RESPONSE OF THE THERMISTOR
USED IN THE AUSTRALIAN RADIOSONDE

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ABSTRACT

The specification for the Australian radiosonde states that the response of the thermistor shall not exceed 5 seconds. This limit, however, only applies at sea level and at high altitudes the response increases considerably to a value of about 18 seconds at 35 km. The response has been measured during a balloon flight using a modified radiosonde. Theoretical values of response based on heat transfer theory are in reasonable agreement with the experimental values. The theoretical determination requires a knowledge of the thermal capacity of the thermistor and this has been measured for temperatures over the range +40°C to -60°C. Examples are included showing the errors due to response for typical situations in a radiosonde flight.

1. INTRODUCTION

The thermistor currently used in the Australian radiosonde is manufactured in the U.S.A. and is the same type as that generally used in the United States Weather Bureau radiosondes. It is a ceramic rod coated with white paint to reflect solar radiation. The overall nominal diameter is 0.05 inch and the nominal length is 2 inch. The response was found during a balloon ascent by heating the thermistor at intervals throughout the flight and measuring its temperature as it cooled. The response is defined as the time required for the difference in temperature between the thermistor and the air to be reduced to 1/e (=0.368) of its initial value provided that the air temperature remains constant. As the air temperature is not constant during the flight, this must be taken into consideration when determining the response.

The response of a similar thermistor has been determined in laboratory experiments (Badgley 1957). The overall diameter of the thermistor used by Badgley is not clear; however, if it is assumed that the diameter quoted refers to the rod only, the diameter when coated would be about 0.033 inch. This corresponds to the diameter of the thermistor used with military radiosondes in the U.S.A. Badgley’s results for a 0.033 inch diameter thermistor can be compared to those found in this experiment for a 0.05 inch diameter thermistor by making a correction for the difference in diameters. On this basis the results are in agreement.
2. EXPERIMENTAL PROCEDURE

A standard 402 MHz radiosonde was modified, as shown in Fig. 1, so that the temperature element is heated by the high tension battery supply via the relay contacts. When the relay is energised the high tension is disconnected and the element is connected to the blocking oscillator. The response of the element can be examined as it cools each time the relay is closed by a baroswitch contact. Both the low reference and humidity baroswitch contacts were connected to the relay so that the response could be examined up to the end of the flight. The low reference was connected in place of the high reference to provide a reference value. The 10k ohm resistor, between the high tension supply and the temperature element, is provided to prevent excessive heating of the temperature element. The radiosonde transmissions were recorded with standard 402 MHz radiosonde ground equipment. The recorder chart speed was increased so that the response could be determined accurately. Height and rate of ascent information were obtained from radar data.

3. THEORETICAL BASIS OF THE EXPERIMENT

Middleton and Spilhause (1953) show that the response is the time taken for the difference in temperature between that of the thermistor and the air to be reduced to 1/e of its initial value, and give the following relationship for response:

\[
\frac{dT_m}{dt} = \frac{1}{\lambda} (T_m - T_a)
\]

...(1)

where:

\[T_m = \text{Temperature of the thermistor}\]

\[T_a = \text{Temperature of the air}\]

\[t = \text{Time}\]

\[\lambda = \text{Response}\]

The air temperature \(T_a\) cannot be considered constant during the time the response is being measured and its variation must be taken into account. It is satisfactory to assume that the air temperature is changing at a constant rate.

Whence \[T_a = T_{ao} + \beta t\]

...(2)

where \(T_{ao}\) is the initial air temperature, and

\[\beta\] is the rate of change of temperature with time.
Fig. 1  Radiosonde circuit for thermistor response experiment.
Substituting equation (2) in equation (1):

$$\frac{dT_m}{dt} = \frac{1}{\lambda} \left[ T_m - (T_{ao} + \beta t) \right]$$

... (3)

Following a similar approach to that used by Middleton and Spilhause (1953) gives the solution:

$$T_e = T_{ao} + \beta t - \beta \lambda + \left( T_{eo} - T_{ao} + \beta \lambda \right) e^{-\lambda t}$$

... (4)

for $t=t_s$, where $t_s$ is at least three times $\lambda$, $T_e = T_{es}$ and equation (4) when rearranged becomes:

$$T_{ao} = T_{es} - \beta t_s + \beta \lambda$$

... (5)

where $T_{es}$ is the steady state temperature.

