bar{G} . Dr. Bullock replied that he did not think this was likely. If it had been possible to allow for variations in H there may have been significant changes in the \bar{G} values, but in general the results should not be sensitive to the other assumptions made. Mr. C.E. Wallington said that it appeared that it may be possible to define the developing stage of a cyclone by looking at changes in the value of the ratio \bar{G}/\bar{D} , with which Dr. Bullock agreed.

T.T.G.

2 October 1969

EVAPORATION, DRAINAGE AND WATER STORAGE OF A FIELD CROP

By C.B. Tanner

Professor Tanner is Head of the Department of Soil Science of the University of Wisconsin. Before embarking on the main subject of his address, Prof. Tanner remarked that, although interest in water balance problems was usually greatest in arid areas, it could be more profitable to concentrate attention on water balance problems in areas with abundant water where greater benefit would follow from optimal use.

Prof. Tanner then described the experimental measurement of terms in the water balance equation for a snap bean crop on well-watered sandy soil in Wisconsin. Drainage below the root zone was estimated from neutron probe measurements of soil moisture using an empirical drainage-soil moisture relationship