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THE ANALYSIS OF WIND GUSTS

By A.M. Hasofer* and K. Sharpe

Department of Statistics, University of Melbourne

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ABSTRACT

Orthogonal components of horizontal wind velocity are assumed to be independent, non-stationary Gaussian Stochastic Processes with negligible means and the same covariance structure. A method for estimating parameters associated with the long-term structure of the wind, based on the frequency of upcrossings at various levels, is given and applied to observed data.

The application of this approach to the prediction of maximum wind velocities and speeds expected over periods of 10 to 100 years is also considered and a comparison made with the more usual approach based on Gumbel's extreme value theory.

1. INTRODUCTION

One of the major factors to be considered in the design of tall structures is wind loading. The traditional approach to this problem has been to consider the wind as a steady uniform flow resulting in static loads. In recent years, however, starting with the work of A.G. Davenport, the accent has been on considering the wind to be stochastic in nature and the resultant response of the structure to be dynamic.

In some recent work by Hasofer (1969a) this problem has been considered from a different approach to that used by Davenport, and in the present paper the structure of the wind is examined in order to obtain estimates of parameters required in Hasofer's model. A method for estimating these quantities from the number of wind gusts in excess of various speeds is given and is applied to data obtained from the Melbourne Weather Bureau.

As a by-product of these procedures, we shall also give a method, different to that currently used, for estimating maximum wind gusts associated with various return periods.

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* Now at Department of Mathematical Statistics, University of N.S.W.

2. ASSUMPTIONS

The estimation methods to be developed are based on the following assumptions:

(i) The wind consists of two uncorrelated orthogonal components, U(t) and V(t), each of which is a Gaussian Process with the same covariance structure and a mean that is negligible compared with the high velocities of interest.

(ii) The covariance function

$$R(t,s) = E\left[V(t)\,V(s)\right],$$

while not strictly stationary, is locally stationary in the sense that for any two points of time u and v in a small neighbourhood of t, say five minutes either side of t, we have approximately

$$R(u,v) = R(t, t\pm(u-v)),$$

that is, R(u,v) depends on t and the difference (u-v), but not on the actual values of u and v.

(iii) R(t,s) is periodic with period one year in the sense that

$$R(t+T_y,\ s+T_y) = R(t,s),$$

where T_y is the length of one year.

(iv) The variance of V(t), R(t,t), is slowly varying in the sense that it will not vary appreciably over periods of, say, one hour; and also that the total variation over a yearly cycle is relatively small.

The assumption that the directional components of the wind can be considered to be Gaussian Processes has been advocated by various authors - for example, Davenport (1961) p. 455, Cramér (1960) p. 13, and the references quoted therein. The remainder of assumption (i) implies that the wind speed W(t), given by $W(t) = \left[U^2(t) + V^2(t)\right]^{\frac{1}{2}}$, is a "Rayleigh Process". In other words the undirected wind speed is a stochastic process for which the distribution at any time instant is Rayleigh. Note that all of the points contained in assumption (i) are needed to obtain a Rayleigh Process, the use of which is suggested by Davenport (1967).

By definition R(t,s) is the expected value of V(t)V(s) with respect to some probability measure defined on a sample space such that the observed wind record forms a single partial observation. This sample space, however, is hypothetical since it is impossible to obtain multiple observations. In order to obtain useful results for stochastic processes of this type it is therefore necessary to introduce some form of stationarity assumption. The most reasonable assumption for our purpose is to consider records for various years to be independent observations from the same process. This implies assumption (iii) above. Assumptions at least comparable to this are made whenever, for example, the maximum gust over a period of years is predicted.

The first part of assumption (iv) is implied to a large extent by the assumption concerning local stationarity which is implicit in all studies of the spectrum of the wind over periods of a few minutes (see Cramér (1960)), while the assumption concerning the small total variation of R(t, t) is supported by the analysis of the observed data discussed below.

3. ANALYSIS OF WIND GUSTS

The most informative, long-term, source of wind data available appears to be the daily Dines anemograph charts recorded by various weather stations. In Australia, for example, charts are currently being recorded at about sixty locations.

In general the scale of these charts is too small to enable us to follow the movements of the pen in detail. However, a suitable statistical analysis can be performed by considering the upcrossings of these charts at levels high enough to enable the number of such upcrossings to be counted.

In this section we shall quote, without proof, a number of results relating to the upcrossings of Gaussian and Rayleigh Processes, applicable to directional wind velocity and undirected wind speed respectively, while in the next section the relevance of these results to the observed data is discussed. For further details of these results, including proofs, see Cramér and Leadbetter (1967) and Hasofer (1969b).

