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THE EFFICIENCY OF CONSERVATION OF ABSOLUTE ANGULAR MOMENTUM IN THE AUSTRALIAN TROPICAL TROPOSPHERE

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ABSTRACT

The mean zonal wind variation with latitude in the Australian high tropical troposphere is consistent with partial conservation of absolute angular momentum requirements. The efficiency of this conservation is discussed in terms of large and small-scale motions. Australian tropical monthly mean efficiency indices are derived for the period 1961 to 1971. Winter is characterised by high efficiency indices and summer by significantly lower values.

INTRODUCTION

Whilst much research has been devoted to parameterisation of the westerly winds of the temperate latitudes, little progress has been made with the tropical wind fields. The distribution of tropical Australian upper wind reporting stations is relatively dense, at least poleward from 10°S. Monthly mean data from these stations affords the opportunity to investigate the extent to which upper tropospheric winds in the Australian tropical region conserve absolute angular momentum.

The efficiency of conservation of absolute angular momentum provides a useful parameter to describe the mean flow aloft. However, the relationship of the efficiency to broadscale weather features and its operational value remain untested. For the purpose of this paper it is convenient to define the Australian tropical region as that area lying equatorward of the 80 percent frequency enclosure of the position of the sub-tropical jetstream core (Weinert, 1968).

THEORETICAL CONSIDERATIONS

The absolute angular momentum (G) of a unit parcel of the atmosphere at latitude ϕ is, for practical purposes, defined by the equation

$$G_{\phi} = (wa \cos\phi + u_{\phi})\, a \cos\phi \qquad \ldots 1$$

where w is the earth's angular velocity and a its radius while u_{ϕ} is the zonal velocity of the parcel relative to the earth.

At the equator

$$G_{o} = (wa + u_{o})a \qquad \ldots 2$$

which can be written

$$G_o = (1 + \frac{u_o}{wa}) wa^2 \qquad \ldots 3$$

Monthly mean values of u_o are small (Ramage, 1969) and it seems reasonable to assume that it rarely exceeds 25 ms^{-1}. Since wa = 460 ms^{-1} then $\frac{u_o}{wa} << 1$, it is permissible to neglect $\frac{u_o}{wa}$.

It follows that a parcel moving from the equator to latitude ϕ under constant absolute angular momentum conditions will have a zonal velocity given by

$$u_\phi = wa(\sec\phi - \cos\phi) \qquad \ldots 4$$

Hence the zonal velocity of an air parcel should conform to eq 4 provided that

- the parcel has crossed the equator with a small relative zonal velocity
- absolute angular momentum is conserved.

Within the troposphere these conditions are most likely to be fulfilled in high level tropical circulation beyond the effect of the earth's frictional torque. Fig 1, adapted from Riehl (1954, 1969), shows results which seem to recur in the high tropical troposphere, namely

- absolute angular momentum is not conserved
- there is a positive linear correlation between mean values of u_ϕ and ($\sec\phi - \cos\phi$).

Palmén (Palmén and Newton, 1969) has interpreted these results as an indication that the upper tropospheric tropical circulation conforms to a partial conservation of absolute angular momentum.

Because of the linear relationship between u_ϕ and ($\sec\phi - \cos\phi$) it is convenient to define an efficiency index (E) such that

$$E = [\text{slope of the linear regression equation between } u_\phi \text{ and } (\sec\phi - \cos\phi)] / wa \qquad \ldots 5$$

It is clear from eq 4 that if absolute angular momentum is fully conserved then E equals one. E represents the efficiency of conservation of angular momentum necessary to reproduce the observed zonal wind distribution which would be given by an equation of the form

$$u_\phi = Eaw(\sec\phi - \cos\phi) \qquad \ldots 6$$

Krishnamurti (1971) collected 200 mb wind data for June, July and August 1967 between 45°N and 45°S using synoptic and aircraft reports. His values of u_ϕ averaged around latitude circles and plotted against ($\sec\phi - \cos\phi$) are shown in Fig 2. Between 5 and 20 degrees a marked linearity is evident in both hemispheres. Correlation coefficients were significant at the 99% level.

