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RAINFALL CORRELATIONS IN THE PASTORAL ZONE OF EASTERN AUSTRALIA

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ABSTRACT

In studying the economics of the spatial diversification of pastoral holdings in eastern Australia, it was found necessary to quantify the similarity between the climates of stations separated by varying distances. This was done by concentrating on the most important climatic element, namely rainfall.

Simple correlations between the square roots of monthly and yearly rainfalls for 42 stations in eastern Australia were related to distance from various master stations through regression analysis. It was found that correlation decreases at a diminishing rate with distance from master station. The rate of decrease depends on direction, being generally most rapid in an E-W direction.

1. INTRODUCTION

Other things being equal, the closer two geographical points are together the higher will be the correlation between their climates. In the pastoral zone of eastern Australia*, the most important climatic variable is rainfall and this is the factor which is primarily responsible for the extreme variability of production and income in this zone.

Rainfall correlation appears to be a neglected field of study in Australia, which is surprising since it is an important aspect of the climate of most regions. This is particularly so in the pastoral zone where the influence of drought is so pervasive. Casual observation suggests that droughts are seldom highly localized but, on the other hand, only rarely affect a large proportion of the pastoral zone at the one time.

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* The pastoral zone of eastern Australia is defined here as those areas of Queensland and New South Wales which receive less than 20 inches of rain on average. The most important industry in the zone is wool production.

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In the present instance, attention is focussed on quantifying the nature of correlation effects because of their potential role in contributing to the benefits of running spatially diversified chains of sheep properties in the pastoral zone. The situation of some establishments in a chain enjoying favourable seasons whilst others are subject to poor or disastrous seasons and thus providing scope for stock movements between establishments, is most readily studied through an analysis of rainfall correlations. Such correlations become important in investigating the benefits of spatial diversification when the analytical approach adopted (Anderson, 1970) involves synthesizing random sequences of rainfall for several stations.

2. THE DATA

Two sets of data were employed in this analysis. In a pilot study relating to 11 stations* in the Queensland pastoral zone, monthly and yearly rainfall data for the 40 year period 1921-1960 were used to compute simple correlations between pairs of stations for specified periods. At this stage no special computer program had been written and the computations involved considerable handling of punch cards so that only the limited number of years and stations were taken.

However, there were sufficient data to indicate the general pattern of correlations from the interpolated isopleths. By taking each station in turn as master station it was ascertained that the general pattern of the isopleths running roughly from N to NW was rather persistent.

Although the rainfall distributions clearly are not normal, this was not regarded as a cause for concern as, in the present context, the specification of the probability distribution only becomes important when significance testing is contemplated. However, in the light of subsequent developments, some of the correlations were recomputed after a square-root transformation of the data. It was ascertained that this had a slight effect of reducing most correlations while leaving the isopleth patterns essentially unchanged.

Having proceeded thus far, it was planned to increase the number of stations throughout the pastoral zone so that quantitative relationships could be established. However, it was discovered through an address by Maher (1969) that comprehensive rainfall correlations for 100 stations across the continent had already been computed in connection with a study of the climatic homogeneity of weather forecasting districts.

The Bureau of Meteorology kindly made these correlation data available to the author. The rainfall data were the majority of those used in a study by Gibbs and Maher (1967) of rainfall deciles as drought indicators and relate to the 65 year period 1900-1964. However, for the correlation analysis all data were square-root transformed in an attempt to normalize the series. This was fairly successful for the annual totals but not so for the monthly totals.

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* The stations were Hughenden, Tangoran, Muttaburra, Aramac, Longreach, Isisford, Blackall, Charleville, Cunnamulla, Bollon and Dirranbandi.

Correlation data were assembled for 42 recording stations in and around the pastoral zone of eastern Australia. In a manner similar to that used in the pilot study, isopleths were sketched by hand but this approach was soon dismissed. The task proved very tedious (say, 4 isopleths times 12 master stations times 13 periods) and cumbersome to report. In retrospect, the methods discussed by Maine, Hincksman and Seaman (1967) for graphic display by computer certainly would have reduced the tedium, but in the absence of such facilities, alternative methods were sought to simplify the analysis.

However, the important points to emerge from the preliminary inspection and plots were that

(a) no negative correlations were found,

(b) for correlations greater than about 0.2, the isopleths tended to be elliptical although somewhat irregular,

(c) widely scattered stations exhibited corresponding isopleths of broadly similar size and disposition,

(d) again, the longest axis of the isopleths tended to N to NW.