For both types of process the upcrossing points constitute a point stochastic process. If the underlying process is Gaussian, with mean zero and locally stationary, under reasonably general conditions the mean number of upcrossings in $(t, t+\Delta t)$ is given by $q(t,x)\,\Delta t$, where

$$q(t,x) = \frac{1}{2\pi}\left[\frac{R_{12}(t,t)}{R(t,t)}\right]^{\frac{1}{2}} \exp\left[-x^2/2\,R(t,t)\right] \qquad \ldots(1)$$

and

$$R_{12}(t,t) = \left[\frac{\partial^2 R(t,s)}{\partial t\;\partial s}\right]_{t=s} .$$

The corresponding expression for the mean number of upcrossings by a locally stationary Rayleigh Process is

$$q'(t,x) = \frac{x}{R(t,t)}\left[\frac{R_{12}(t,t)}{2\pi}\right]^{\frac{1}{2}} \exp\left[-x^2/2\,R(t,t)\right] \qquad \ldots(2)$$

where $R(t,t)$ and $R_{12}(t,t)$ again refer to the covariance function of the component wind.

For the wind model the assumption that R(t, s) is periodic implies that q(t, x) and q'(t, x) will likewise be periodic, each with a one year period. Hence the mean total number of upcrossings at level x in N years will be given by Q(N, x) and Q'(N, x), respectively, where

$$Q(N, x) = N \int_{o}^{T_y} q(t, x)\, dt,$$

and

$$Q'(N, x) = N \int_{o}^{T_y} q'(t, x)\, dt.$$

If now we invoke the assumption that R(t, t) remain reasonably constant, it follows that, to a first approximation,

$$Q(N, x) = \frac{NT_y}{2\pi} \left[\frac{R_{12}}{\sigma^2}\right]^{\frac{1}{2}} \exp\left[-x^2/2\sigma^2\right] \qquad \ldots(3)$$

and

$$Q'(N, x) = \frac{xNT_y}{\sigma^2} \left[\frac{R_{12}}{2\pi}\right]^{\frac{1}{2}} \exp\left[-x^2/2\sigma^2\right] \qquad \ldots(4)$$

where:

$$\sigma^2 = \frac{1}{T_y} \int_{o}^{T_y} R(t, t)\, dt,$$

and

$$R_{12} = \left\{ \frac{1}{T_y} \int_{o}^{T_y} \left[R_{12}(t, t)\right]^{\frac{1}{2}} dt \right\}^2 .$$

4. ANEMOMETER RESPONSE

The instrument used to record the data to be analysed was a Dines pressure tube anemometer. While these instruments are usually scaled in knots or miles per hour, the wind property actually measured is pressure which is proportional to velocity squared.

For a directional component of the wind, V(t) say, the pressure $P_V(t)$ will be of the form

$$P_V(t) = \alpha V(t) \, |V(t)|$$

taking direction into account. Since this is a one-to-one relationship, we can immediately deduce an expression analogous to equation (1) for the mean number of upcrossings at level x of $P_V(t)$, namely

$$q_P(t,x) = \frac{1}{2\pi} \left[\frac{R_{12}(t,t)}{R(t,t)}\right]^{\frac{1}{2}} \exp\left[-x/2\alpha R(t,t)\right]. \qquad \ldots(5)$$

Similarly we have that for the magnitude of the pressure $P_W(t)$ of the non-directional wind speed W(t),

$$P_w(t) = \alpha W^2(t)$$

and

$$q'_P(t,x) = \frac{1}{R(t,t)} \left[\frac{x R_{12}(t,t)}{2\alpha\pi}\right]^{\frac{1}{2}} \exp\left[-x/2\alpha R(t,t)\right]. \qquad \ldots(6)$$

It is clear therefore that if we measure pressure and then convert back to miles per hour, the mean number of upcrossings at various levels as given by formulae (1) and (2) remains valid.

Anemometers, however, do not have an instantaneous response and the pressure values undergo some form of averaging before they are converted into velocities. Pasquill (1962) (see in particular page 11) suggests that this averaging process can satisfactorily be represented by a simple moving average of the form

$$\frac{1}{T} \int_0^T P(t-u) \, du, \qquad \ldots(7)$$

where T is of the order of three seconds for a Dines anemometer.

While the effect of such a moving average on the distribution function of the pressure is not known exactly, we shall assume that it is negligible in that the form of equations (5) and (6) remains unaltered, though the values of $R(t,t)$ and $R_{12}(t,t)$ may change. The justification for this assumption is that:

(i) we are averaging over highly correlated values for a short time interval,

(ii) the conclusions arrived at by various authors regarding the Gaussian and Rayleigh nature of the wind have been based on observations taken by anemometers of this type.