Substituting equation (5) in equation (4) and rearranging gives:

$$\lambda = \frac{t}{\ln \left[ \frac{\beta (t_s - t) + (T_e - T_{es})}{T_{eo} - T_{es} - t_s} \right]}$$

... (6)

Values of $T_e$, $T_{eo}$, $T_{es}$, $t_s$, $t$ and $\beta$ are found from the recorder trace and substituted in equation (6) to find the response. A typical section of the recorder trace is shown in Fig. 2. The experimental values are plotted on Fig. 3.

4. THEORETICAL RELATIONSHIP FOR RESPONSE

The response of the thermistor is determined by the thermal capacity $Q$ divided by the heat transfer coefficient $h$:

$$\lambda = \frac{Q}{h}$$

... (7)

Hind (1965) gives a relationship for the average heat transfer coefficient $h_{av}$:

$$h_{av} = \frac{N_u k}{d}$$

... (8)

where $N_u = \text{Nusselt number}$

$k = \text{Thermal conductivity of air}$

$d = \text{Diameter of thermistor}$.
Fig. 3. Response of Australian radiosonde thermistor during experimental balloon test flight.

Fig. 2. Typical section of trace on recording chart.

\[ T_e = \text{temperature of thermistor at time } t \]
\[ T_{e0} = \text{temperature of thermistor at time } t = 0 \]
\[ T_{eS} = \text{temperature of thermistor at time } t = t_s \]
\[ \Delta T = \text{rate of change of air temperature} \]
\[ \Delta t = \text{time} \]
\[ \theta_1 = \text{angle} \]
\[ \theta_2 = \text{angle} \]
The total heat transfer coefficient is obtained by multiplying the average heat transfer coefficient by the surface area. Therefore

\[ h = \frac{N_u k}{d} \pi d l = N_u k \pi l \]  \hspace{1cm} \ldots (9)

where \( l \) = length of the thermistor.

Eckert and Drake (1959) show that for a cylinder with the air flow perpendicular to its axis, the Nusselt number is given by

\[ N_u = 0.43 + C(\text{Re})^m \]  \hspace{1cm} \ldots (10)

where \( \text{Re} = \text{Reynolds number} \),

and that for \( \text{Re} \) between 1 and 4000

\[ C = 0.48 \]
\[ m = 0.5. \]

Also

\[ \text{Re} = \frac{V \rho d}{\mu} \]  \hspace{1cm} \ldots (11)

where \( V = \text{Velocity of air over the thermistor} \),
\( \rho = \text{Density of air} \),
\( \mu = \text{Coefficient of viscosity of air} \).

Substituting equations (8), (9), (10) and (11) in equation (7)

\[ \lambda = \frac{Q}{(0.43 + 0.48 \left( \frac{V \rho d}{\mu} \right)^{0.5}) k \pi l} \]  \hspace{1cm} \ldots (12)

5. DETERMINATION OF THERMAL CAPACITY OF THERMISTOR

The thermal capacity \( Q \) is given by the heat input divided by the temperature rise:

\[ Q = \frac{\text{Heat Input}}{\text{Temperature Rise}} \]  \hspace{1cm} \ldots (13)

The thermal capacity has been measured by heating the thermistor with a known amount of electrical energy and measuring the temperature rise by monitoring its resistance. The heat must be applied and the resistance measured in a very short time to prevent errors caused by heat loss. Provided the temperature rise is not more than a few degrees the heat loss due to radiation is not significant. The heating period was limited to one second.
Fig. 4 shows the experimental set up. Initially switch "S" is in position (a) and the storage oscilloscope is triggered on a slow time base for a single sweep. The storage oscilloscope indicates a voltage from which the resistance of the thermistor can be derived. The value of resistance $R_S$ is large enough to prevent any significant heating. After a short interval switch "S" is changed to position (b) and the thermistor receives heat according to the relation:

$$\text{Heat input} = \frac{V^2 t}{R_{av}} \ldots (14)$$

where $V$ = the supply voltage

$R_{av}$ = the average resistance of the thermistor

$t$ = the time the heating voltage is applied.

