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CIRCULAR ECHOES OBSERVED ON SHIPS' RADAR

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Anomalous radar echoes near Cape Schanck on 19 January 1973 have been reported from the Australian observing ship *Cathay*. Two echoes, one roughly circular at about 11 km radius around the ship and a concentric arc about 27 km were observed; a small craft near one of them returned an intense echo, and visually the sea horizon was undulating, shimmering and banded. One of the observations is sketched in Fig 1; later the inner echo become completely closed but its asymmetry - nearest on the port (coastward) side - persisted. Circular echoes on ships' radar have been reported before, but *Cathay's* echo detail and concurrent temperature and wind reports prompted analysis.

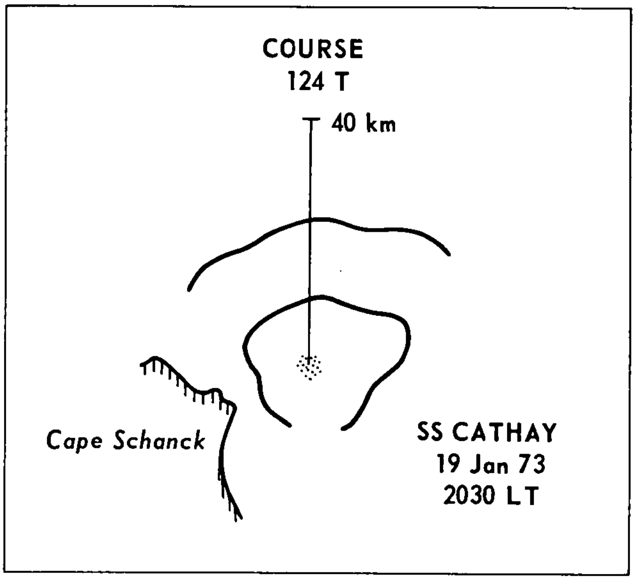

Fig 1

These observations appear consistent with super-refraction in a layer at about the height of the scanner and propagation in hops. The energy back-scattered from other than rough seas is minute, but detection beyond a few miles would be possible if there is significant focusing.

Consider the following model. Assume a strongly refracting layer with base at height H within which rays follow a path of constant curvature. Below the layer, curvature is much less, and for simplicity assumed equal to earth curvature. Analysis is facilitated by considering the earth flat. For the case where the layer, base height H, is above the scanner, height h, we can say, with negligible error, the distance to the far echo

$$S' = 4H/a - h/a + 4ra,$$

using the symbols of Fig 2. For focusing to occur, S' would be stationary with respect to a, that is

$$dS'/da = -4H/a^2 + h/a^2 + 4r$$
$$= 0.$$

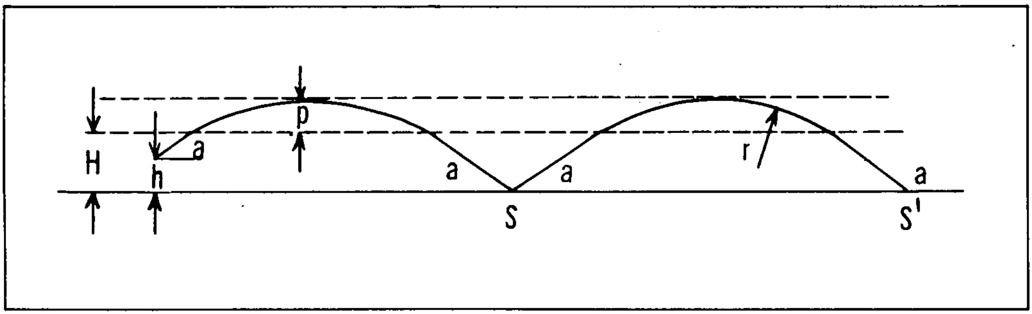

Fig 2

Solution yields

$$r = S'^2/16(4H - h), \text{ and}$$
$$a = 2(4H - h)/S'.$$

The penetration of the ray into the layer

$$p = ra^2/2$$
$$= (4H - h)/8,$$

and this is the minimum possible thickness of the layer.

The asymmetry of the echo is likely due to the layer varying slightly in height. The approximate gradient of the base of the layer

$$b = a(S_s - S_c)/(S_s + S_c),$$

where S_s and S_c are the distances to the further (seaward) and nearer (coastward) part of the inner echo. (Observed values suggest that the height of the layer would be increased at a little over one in one thousand, seaward.)