In order to investigate the effect of the moving average on the quantities $R(t,t)$ and $R_{12}(t,t)$, it is convenient to express them in terms of the covariance functions of $P_V(t)$ and $P_W(t)$.

Using the results derived by Hasofer (1969a) it follows that the covariance function of $P_V(t)$, $R_P(t,s)$ is given by

$$R_P(t,s) = 4\alpha^2 R(t,t)\,R(s,s) \sum_{n=1}^{\infty} K_{2n-1} \left[\frac{R(t,s)}{\{R(t,t)\,R(s,s)\}^{\frac{1}{2}}}\right]^{2n-1} \quad \ldots (8)$$

where

$$K_1 = 2/\pi, \quad K_3 = 1/3\pi, \quad K_5 = 1/60\pi,$$

and

$$K_{2n-1} = \frac{1}{\pi\,[(2n-1)!]\;2^{2n-7}} \left[\frac{(2n-5)!}{(n-3)!}\right]^2 \quad \text{for } n > 3,$$

while for $P_W(t)$, direct evaluation of

$$E\left[W^2(t)\,W^2(s)\right] - E\left[W^2(t)\right] E\left[W^2(s)\right]$$

gives

$$R_{P'}(t,s) = 4\alpha^2 \left\{R(t,s)\right\}^2 \quad \ldots (9)$$

Putting $t = s$ in equations (8) and (9) we find

$$\mathrm{Var}\left[P_V(t)\right] = R_P(t,t) = 3\alpha^2 \left[R(t,t)\right]^2, \quad \ldots (10)$$

and

$$\mathrm{Var}\left[P_W(t)\right] = R_{P'}(t,t) = 4\alpha^2 \left[R(t,t)\right]^2, \quad \ldots (11)$$

and hence $R(t,t)$ is expressible simply in terms of the variance of $P_V(t)$ and $P_W(t)$.

In order to obtain simple relationships between $R_{12}(t,t)$ and the two pressure processes we will use the local stationarity of $R(t,s)$ and assume

$$\left.\frac{\partial R(t,s)}{\partial t}\right|_{t=s} = \left.\frac{\partial R(t,s)}{\partial s}\right|_{t=s} = \frac{\partial R(t,t)}{\partial t} = 0$$

Under these circumstances it follows that

$$\left.\frac{\partial^2 R_p(t,s)}{\partial t \ \partial s}\right|_{t=s} = 4 \alpha^2 R(t,t)\, R_{12}(t,t) \sum_{n=1}^{\infty} (2n-1)\, K_{2n-1}, \qquad \ldots(12)$$

$$= 4 \alpha^2 R(t,t)\, R_{12}(t,t),$$

and

$$\left.\frac{\partial^2 R_{p'}(t,s)}{\partial t \ \partial s}\right|_{t=s} = 8 \alpha^2 R(t,t)\, R_{12}(t,t). \qquad \ldots(13)$$

Hence the effect of the moving average on the quantities $R(t,t)$ and $R_{12}(t,t)$, as they appear in equations (5) and (6), can be determined by considering the effect of the moving average on (10), (11), (12) and (13).

Consider now a stationary process $\zeta(t)$ with covariance function $\gamma(\tau)$ and let

$$\overline{\zeta(t)} = \frac{1}{T} \int_0^T \zeta(t-u)\, du.$$

The covariance function of $\overline{\zeta(t)}$ will then be of the form

$$\overline{\gamma(\tau)} = \frac{1}{T^2} \int_{v=\tau}^{\tau+T} \int_{w=0}^{T} \gamma(v-w)\, dv\, dw,$$

from which we find

$$\overline{\gamma}(0) = \frac{1}{T^2} \int_0^T \int_0^T \gamma(v-w)\, dv\, dw \qquad \ldots(14)$$

$$= \frac{2}{T} \int_0^T \left(1-\frac{y}{T}\right) \gamma(y)\, dy,$$

and

$$\overline{\gamma_{12}(0)} = -\frac{1}{T^2} \int_0^T \gamma'(T-w)\, dw. \qquad \ldots(15)$$