After the heating period, switch "S" is returned to position (b) and the storage oscilloscope indicates a voltage from which the new resistance of the thermistor can be derived. The temperature rise is found from the resistance values using resistance ratio tables prepared by Callus (1968). The value of the temperature rise and the heat input obtained from equation (14) are substituted in equation (13) to find the thermal capacity. The thermal capacity was measured at various temperatures between $+40^\circ$ and $-60^\circ\text{C}$ and the results are graphed in Fig. 5. The smooth curve drawn on this graph is the basis of the thermal capacity values used to determine the response.

It was thought that the heat may not transfer from the resistance material to the white coating quickly enough for the resistance of the temperature element to represent the temperature rise of the whole thermistor. In practice the heat transfers very quickly to the white coating and the thermistor is at a uniform temperature when the resistance measurement is made. (Using the same equipment the time constant for heat transfer between the resistance material and the white coating was measured as 0.04 second.) The accuracy of the thermal capacity measurements are limited due to heat loss, the accuracy of time and voltage measurements on the storage oscilloscope, and uneven heat distribution at the ends of the thermistor, and these values are not considered to be better than 10 percent.

6. THEORETICAL VARIATION OF RESPONSE WITH HEIGHT

Table 1 shows the theoretical response obtained using equation (12). The values of thermal capacity were taken from Fig. 5 for temperatures which correspond to those found in the experimental flight. The ascent rates from the experimental flight were also used. The values of thermal conductivity of air, coefficient of viscosity and density were taken from the U.S. Standard Atmosphere (1964). Experimental values of response obtained from the curve in Fig. 3 are included in Table 1 for comparison.
Table 1. Experimental and Theoretical Thermistor Responses for Radiosonde test flight

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Velocity (m/s)</th>
<th>Air Temp. (°C)</th>
<th>Thermal Cap. of Thermistor (kcal/°C)</th>
<th>Theoretical Response (sec)</th>
<th>Experimental Response (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5</td>
<td>+22</td>
<td>$21 \times 10^{-6}$</td>
<td>3.2</td>
<td>3.7</td>
</tr>
<tr>
<td>5</td>
<td>4.4</td>
<td>-10</td>
<td>$19 \times 10^{-6}$</td>
<td>3.0</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>5.5</td>
<td>-48</td>
<td>$18 \times 10^{-6}$</td>
<td>3.6</td>
<td>4.8</td>
</tr>
<tr>
<td>15</td>
<td>6.6</td>
<td>-61</td>
<td>$18 \times 10^{-6}$</td>
<td>4.7</td>
<td>6.4</td>
</tr>
<tr>
<td>20</td>
<td>7.2</td>
<td>-58</td>
<td>$18 \times 10^{-6}$</td>
<td>6.6</td>
<td>8.0</td>
</tr>
<tr>
<td>25</td>
<td>7.7</td>
<td>-50</td>
<td>$18 \times 10^{-6}$</td>
<td>8.9</td>
<td>10.1</td>
</tr>
<tr>
<td>30</td>
<td>9.0</td>
<td>-42</td>
<td>$18 \times 10^{-6}$</td>
<td>11.2</td>
<td>12.9</td>
</tr>
<tr>
<td>35</td>
<td>9.2</td>
<td>-18</td>
<td>$18 \times 10^{-6}$</td>
<td>15.3</td>
<td>18.1</td>
</tr>
</tbody>
</table>

7. DISCUSSION

The experimental response times are generally about 15 percent longer than those predicted by theory. This difference is probably due mainly to inaccuracy in the determination of the thermal capacity; however, errors in the experimental response and the fact that the flight was not carried out in a standard atmosphere are possibly contributing factors. The scatter in the experimental results (see Fig. 3) is due to the fact that the temperature is not always varying at a constant rate during the time the response measurement is made. This problem could be reduced by switching a second thermistor into circuit, during the period that the thermistor is being heated, to give a more accurate indication of the rate of change of air temperature.

Response times can be determined from equation (12) with sufficient accuracy for most practical purposes. Table 2 gives the response times at various ascent rates and heights, based on the measured values of thermal capacity (see Fig. 5) and data from the U.S. Standard Atmosphere. If the values of response are required to an accuracy of better than 15 percent, then it would be necessary to determine the thermal capacity more accurately.
Fig. 4 Equipment set up for measurement of thermal capacity.