The northern hemisphere summer efficiency index was 0.17 corresponding to an equation

$$u_\phi = 76(\sec\phi - \cos\phi) - 10 \; ms^{-1} \qquad \ldots 7$$

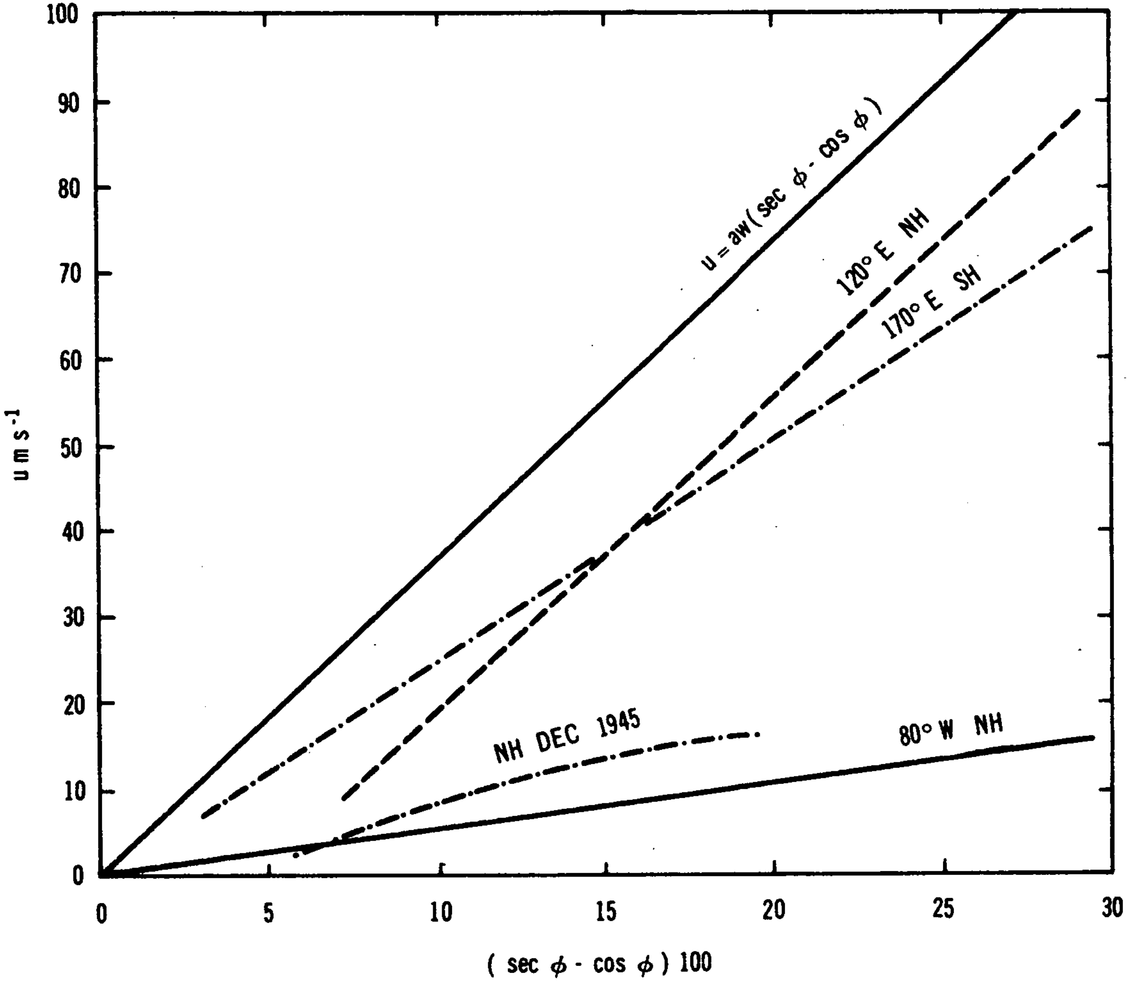

Fig 1 Profiles of west wind component in the high troposphere in winter for selected longitudes, (*after Riehl*).