Following Hasofer (1969a) we shall now assume that R(t, s) has the form

$$R(t,s) = R(t,t)\left[p + q\, w(t-s)\right]$$

for values of $|t-s|$ less than about five minutes, where $p + q = 1.0$, $0 \leq w(t-s) \leq 1$, $w(0) = 1$ and observational evidence suggests that p is of the order of 0.94. Briefly the reason why this form for R(t, s) differs slightly from the form usually considered is that we have defined R(t, s) to be $E\left[V(t)\, V(s)\right]$ whereas the covariance function is usually defined to be $E\left[(v'(t)\, v'(s)\right]$, where $v'(t) = V(t) - \overline{V}$, and $\overline{V}$ is the average velocity over the relatively short time interval considered. Substituting this form for R(t, s) in expressions (8) and (9) we obtain

$$R_P(t,s) = 4\alpha^2 \left[R(t,t)\right]^2 \sum_{n=1}^{\infty} K_{2n-1} \left[p + q\, w(t-s)\right]^{2n-1}$$

and

$$R_{P'}(t,s) = 4\alpha^2 \left\{R(t,t)\left[p + q\, w(t-s)\right]\right\}^2 .$$

Since R(t, t) can be taken to be constant for periods well in excess of the averaging period of the anemometer, we can consider $R_P(t,s)$ and $R_{P'}(t,s)$ to be stationary and apply formulae (14) and (15) to determine the effect of the anemometer.

If now we put $p = 0.94$ and consider the variances of the averaged processes $\overline{P_v(t)}$ and $\overline{P_w(t)}$, using equation (14), we obtain the inequalities

$$2.76\, \alpha^2 \left[R(t,t)\right]^2 \leq \mathrm{Var}\left[\overline{P_v(t)}\right] \leq 3\, \alpha^2 \left[R(t,t)\right]^2$$

and

$$3.53\, \alpha^2 \left[R(t,t)\right]^2 \leq \mathrm{Var}\left[\overline{P_w(t)}\right] \leq 4\, \alpha^2 \left[R(t,t)\right]^2$$

by considering the two extreme cases $w(t-s) = 0$ and $w(t-s) = 1$. Hence the effect of the anemometer on the value of R(t, t) as it appears in equations (5) and (6) will be less than 4% and 6% respectively.

In order to determine the effect of the anemometer on $R_{12}(t,t)$, using equation (15), we require knowledge concerning the derivative of $w(t-s)$. For wind loading, however, our major concern is with the value of $R(t,t)$, the variance of the component wind, and it is sufficient for our purposes to know that the anemometer has little effect on this quantity and to ignore the effect on $R_{12}(t,t)$.

5. DATA ANALYSIS

The data to be analysed was obtained directly from the daily anemograph charts recorded at Melbourne by the Bureau of Meteorology for the period 1940-1968, excluding 1954-1955 and 1958-1960 – a total of 24 years. The charts for the excluded years could not be located.

All gusts in excess of 53 mph were recorded to the nearest lower mph, together with an estimate of direction. From this information we also obtained data on gusts from the north direction that were in excess of 53 mph. The reason for considering only gusts in excess of 53 mph was the difficulty at times encountered in distinguishing between gusts below this level. In general this lower limit will depend upon the scale of the anemograph and the character of the wind, and hence will vary with location.

Two further problems associated with the collection of data were:

(i) During high winds the pen drawing the charts occasionally ran short of ink.

(ii) During high winds the direction plot, as an angle between 0° and 360°, appears as a solid band approximately 45° in width. The direction of the recorded gusts was taken to be the midpoint of this band to the nearest $22\frac{1}{2}^{\circ}$.

To the estimated north component of the gusts was fitted the model

$$Q(x) = \frac{1}{2\pi}\left(\frac{R_{12}}{\sigma^2}\right)^{\frac{1}{2}} e^{-x^2/2\sigma^2}, \qquad \ldots(16)$$

while to the undirected wind speed data was fitted the model

$$Q'(x) = \frac{x}{\sigma^2}\left(\frac{R_{12}}{2\pi}\right)^{\frac{1}{2}} e^{-x^2/2\sigma^2}, \qquad \ldots(17)$$

where $Q(x)$ and $Q'(x)$ refer to the expected number of upcrossings at level x per year.