To provide a more readily comprehensible analysis of these data and interim conclusions, a more compact and quantitative analytical procedure seemed desirable. This was tackled firstly by concentrating on the effect of distance alone, and secondly by examining the over-riding influence of direction on distance effects.

3. CORRELATION AND DISTANCE

Quantification of the effect of distance from master station on correlation was regarded as an important step for numerical specification in simulation models of spatially diversified enterprises in the pastoral zone. Inspection of isopleths revealed that correlation decreased at a diminishing rate with increasing distance.

Many functional forms could be used to describe such a relationship over a restricted range of distance, but the simplest - a quadratic polynomial - was selected in the first instance, namely $C = b_o + b_1D + b_2D^2$; where C denotes correlation and D distance from master station in hundreds of miles. As the stations were chosen originally to provide a fairly even spread over the entire continent, the distance between stations generally exceeded 100 miles. Since it was desired to interpolate predictions for distances of less than 100 miles, it was considered justifiable to introduce some additional information, i.e. a constraint, in the fitting. This was the constraint that at $D = 0$, predicted correlation must be unity. Introducing such a perfectly known coefficient, $b_o = 1$, has the effect of tightening-up the confidence bands about the remaining unknown coefficients and is equivalent to fitting the new dependent variable $(C - 1)$ and 'forcing the equation through the origin', as $(C - 1) = b_1D + b_2D^2$.

On the basis of significance tests on the overall regressions and the individual coefficients, and inspections of residuals from the regressions, the fitted quadratic equations proved fairly adequate. For instance, in 242 of 269 cases, the fitted quadratic explained more than 95 percent of the variation. The quadratics proved at least as good as the alternatively transformed second-degree polynomials which were fitted. Regression equations for a cross-section of rainfall stations and for rainfall recorded by month and calendar year are summarized in Table 1.

Table 1. Quadratic equations for the rainfall[1] correlation (C) predicted for varying distance (D)[2] from specified stations

$$C = 1.0 + b_1 D + b_2 D^2$$

Period	Cloncurry b_1	Cloncurry b_2	Aramac b_1	Aramac b_2	Charleville b_1	Charleville b_2	Bourke b_1	Bourke b_2	Coonamble b_1	Coonamble b_2	Menindee b_1	Menindee b_2	Condobolin b_1	Condobolin b_2
	Master Station													
Jan	-.2105[3]	.0135	-.2464	.0231	-.2897	.0316	-.2605	.0239	-.2698	.0250	-.2129	.0177	-.2447	.0202
Feb	-.1768	.0083	-.2268	.0179	-.2158	.0160	-.2292	.0175	-.2498	.0201	-.2374	.0151	-.2243	.0148
Mar	-.1891	.0110	-.1953	.0144	-.2267	.0215	-.2090	.0164	-.2365	.0168	-.1702	.0102	-.2275	.0171
Apr	-.2276	.0146	-.2206	.0143	-.2515	.0200	-.2410	.0178	-.2810	.0226	-.1935	.0112	-.2391	.0181
May	-.1777	.0092	-.1662	.0085**	-.1741	.0104**	-.1709	.0105	-.1972	.0133	-.1875	.0124	-.1914	.0118
Jun	-.1726	.0080	-.1630	.0106	-.1175	<u>.0009</u>	-.2066	.0146	-.2052	.0157	-.1908	.0118	-.2100	.0137
Jul	-.1615	.0086	-.1260	.0053x	-.1251	.0007	-.2293	.0238	-.2512	.0235	-.2106	.0117	-.2823	.0222
Aug	-.1698	.0085	-.1948	.0098	-.1906	.0126**	-.2249	.0187	-.2290	.0160	-.2208	.0153	-.2327	.0167
Sep	-.1573	.0086	-.1354	.0050x	-.1493	.0089	-.2263	.0195	-.2089	.0179	-.1749	.0106	-.2294	.0197
Oct	-.1708	.0100	-.1789	.0103**	-.1746	.0129**	-.2288	.0193	-.2298	.0200	-.2221	.0183	-.2202	.0163
Nov	-.1868	.0113	-.2091	.0186	-.2419	.0233	-.2252	.0207	-.2194	.0182	-.2316	.0195	-.1834	.0130
Dec	-.1977	.0111	-.2500	.0202	-.2402	.0209	-.2734	.0248	-.3179	.0280	-.2644	.0182	-.2517	.0174
Year	-.1507	.0070	-.1536	.0098	-.1498	.0113**	-.1625	.0101	-.1783	.0129	-.1654	.0101	-.1742	.0115

(1) Rainfalls have been square-root transformed before correlations were estimated.