Fig. 5 Thermal capacity of Australian radiosonde thermistor.
Fig. 6 Response of radiosonde thermistor at an inversion
Table 2. Theoretical response times in seconds for various heights and ascent rates

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Ascent rates (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2.9</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>10</td>
<td>4.7</td>
</tr>
<tr>
<td>15</td>
<td>6.7</td>
</tr>
<tr>
<td>20</td>
<td>9.4</td>
</tr>
<tr>
<td>25</td>
<td>13.7</td>
</tr>
<tr>
<td>30</td>
<td>17.1</td>
</tr>
<tr>
<td>35</td>
<td>21.5</td>
</tr>
</tbody>
</table>

8. ERRORS DUE TO THERMISTOR RESPONSE

If the air temperature is changing at a constant rate, $\beta$ degrees per second, Middleton and Spilhause (1953) show that after a long time (at least three times the response) the thermistor temperature lags behind the air temperature by $\beta \lambda$ degrees. Typical values of lag are shown in Table 3.

Table 3. Lag in °C for an ascent rate of 6m/s

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Response (sec)</th>
<th>Lapse rate in °C/km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>2.1</td>
<td>.01</td>
</tr>
<tr>
<td>20</td>
<td>7.0</td>
<td>.04</td>
</tr>
<tr>
<td>35</td>
<td>17.1</td>
<td>.20</td>
</tr>
</tbody>
</table>

The indicated temperature at an inversion can be found from equation (4). Fig. 6 shows an inversion of 5°C in 200 feet and the corresponding indicated temperatures caused by the response at heights of 0, 20 and 35 km.

Due to response the measured temperature below the tropopause will be slightly higher than the true temperature, and above the tropopause slightly lower than the true temperature. The resultant errors in the determination of pressure height levels can be determined by substituting the temperature lag errors in the
following relationship given by the U.S. Air Weather Service (1955):

\[ \Delta h = c \left( \ln \frac{P_1}{P_2} \right) \frac{\bar{T}}{T} \]

where \( c \) is a constant (29.29 for \( h \) in meters)

\( P_1 = \) the pressure at the lower boundary of the layer

\( P_2 = \) the pressure at the upper boundary of the layer

\( \bar{T} = \) the mean temperature error over the layer due to lag

\( \Delta h = \) the error in thickness over the layer.

The thickness errors \( \Delta h \) due to response for various layers in a U.S. Standard Atmosphere (1964) are shown in Table 4. The height error at the top of each layer is the algebraic sum of the thickness errors for all the layers below.

<table>
<thead>
<tr>
<th>Layer (mb)</th>
<th>Average Response (sec)</th>
<th>Average Velocity (m/s)</th>
<th>Temperature Lag (°C)</th>
<th>Thickness Error (meters)</th>
<th>Height Error (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000-700</td>
<td>3.4</td>
<td>4</td>
<td>0.085</td>
<td>-0.93</td>
<td>-0.93</td>
</tr>
<tr>
<td>700-500</td>
<td>3.2</td>
<td>4</td>
<td>0.101</td>
<td>-0.98</td>
<td>-1.91</td>
</tr>
<tr>
<td>500-300</td>
<td>3.4</td>
<td>5</td>
<td>0.110</td>
<td>-1.67</td>
<td>-3.58</td>
</tr>
<tr>
<td>300-200</td>
<td>3.6</td>
<td>5</td>
<td>0.080</td>
<td>-0.93</td>
<td>-4.51</td>
</tr>
<tr>
<td>200-100</td>
<td>4.2</td>
<td>6</td>
<td>0.000</td>
<td>0.00</td>
<td>-4.51</td>
</tr>
<tr>
<td>100-50</td>
<td>6.2</td>
<td>6</td>
<td>0.005</td>
<td>+0.13</td>
<td>-4.38</td>
</tr>
<tr>
<td>50-25</td>
<td>8.0</td>
<td>7</td>
<td>0.055</td>
<td>+1.12</td>
<td>-3.26</td>
</tr>
<tr>
<td>25-10</td>
<td>10.1</td>
<td>8</td>
<td>0.084</td>
<td>+2.25</td>
<td>-1.01</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT

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REFERENCES


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