In order to estimate the parameters in the regressions (16) and (17), it is necessary to take into account the high correlations between the observations. Ideally we require the theoretical form of the variances and covariances of the observations at various levels. Unfortunately these quantities cannot be readily calculated theoretically, and the best we could do was to estimate them from the observations available. Regressions (16) and (17) were then fitted to the total number of upcrossings at various levels for the 24 years, using the methods for dealing with correlated variables - see, for example, Rao (1965) p. 179 applied to exponential regression as given by Williams (1959) p. 62. The estimates of σ^2 and R_{12} obtained were as follows. For the north component, using equation (16)

$$\sigma^2 = 129 \ (\text{mph})^2$$

$$R_{12} = 4.23 \ (\text{mph})^2 \ (\text{sec})^{-2},$$

while for all the data, using equation (17),

$$\sigma^2 = 143 \ (\text{mph})^2$$

$$R_{12} = 0.049 \ (\text{mph})^2 \ (\text{sec})^{-2}.$$

A statistical test of the appropriateness of these models gave the following results. For the north component, a comparison of the estimated residual variance with the theoretical value (taking the estimated covariance matrix to be theoretically correct) showed no significant departure from the assumed model; and the estimate of the correlation ratio was 0.90. Using all the data, the estimated correlation ratio was 0.82, but a comparison of the residual variance with the theoretical value did indicate a significant departure from the assumed model. It should be noted, however, that the model fitted well enough for us to be able to use it for prediction purposes.

A possible explanation for the lack of fit of the Rayleigh Process is the existance of "prevailing" winds. This may also account for the estimate of R_{12} being so much larger for the north component, since the reason for choosing this direction for analysis was that the majority of gusts in excess of 53 mph came from that direction.

The data points and regression curves associated with equations (16) and (17) are plotted in Figs. 1 and 2 respectively. The graphs show up a rather unexpected feature, namely that all of the data points lie on the same side of the fitted regression line. This apparent anomaly is discussed in the Appendix 1.

In order to investigate the variation of the quantities $R(t,t)$ and $R_{12}(t,t)$ throughout the year, equation (17) was fitted to the relevant data for each of the four seasons, using the estimate of the covariance matrix obtained from all the data. The following estimates were obtained:

	Summer	Autumn	Winter	Spring
$\sigma^2 (\text{mph})^2$	127	121	146	139
$R_{12} (\text{mph})^2 (\text{sec})^{-2}$	0.093	0.087	0.051	0.178

Of particular interest is the small variation in the estimates of σ^2, which supports assumption (iv) of Section 2.

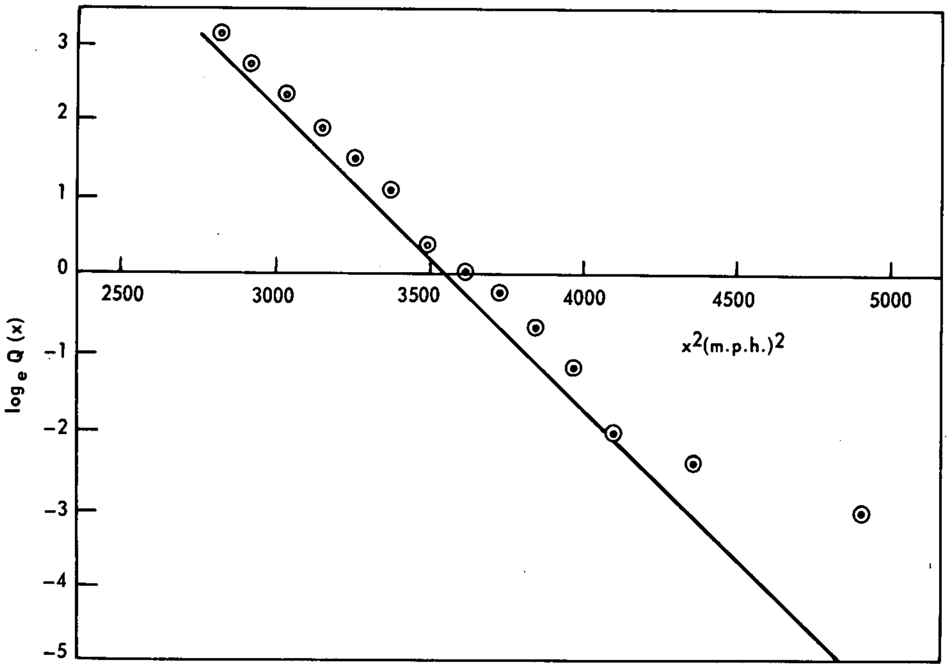

Fig. 1 Regression line and points relating $\log_e Q(x)$ to x^2 where $Q(x)$ is the average number of wind gusts, per year, whose north component is in excess of x m.p.h.