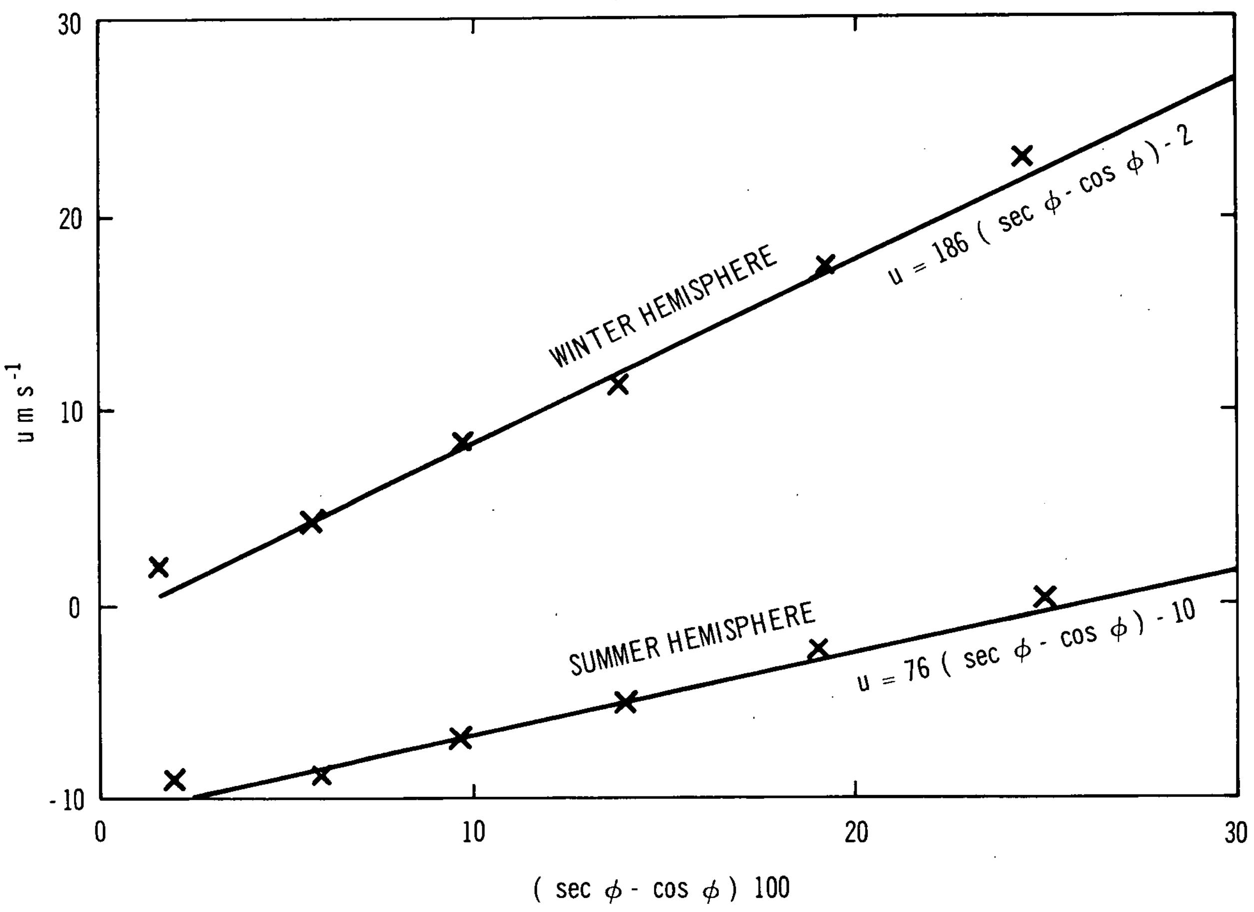

Fig 2 Mean 200 mb zonal wind profile between 20°S and 20°N for June, July and August, 1967.

In the southern hemisphere winter the efficiency index was 0.40 and the regression equation

$$u_\phi = 186(\sec\phi - \cos\phi) - 2 \text{ ms}^{-1} \qquad \ldots 8$$

The linearity obviously cannot be extended to the equator.

EFFICIENCY INDEX AND LARGE-SCALE MOTION

In the following arguments it will be assumed that in the high tropical tropospheric flow u>>v, where v is the meridional velocity component, and that longitudinal gradients are negligible compared to latitudinal gradients.

As the upper zonal flow can be represented by eq 6 then, since $\frac{\partial u}{\partial y} = \frac{1}{a}\frac{\partial u}{\partial \phi}$,

$$\frac{\partial u}{\partial y} = Ew(\sec\phi \tan\phi + \sin\phi) \qquad \ldots 9$$

The absolute angular vorticity Q_ϕ under the assumptions stated becomes

$$Q_\phi = 2w(1-E) \sin\phi \qquad \ldots 10$$

If the parcel has a horizontal cross-sectional area A_ϕ and is bounded by the tropopause (assumed horizontal) on its upper surface and a horizontal lower surface, then the time derivative of the absolute vorticity $(\dot{Q}_\phi)$ is given by

$$\dot{Q}_\phi = -\left(\frac{Q_\phi}{A_\phi}\right)\dot{A}_\phi - \left[\frac{\partial W}{\partial x}\cdot\frac{\partial v}{\partial z} - \frac{\partial W}{\partial y}\cdot\frac{\partial u}{\partial z}\right] - \left[\frac{\partial \alpha}{\partial x}\cdot\frac{\partial p}{\partial y} - \frac{\partial \alpha}{\partial y}\cdot\frac{\partial p}{\partial x}\right] + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right] \qquad \ldots 11$$

where W is the vertical velocity component, p pressure, α specific volume and F_x and F_y are the horizontal components in the x- y- directions respectively of the viscous and frictional forces.