(2) D is measured in 100's of miles

(3) Unless otherwise specified, coefficients are significantly different from zero at less than .001% probability. Otherwise,
** denotes .011 ⩽ P% < 1, * denotes 1 ⩽ P% < 5,
x denotes 5 ⩽ P% < 15 and underline denotes non-significance.

Most of the estimated coefficients are highly significantly different from zero - the exceptions being the quadratic terms for some relatively dry months at Aramac and Charleville. The trend most readily apparent from Table 1 is the confirmation that correlation decreases at a diminishing rate over the range of observations. There is a suggestion that the rate of decrease is greatest in the months of highest rainfall for any particular station, but this is not well defined.

4. CORRELATION AND DIRECTION

The observation that most hand-plotted isopleths tended to be elliptical suggested a convenient method for determining the direction in which correlation decreases most rapidly. If a second-degree equation in two variables (distance E-W and distance N-S) is fitted to correlation data for a given master station and period, for any given correlation this may define an elliptical isopleth. Intuitively, this will occur when the equation describes a regular dome-shaped surface and this surface is cut by a horizontal plane. More formally, for the quadratic surface $C = c_o + c_1N + c_2E + c_3N^2 + c_4E^2 + c_5NE$, where N now denotes hundreds of miles north (positive) or south (negative) and E now denotes hundreds of miles east (positive) or west (negative), let $C = C^*$, a fixed level of correlation, and rearrange as the typical quadratic equation,

$$c_3N^2 + c_5NE + c_4E^2 + c_1N + c_2E + c_o - C^* = 0.$$

Then the curve is an ellipse if the discriminant is negative, i.e., $c_5^2 - 4c_3c_4 < 0$. The directions of the major and minor axes are found by rotating the axes through an angle α determined from $\cot 2\alpha = (c_3 - c_4)/c_5$ (Thomas, 1968, pp. 350-352).

It was found that the second-degree equation provided a reasonable fit in terms of explaining variation in these data although the fits were not as close as for the previous regressions. However, 265 of the 269 regressions fitted explained more than 50 percent of variation. For each master station the new coordinates for the other stations were computed from the respective paired latitudes and longitudes. In fitting the regression equations, c_o was this time not constrained to unity as there was no interest in extrapolating beyond the range of observations. Choice of isopleth is arbitrary but $C^* = .5$ was chosen for the present discussion. An ellipse is conveniently summarized by the length and direction of its axes. These were found from the fitted equations by computing the angle α and letting the axes cut the ellipse. Axes were found as straight lines with slopes $\tan \alpha$ and $\tan (\alpha + 90)$ which pass through the centre of the ellipse.

This procedure enables rapid assessment of the general influence of direction on the decline of correlation with distance. In most cases it worked fairly well as, for example, is illustrated in Fig. 1 which compares a hand-sketched isopleth with a computed elliptical isopleth.