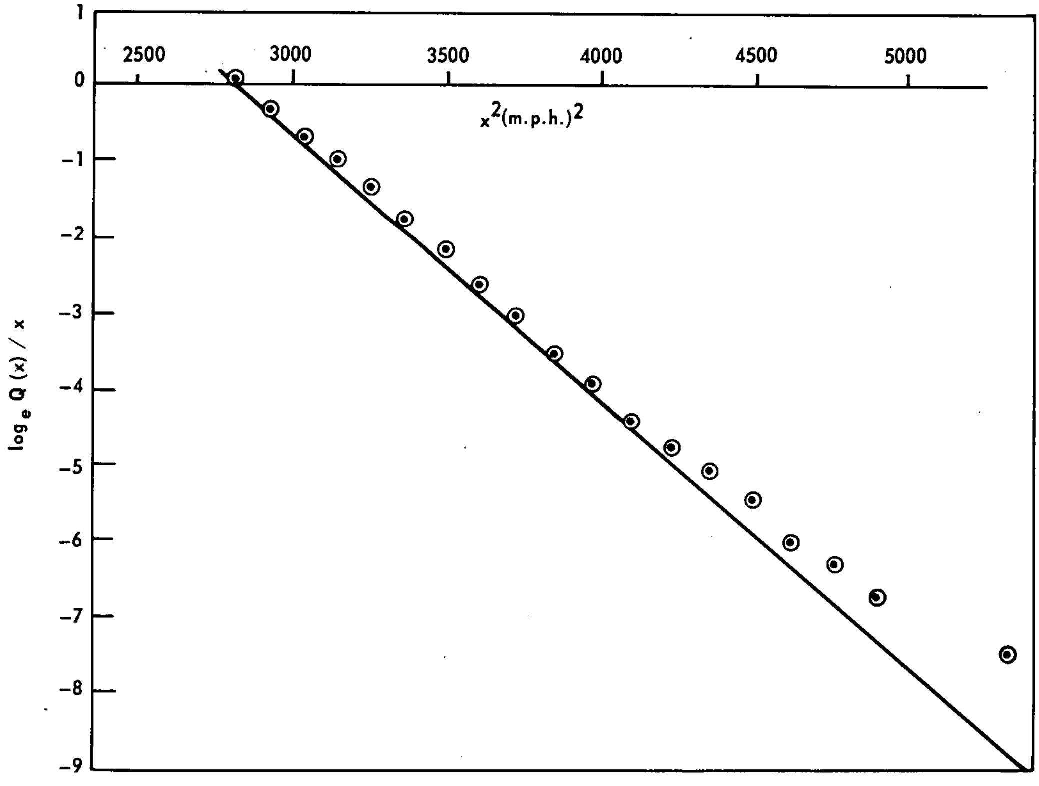

Fig. 2 Regression line and points relating $\log_e Q'(x) / x$ to x^2 where $Q'(x)$ is the average number of wind gusts, per year, in excess of x m.p.h.

6. PREDICTION OF MAXIMUM WIND GUSTS

One of the quantities that has been basic to the traditional approach to wind loading is the maximum wind gust expected over a number of years. In the past, these maximum gusts have been estimated using Gumbel's extreme value theory applied to the yearly maximum gust, see for example Whittingham (1964).

We shall now demonstrate how the wind model discussed above can be used to estimate these maximum gusts, thereby making fuller use of the information available.

Cramér and Leadbetter (1967) have shown that if the underlying stochastic process is Gaussian and Stationary then, asymptotically, the upcrossings process is Poisson. On the basis of this result it is reasonable to suppose that for our "locally stationary" Gaussian Process of the component wind, the upcrossings process will be non-stationary Poisson which could be made stationary by a suitable change in time scale. In particular, if we restrict consideration to the number of upcrossings in yearly periods, then for high enough wind velocities the associated distribution will be approximately Poisson. It is further conjectured that a similar result will hold for the distribution of upcrossings, at high levels, of the locally stationary Rayleigh Process associated with wind speed.

If we let $M(NT_y)$ denote the maximum gust during a period of N years then, for large enough x, we have that for a directional component

$$\Pr\left[M(NT_y) \leq x\right] = \exp\left[-NQ(x)\right],$$

while for the undirected wind speed

$$\Pr\left[M(NT_y) \leq x\right] = \exp\left[-NQ'(x)\right],$$

where $Q(x)$ and $Q'(x)$ are given by equations (16) and (17) respectively.

Note, however, that these expressions give only an approximation to the distribution of $M(NT_y)$ for values of x large enough to invoke the Poisson assumption, and do not, in themselves, represent distributions.

In Table 1 values of the ratio of estimated variance to estimated mean for the number of upcrossings per year at various levels are set out for both the north component and the actual value of the gusts. If the underlying distribution is Poisson these ratios should lie within the range 0.57 - 1.53, with a probability of approximately 0.90. Hence we find that for the undirected gusts the Poisson assumption is not refuted for speeds in excess of 65 mph, while for the north component a velocity in excess of 62 mph will suffice.