In the upper tropical troposphere u assumes a maximum value and so $\frac{\partial u}{\partial z}$ is negligible and, neglecting longitudinal gradients, eq 11 reduces to

$$\dot{Q}_\phi = -\left(\frac{Q_\phi}{A_\phi}\right)\dot{A}_\phi - \frac{\partial F_x}{\partial y} \qquad \ldots 12$$

Evidence is presented below to suggest that F_x is proportional to E and, since E is independent of ϕ then $\frac{\partial F_x}{\partial y}$ is zero. Therefore conservation of potential vorticity holds and

$$A_\phi(1-E)2w \sin\phi = \text{constant} \qquad \ldots 13$$

As eq 13 is only valid over the range of validity of eq 6 it does not include the equator, as Fig 2 shows, and so the constant need not be zero.

It follows, from the presence of the tropopause at the upper boundary, that poleward flowing air will be convergent and subside, and that equatorward flow will be divergent and be accompanied by rising air in the layer below. The larger the value of E, the smaller will be this effect.

EFFICIENCY INDEX AND SMALL-SCALE MOTION

Since E is a measure of the depletion of the parcel's absolute angular momentum then, from Newton's second law of motion, it is also a measure of the external torques acting on the parcel that effect this depletion. The torques are caused by the longitudinal pressure gradient $\frac{\partial p}{\partial x}$ and the eddy frictional force F_x according to

$$\dot{G}_\phi = (-\alpha \frac{\partial p}{\partial x} + F_x) \text{ a } \cos\phi \qquad \ldots 14$$

Longitudinal gradients of pressure are usually very small in the tropics and their contribution will be neglected in accordance with previous assumptions. Hence F_x should be the main factor governing the value of E.

Since the presence of eddies also governs the value of the constancy (C) of the wind, where

C = (speed of vector mean wind)/(mean of scalar speeds) ...15
E and C should be related.

If eddies were absent C would equal 1 and, from eq 14, since F_x would be zero, G_ϕ would be constant and E should therefore be 1. Thus a positive correlation between E and C is predicted. The variation of C and E based on the monthly mean statistics at 40,000 ft (12,200 m) for the Australian tropical regions stations (Table 1) for the period 1961 - 1971 is shown in Fig 3. The regression equation relation E and C at 40,000 ft (12,200 m) for these stations is

$$E = 1.3C - 0.41 \qquad \ldots 16$$

with a sample correlation coefficient of 0.94 and a standard error of estimate of E of 0.002. Although the correlation is not proof that E is determined by F_x such an explanation is the simplest and most probable.

INTERACTION BETWEEN LARGE AND SMALL-SCALE MOTIONS

F_x may be considered as acting in two parts, F_{xy} which acts on the vertical faces of the parcel and F_{xz} which acts on the horizontal faces and is determined in part by vertical eddies. A large-scale poleward directed flow aloft has been shown to be accompanied by subsidence. This creates a subsidence inversion at lower levels and thereby protects the upper layers from vertical eddies. The contribution to F_x by F_{xz} should be minimised under these conditions and, from the considerations in the section on small-scale motion, a maximum value of E would therefore be expected. An equatorward flow would be accompanied by rising air below, thus enhancing vertical eddy development, increasing F_{xz} and F_x, hence decreasing the value of E.

An examination of monthly mean meridional components at 40,000 ft (12,200 m) for the Australian tropical stations equatorward of 20°S indicated a significant positive correlation (0.68) between monthly mean poleward wind and efficiency index. Equatorward of 15°S the sample correlation coefficient was 0.89 with the same sample size (12). The monthly mean variation of the poleward wind component for these stations is included in Fig 3. The available evidence therefore tends to support the interpretation of the efficiency index values presented so far.