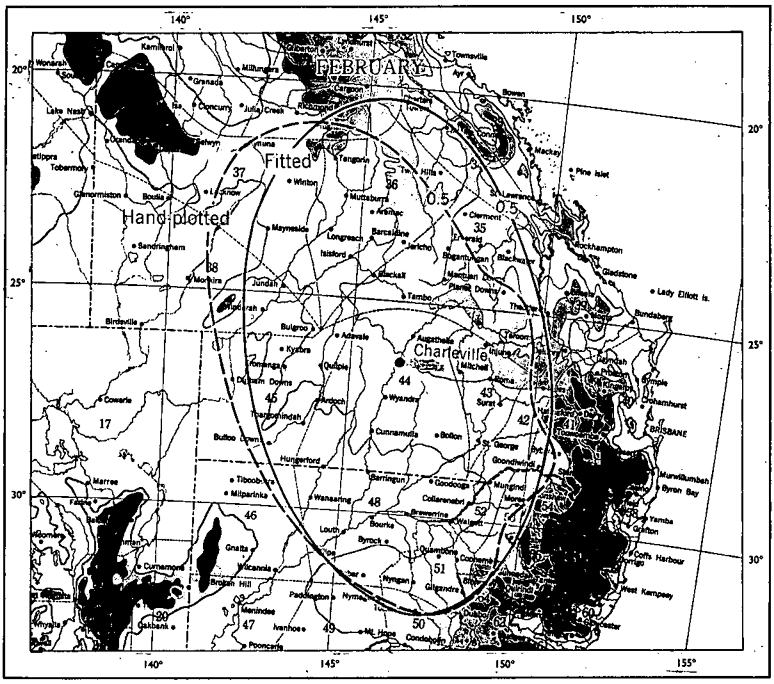

Fig. 1 Comparison of hand-plotted and fitted isopleths.

The ellipses computed for five representative master stations are summarized in Table 2 by the direction of the minor axis and the half lengths of minor and major axes. These data allow a direct interpretation, e. g. for Charleville in February the closest point towards the Pacific Ocean with a predicted correlation of .5 lies about 258 miles away at 84^{O}. However, along the major axis (174^{O} or 354^{O}) this same correlation is reached at about 427 miles distance.

The data of Table 2 reinforce the earlier observations on the configuration of the isopleths. Generally, correlation diminishes most rapidly in the E to ENE and W to WSW directions. Maher (1969) has speculated that this is connected with the usual tracks of moving pressure systems in the Australian region. An exception in Table 2 in Coonamble. For half the months the minor axis runs closer to N-S than E-W. However, in these months the ellipses tend to be circular implying that direction is then unimportant.

Table 2. Summaries of elliptical isopleths of 0.5 correlation of square-root rainfall for specified stations

Period	Master Station														
	Aramac			Charleville			Bourke			Coonamble			Condobolin		
	Dir.(1)	Min.(2)	Maj.(3)	Dir.	Min.	Maj.	Dir.	Min.	Maj.	Dir.	Min.	Maj.	Dir.	Min.	Maj.
Jan	-(4)	-	-	-	-	-	80	220	317	154	277	327	-	-	-
Feb	88	320	754	84	258	427	80	257	384	74	280	356	78	615	1216
Mar	-	-	-	-	-	-	83	332	483	167	319	321	77	332	568
Apr	84	431	954	82	202	379	83	220	360	71	251	289	78	298	516
May	156	342	418	84	324	569	84	316	524	79	289	495	76	620	1041
Jun	77	436	555	77	423	485	70	321	338	72	367	407	78	248	378
Jul	143	359	495	70	410	442	74	359	436	157	343	365	-	-	-
Aug	69	330	476	87	300	464	71	347	370	155	327	346	74	286	391
Sep	66	425	838	89	396	783	76	286	387	76	333	474	80	279	583
Oct	86	323	550	85	334	537	79	258	424	77	316	436	78	286	429
Nov	63	464	967	72	248	289	156	328	343	154	353	399	72	434	564
Dec	69	228	248	75	199	261	76	230	263	155	186	200	83	344	702
Year	85	416	737	80	387	655	79	354	486	78	344	503	90	299	764

(1) Direction of minor axis, degress east of north

(2) Half-length of minor axis, miles.

(3) Half-length of major axis, miles.

(4) Dash indicates that the fitted second-degree equation did not result in an ellipse.

5. SUMMARY

An aspect of the climate of the pastoral zone which is of particular importance in studying the economics of spatial diversification is the manner in which similarity of climate varies between spatially separated stations. The foregoing analysis has concentrated on simple correlations of monthly and annual rainfall - the former being especially useful in simulation models of diversified production. Other features of rainfall which must be considered along with correlations are, of course, the annual total and the seasonal distribution, both of which vary substantially through the pastoral zone.

Correlation of rainfall in the pastoral zone of eastern Australia decreases at a diminishing rate with distance from a master station. While this holds generally, there is usually an additional influence of direction on the rate of decrease. Very broadly, correlation decreases most rapidly in an E-W direction - a finding which is relevant for pastoralists contemplating spatial diversification as a drought mitigating strategy. Close analysis of any particular case of spatial diversification would probably have to make use of hand-plotted isopleths based on appropriately chosen recording stations which may reveal exploitable local irregularities in correlations.

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