The corresponding average number of upcrossings per year for these two situations is 0.542 and 0.500 respectively. It would thus appear that the Poisson distribution provides a satisfactory approximation when the average number of upcrossings is ≤ 0.5 per year. Return periods of 10-100 years, which are of interest in Civil Engineering Design, correspond to average numbers of upcrossings from 0.1 to 0.01 per year.

Table 1. Ratio of Estimated Variance to Estimated Mean

Velocity mph	All Gusts	North Component
53	22.7	17.13
54	18.5	11.88
55	15.8	10.04
56	12.9	7.67
57	9.73	5.89
58	7.45	3.87
59	5.90	2.68
60	5.27	1.65
61	3.04	1.53
62	2.20	1.22
63	1.72	1.03
64	1.88	0.88
65	1.12	0.96
66	1.03	0.96
67	1.03	1.00
68	0.87	1.00
69	0.96	1.00
70	0.96	1.00
73	1.00	

Table 2. Predicted maximum velocities (all data)

Return Period	Wind Speed		North Component	
	Upcrossings	Gumbel	Upcrossings	Gumbel
10	68.3	70.4	64.2	66.7
20	69.8	73.3	65.7	69.9
50	71.7	76.9	67.5	74.0
100	73.2	79.7	68.8	77.0

Table 3. Predicted maximum velocities (omitting max. observed gust)

Return Period	Wind Speed		North Component	
	Upcrossings	Gumbel	Upcrossings	Gumbel
10	68.3	69.4	64.2	65.5
20	69.8	71.9	65.7	68.3
50	71.7	75.2	67.4	71.9
100	73.2	77.7	68.7	74.7

In Table 2 the wind velocities and speeds associated with various return periods are given for the directional and non-directional data using the methods described above. Also given are the analogous values obtained from a Gumbel analysis of the yearly maxima. Table 3 sets out the results of a similar analysis carried out on the same data, with the exception that the largest recorded gust is omitted. As expected, the Gumbel analysis is considerably more sensitive to this one observation than the alternative method proposed.

7. CONCLUSION

We have shown that, by making a number of assumptions regarding the underlying statistical nature of wind velocity, simple expressions can be obtained for the expected number of upcrossings at various levels for both a directional component of wind velocity and the undirected wind speed. An overall justification for the assumptions made is that the expressions obtained provide an acceptable fit to the data considered. The advantage of the methods proposed, over the more usual approaches to the same problems, is that they enable us to make much fuller use of the information available.

REFERENCES

Cramér, H.E., 1960 "Use of power spectra and scales of turbulence in estimating wind loads," Meteorological Monographs, 4, No. 22, pp. 12-18.

Cramér, H.E. and Leadbetter, M.R., 1967 "Stationary and related stochastic processes." John Wiley and Sons, New York.

Davenport, A.G., 1961 "The application of statistical concepts to the wind loading of structures," Proc. Inst. of Civil Eng., 19, pp. 449-72.

1967 The treatment of wind loading on tall buildings. "Tall Buildings" edited by A. Coull and B. Stafford-Smith. Pergamon Press. pp. 3-44.

Hasofer, A.M. 1969a "A non-stationary stochastic model for wind-load design," (To be published).

1969b "On the derivative and the upcrossings of the Rayleigh Process," (To be published).

Pasquill, F. 1962 "Atmospheric Diffusion", Van Nostrand, Princeton.

Rao, C.R. 1965 "Linear statistical inference and its applications." John Wiley and Sons, New York.

Whittingham, H.E. 1964 "Extreme wind gusts in Australia," Commonwealth of Australia, Bureau of Meteorology, Bulletin No. 46.

Williams, E.J. 1959 "Regression analysis," John Wiley and Sons, New York.

APPENDIX 1

The following comments regarding the possibility of observing the residuals from a fitted regression to be all of the same sign are largely due to Professor E.J. Williams.

Consider the linear regression

$$E\left[\underline{Y} \mid \underline{X}\right] = \underline{X}\,\underline{\beta}$$

where $\underline{Y}$ is a n- vector of observations, β is a p - vector of regression coefficients and $\underline{X}$ is a n x p matrix of independent variates.