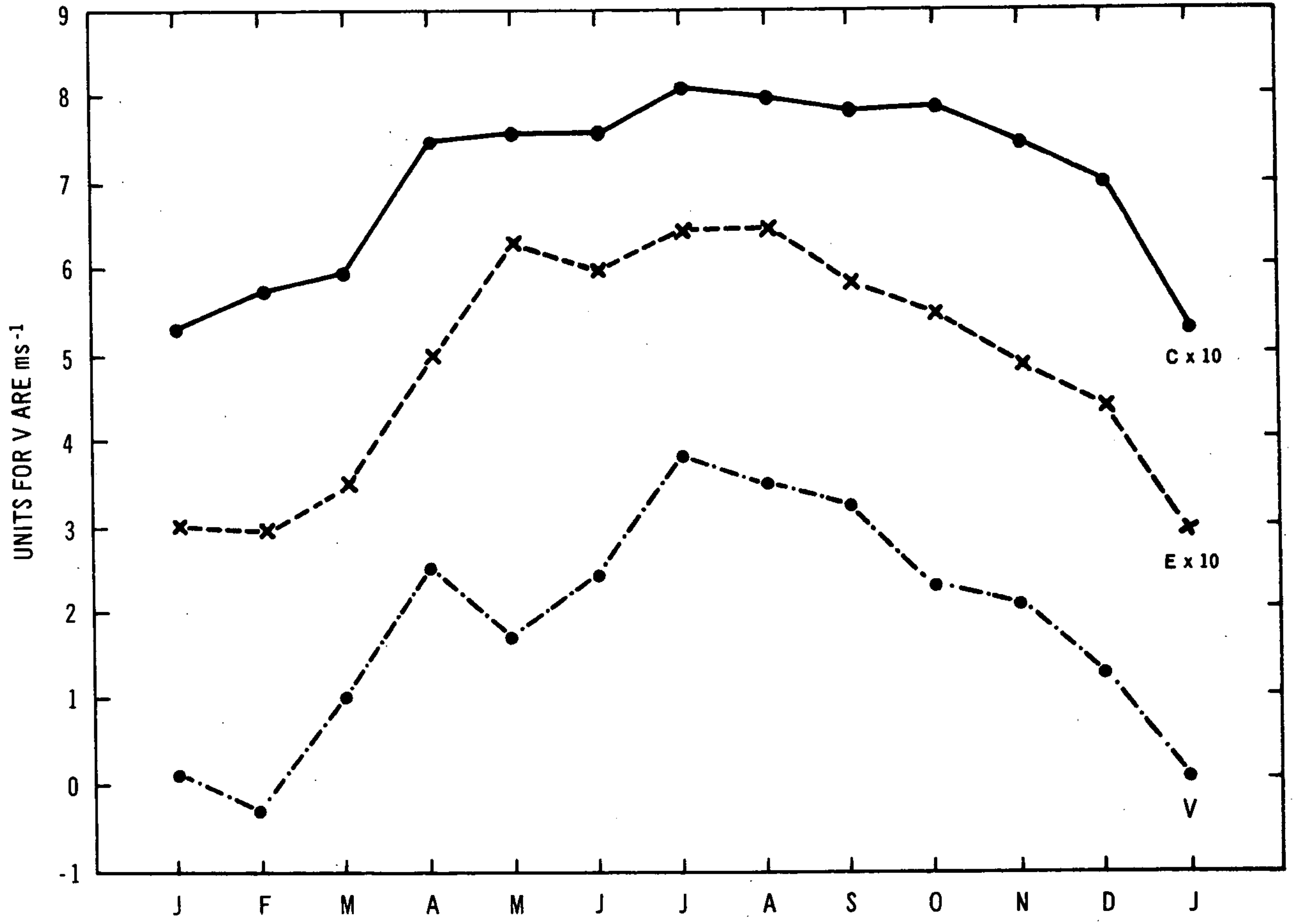

Fig 3 Monthly means of constancy of wind, efficiency index and pole ward wind component at 40,000 ft over the Australian tropics.

EFFICIENCY INDEX OVER THE AUSTRALIAN TROPICAL REGION

Data

Monthly mean values of the zonal wind components for the years 1961 to 1971 were used for the Australian tropical region stations (defined in the introduction). As the monthly mean latitude of the sub tropical jetstream has a slight monthly variation so does the number of tropical stations vary monthly. The number of observations at 40,000 ft (12,200 m) from which each tropical station monthly mean zonal wind was computed together with the number of monthly means available each month to calculate the monthly 40,000 ft (12,200 m) efficiency index is shown in Table 1.

Table 1 Sample sizes in computing 40,000 ft zonal means and the corresponding efficiency indices ('X' denotes that the station is not tropical according to text definition).

Station	Latitude °S	Longitude °E	Number of Observations at 40,000 ft level											
			Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Lae	06-44	147-00	266	261	265	282	285	275	278	280	271	247	244	248
Pt Moresby	09-26	147-13	123	127	144	143	155	149	151	134	129	132	144	139
Cocos Island	12-12	96-50	246	259	288	278	293	285	264	271	274	280	266	251
Darwin	12-26	130-50	266	257	304	291	306	297	308	273	295	293	251	294
Daly Waters	16-16	133-22	45	45	72	100	84	108	92	95	82	75	46	38
Willis Island	16-18	149-59	120	119	126	130	118	104	114	128	117	115	105	131
Cairns	16-55	145-44	277	264	288	278	287	267	263	253	262	290	280	257
Broome	17-57	122-13	108	65	175	88	205	87	225	207	196	198	191	187
Halls Creek	18-14	127-40	74	50	56	57	90	55	98	80	102	97	94	104
Townsville	19-16	146-46	271	275	304	297	302	266	305	268	290	301	285	264
Port Hedland	20-23	119-37	247	249	276	256	267	267	270	249	254	253	X	279
Cloncurry	20-40	140-30	253	261	280	288	291	275	297	287	287	288	X	248
Mackay	21-07	149-10	298	274	302	285	294	285	303	270	291	281	X	289
Onslow	21-40	115-07	75	63	72	X	X	31	55	26	49	33	X	5
Longreach	23-26	144-15	X	X	X	X	X	43	X	55	X	X	X	X
Alice Springs	23-48	133-53	253	265	X	X	X	X	301	297	X	290	272	282
Gladstone	23-51	151-16	X	X	X	X	X	X	291	297	X	X	X	X
Carnarvon	24-53	113-39	X	259	X	X	X	247	285	283	277	X	X	X
Tropical monthly means			15	16	14	13	13	16	17	18	15	15	11	15