The actual curvature of the ray, $1/r'$, is greater than 'flat earth' curvature:

$$1/r' = 1/r + 1/R,$$

where R is earth radius. For almost horizontal rays the curvature is related to the refractive index n, and its vertical gradient by

$$1/r' = 1/n \cdot dn/dh .$$

The index n is close to unity, about 1.0003 at the surface. Refractivity is usually expressed in N units ($N = (n - 1)10^6$), so the refractivity gradient is (minus) $10/r'$ N units per metre, approximately. Refractivity is a function of pressure, temperature and water vapour content; for the present purposes dry and wet bulb temperatures provide a suitable measure.

An estimate of echo enhancement may also be made. The distance to the inner echo,

$$S = (H - h)/a + 2ra + H/a.$$

Its derivative, dS/da, defines the small angle over which rays intercept a small element ΔS at S:

$$\Delta a = \Delta S \cdot 8(4H - h)^2/hS'^2.$$

If the same element is illuminated by unrefracted rays, the angle subtended by ΔS at the radar scanner,

$$\Delta a_0 = h\Delta S/S^2,$$

so that the 'angular' gain enhancement due to refraction

$$(\Delta a/\Delta a_0)_0 = 8S^2(4H - h)^2/S'^2h^2 .$$

At S a similar consideration of the back-scattered energy intercepted by the scanner leads to

$$(\Delta a/\Delta a_0)_S = (8H - 3h)/h .$$

For there-and-back transmission, the radar case, the enhancement

$$\begin{aligned} G &= (\Delta a/\Delta a_0)^2_0 \,.\, (\Delta a/\Delta a_0)_S \\ &= 64 \,.\, (S/S')^4 \,.\, (4H - h)/h^4 \,.\, (8H - 3h)/h . \end{aligned}$$

Any similar estimate for the distant echo would have to include assumptions about the reflection at the sea surface.

For the case $H<h$,

$$S' = r(a^2 - 2h/r + 2H/r)^{\frac{1}{2}} + 3ar + 3H/a \text{ , and}$$

$$dS'/da = ra(a^2 - 2h/r + 2H/r)^{-\frac{1}{2}} + 3r - 3H/a^2 .$$

This is not amenable to general solution, but some particular cases are of interest. When $H = 0$, $dS'/da \neq 0$, and no focusing occurs. The first real solution occurs when $a^2 = 2h/3r$, and $H = 22h/27$, and values for curvature, N gradient, *etc*, can be calculated.

Setting in a typical value of S', 26,000 m, known h, 26 m, and assuming refractivity at the bottom of the layer is close to that indicated by ship temperatures (at 19 m) - dry bulb 24.9°C/wet 20.7°C = 349.6 N units - the following values are found:

Layer base	H	22h/27	h	3h/2	2h
Rel. distance	S/S'	0.36	0.42	0.45	0.47
Enhancement dB	G	2	29	42	50
Thickness m	p	17	10	17	23
Gradient	N/m	-1.8	-2.0	-3.2	-4.5
Refractivity at top of layer	N	319.0	329.6	295.6	245.6

Inspection of this table strongly suggests that the bottom of the layer would be at or just above scanner height: S/S' matches the observations, significant echo enhancement rapidly vanishes when the layer is below scanner height, and if the layer is much above the scanner the refractivity corresponds to quite unrealistic regimes, for example 245.6 N units, H = 2h case, demands dry air at 43°C, or higher temperatures. The actual temperatures at the top of the layer cannot be deduced from the ship observations. Other reports indicate that at the time of the observation the surface air blowing off shore would be about 35°C/23°C (325 N units), and, it seems, some wasarriving at the top of the layer virtually unmodified.

Two other points may perhaps be mentioned. The echo enhancement, about 30 dB, is not unrealistic: power back-scattered from the sea is approximately proportional to the inverse cube of the range, so that in the absence of focusing sea echoes would be seen to at least 1 km. Some sea clutter was indicated to about 2 km The echoes from ships or other objects near the sea echoes would, of course, be unusually intense. Similar refractivity values hold for optical frequencies, but second hop visual images seem unlikely. A band of differing brightness, corresponding to the sea echo, could be expected at an elevation comparable to the radar ray, that is at a little under half a degree above the horizon. Between the horizon and this band it is possible that trans-horizon objects could be seen. Shimmer and roughness are likely since stationary or perfectly smooth stratification of temperatures is most improbable.

The picture emerging is that of a hot off-shore breeze flowing over cooled, mixed and slightly moistened air, of increasing depth seaward, with a very sharp transition between. This situation is probably not uncommon. The radar (and visual) observations, rather rarer, appear to be critically dependent on the depth of the mixed air being close to the radar scanner (and observer) height. The analysis is perhaps trivial, but illustrates, by way of a sample, a phenomena important to microwave link engineering with propagation paths over the sea.