If the observations vector $\underline{Y}$ has an associated covariance matrix $\underline{\Sigma}$, then the vector $\underline{\beta}$ is estimated by minimizing

$$(\underline{Y} - \underline{X}\,\underline{\beta})' \,\underline{\Sigma}^{-1} (\underline{Y} - \underline{X}\,\underline{\beta}),$$

and the estimating equations (using $\underline{\hat{\beta}}$ for the estimate vector) are

$$(\underline{Y} - \underline{X}\,\underline{\hat{\beta}})' \,\underline{\Sigma}^{-1}\, \underline{X} = 0,$$

giving

$$\underline{\hat{\beta}} = (\underline{X}'\,\underline{\Sigma}^{-1}\,\underline{X})^{-1}\,\underline{X}'\,\underline{\Sigma}^{-1}\,\underline{Y}\,.$$

If the n- vector of residuals $(\underline{Y} - \underline{X}\,\underline{\hat{\beta}})$ is denoted by $\underline{e}$, then the vector satisfies the p equations

$$\underline{X}'\,\underline{\Sigma}^{-1}\,\underline{e} = 0.$$

Hence if the components of $\underline{e}$ are to be of the same sign, each row of $\underline{X}'\,\underline{\Sigma}^{-1}$ must have both positive and negative components. This is clearly a necessary but not a sufficient condition.

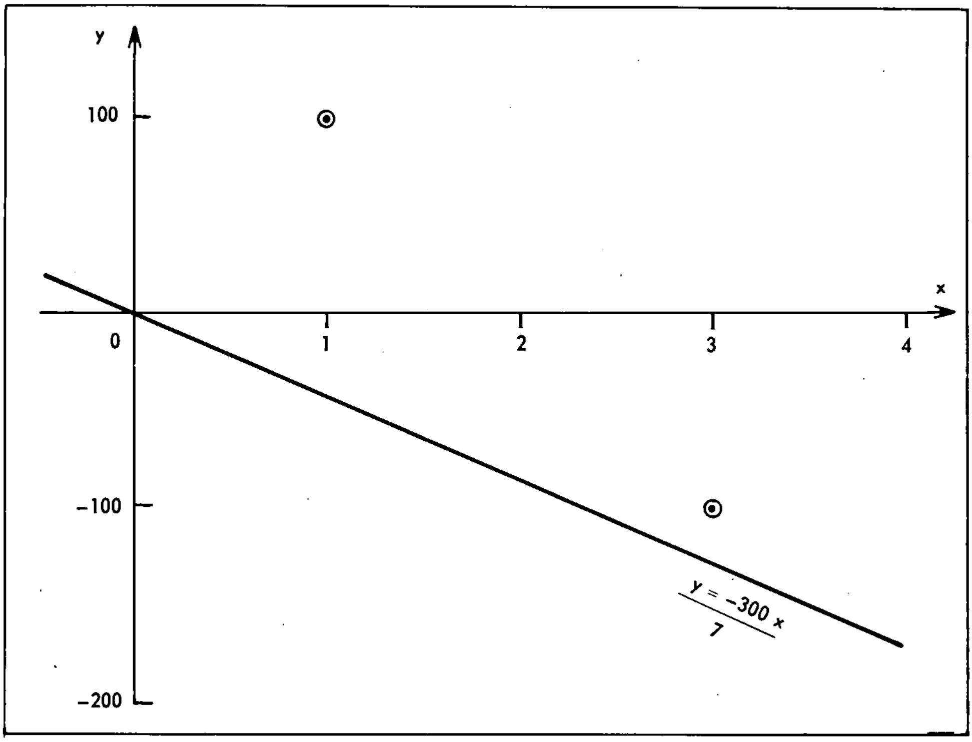

Fig. 1 Regression line and points for example in Appendix 1.

Example:

The simplest possible case is when p = 1, n = 2. If we take

$$\underline{X} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } \underline{\Sigma} = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}, \quad \underline{X}' \underline{\Sigma}^{-1} = \begin{bmatrix} -2/3 \\ 10/3 \end{bmatrix}.$$

Hence, regardless of the values of $\underline{Y}$, $\underline{X}' \underline{\Sigma}^{-1} \underline{e} = 0$ implies the two values in $\underline{e}$ are either both zero or both of the same sign. For example, if we take

$$\underline{Y} = \begin{bmatrix} 100 \\ -100 \end{bmatrix}$$

then

$$\hat{\underline{\beta}} = \underline{X}' \underline{\Sigma}^{-1} \underline{Y} / \underline{X}' \underline{\Sigma}^{-1} \underline{X}$$

$$= -300/7$$

and

$$\underline{e} = (\underline{Y} - \underline{X}\ \hat{\underline{\beta}}) = \begin{bmatrix} 142\,^6/_7 \\ 28\,^4/_7 \end{bmatrix}$$

The corresponding regression line is shown in Fig. I (Appendix 1).