Procedure and Results

For each month the parameter pairs, station mean 40,000 ft (12,200 m) zonal wind and the corresponding (sec ϕ - cos ϕ), were plotted and the linear regression line with (sec ϕ - cos ϕ) as the independent variable computed. The plotted data and regressions lines for January and July are shown in Fig 4. From the

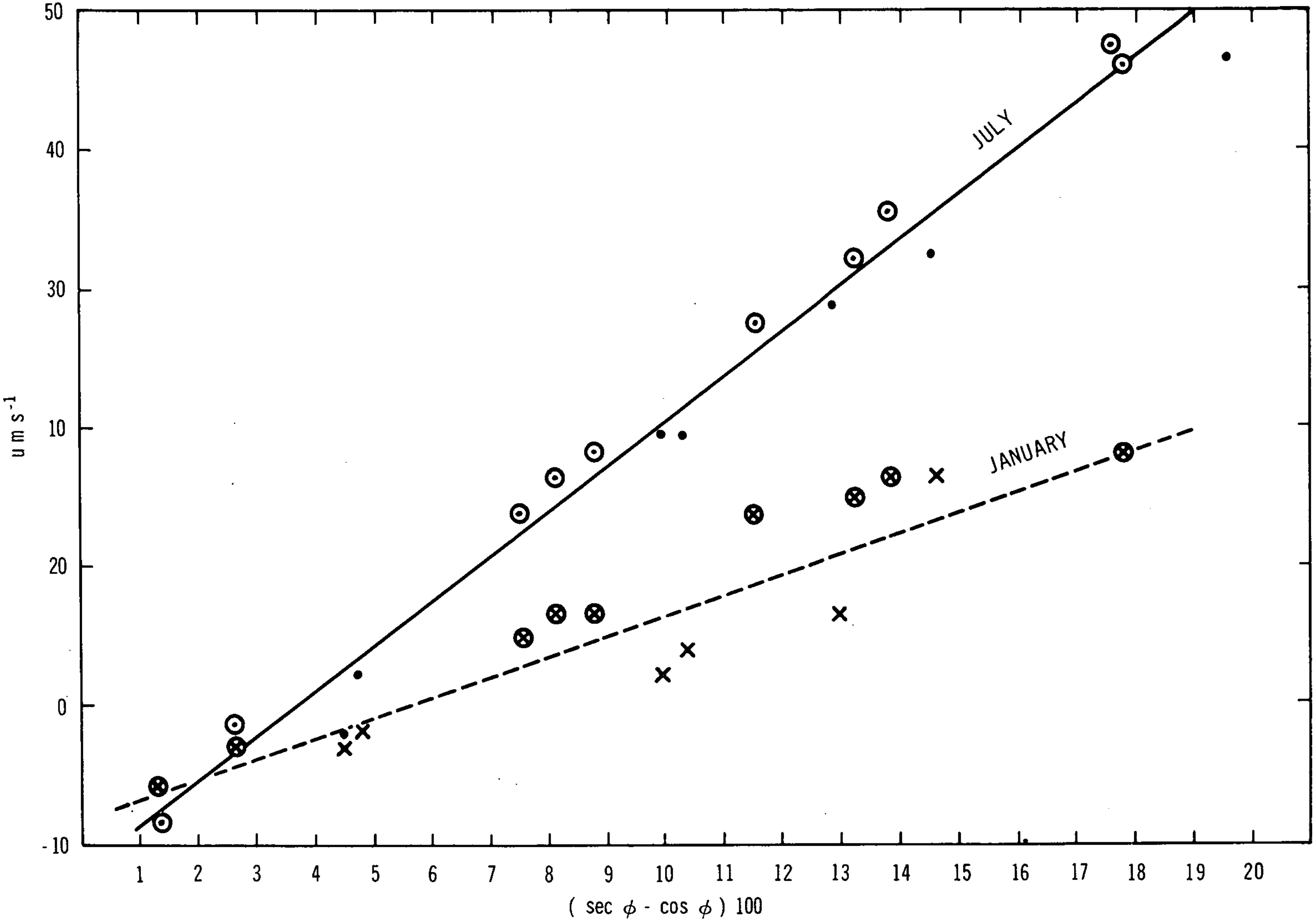

Fig 4 Profiles of 40,000 ft zonal wind component over tropical Australia during January (crosses) and July (dots). Data from east of 130° E are encircled.

regression coefficient the efficiency index was calculated by dividing by"aw". The results of the computations are shown in Table 2. The sample correlation coefficient and the standard error of estimate of the zonal wind from the adopted regression equation are included to indicate the goodness of fit of the regression equation.

Table 2 Monthly mean values of 40,000 ft efficiency index in the Australian tropical region

Month	Sample Size	Efficiency Index (E)	Standard Error Of (E)	Sample Correlation Coefficient	Standard Error Of Estimated u_ϕ ms^{-1}
Jan	15	0.302	0.037	0.920	2.99
Feb	16	0.295	0.034	0.920	3.13
Mar	14	0.352	0.040	0.930	2.85
Apr	13	0.501	0.019	0.992	1.23
May	13	0.631	0.032	0.986	2.10
Jun	16	0.602	0.038	0.973	3.50
Jul	17	0.649	0.023	0.991	2.28
Aug	18	0.649	0.029	0.985	2.99
Sep	15	0.583	0.036	0.976	3.10
Oct	15	0.545	0.036	0.973	2.93
Nov	11	0.489	0.039	0.972	2.63
Dec	15	0.442	0.026	0.979	2.07

For the mid-season months a similar procedure was adopted for other layers in the atmosphere to obtain a vertical profile of the efficiency index. The results are shown in Table 3 and Fig 5.

Summary of Results

The monthly mean 40,000 ft (12,200 m) efficiency index varies regularly through the year. There is a significant change from the winter to the summer value.

(i) Winter. The mean value of the 40,000 ft (12,200 m) efficiency index for the months May to September exceeds 0.55. Vertical profiles indicate that maximum values of the efficiency index occur near this level. The mean height of the tropical tropopause is near 55,000 ft (16,750 m). Above it the efficiency index falls rapidly and the correlation coefficient becomes statistically insignificant.

The deep tropospheric layer from 15,000 to 55,000 ft (4550 to 16,750 m) shows a correlation coefficient exceeding 0.9 and above 20,000 ft (6100 m) the efficiency index increases to far beyond the summertime maximum value. At any level in the troposphere the winter efficiency index is higher than that corresponding to summer.

(ii) Summer. January, February and March are typified by a mean 40,000 ft (12,200 m) efficiency index below 0.4. In the vertical profile the layer of significant correlation coefficients and higher efficiency indices is within the layer 30,000 to 55,000 ft (9150 to 16,750 m). Above and below this layer angular momentum conservation would not appear to have any significant influence on the zonal wind structure. However, in the layer 3,000 to 7,000 ft (925 to 2125 m) there is slight evidence that the strength of the monsoonal westerlies decreases proportionally to (sec ϕ - cos ϕ), presumably by frictional loss to the earth's surface.

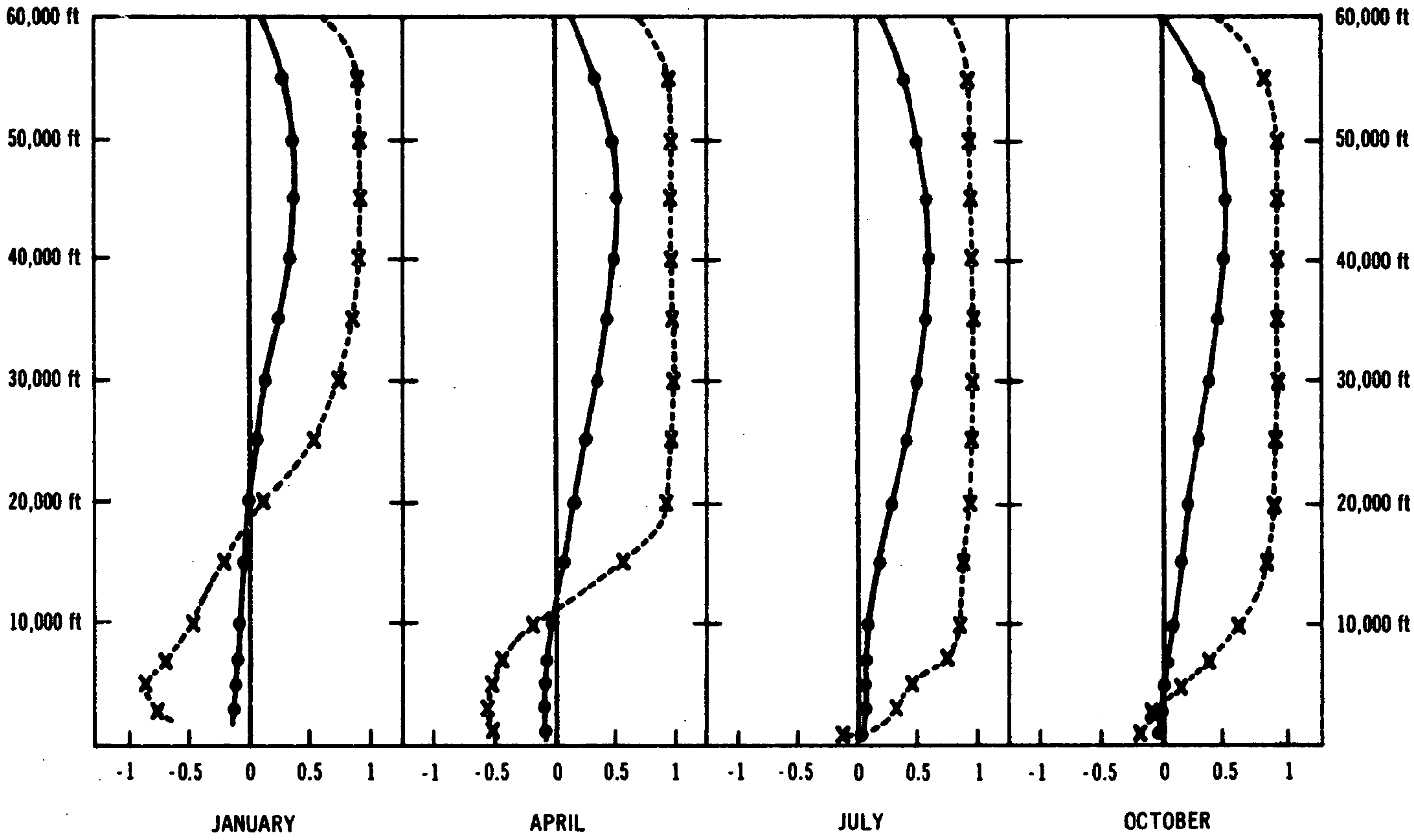

Fig 5 Vertical profiles of efficiency indices (solid lines) and correlation coefficients over the Australian tropics in mid-season months.

Table 3 Monthly mean values of efficiency indices (E) and correlation coefficients (r) (both multiplied by 100)

Height above MSL in thousands of feet		1	3	5	7	10	15	20	25	30	35	40	45	50	55	60
Jan	E	- 5.6	-10.8	-11.3	- 9.2	- 6.4	- 2.3	2.0	7.8	15.7	23.1	30.2	35.0	36.1	28.7	9.5
	r	-49.5	-74.6	-80.6	-65.8	-42.1	-15.2	15.8	54.3	79.2	88.1	92.0	93.7	93.6	90.9	56.8
Apr	E	- 6.1	- 8.4	- 8.5	- 5.7	- 0.8	5.8	15.3	25.2	34.4	42.6	50.1	50.7	44.5	28.5	10.3
	r	-47.5	-58.0	-51.7	-40.3	- 8.7	58.2	92.9	98.3	98.4	99.0	99.2	99.0	97.4	92.6	61.6
Jul	E	- 0.8	4.1	4.5	7.6	11.4	20.6	32.4	45.6	54.8	61.4	64.9	59.2	51.8	37.2	18.8
	r	- 7.2	36.0	46.5	79.3	89.9	92.6	96.7	98.1	98.6	99.1	99.1	97.6	96.9	94.3	75.1
Oct	E	- 0.1	- 0.6	1.7	5.3	9.5	11.9	19.7	29.1	38.9	47.8	54.5	53.8	47.4	27.7	5.5
	r	-17.4	- 5.9	15.0	40.8	67.1	81.1	91.5	92.8	94.6	96.3	97.3	97.6	96.4	84.4	47.3

(iii) Transition Months. Between the extreme seasons there appears both a short autumn transition in April and a longer spring one through October, November and December. In the vertical profiles, particularly in the lower levels, there is a greater similarity between April and January than between April and July whilst October resembles July more than January. At each tropospheric level the October efficiency index exceeds the corresponding April value.

(iv) Longitudinal Gradient of Efficiency Index. Riehl's results (Fig 1) suggest that considerable longitudinal variation of the efficiency index exists. In general such variation was not apparent over the Australian region except perhaps in the summer months.

Examination of the 40,000 ft (12,200 m) plotted data such as Fig 4 indicates that it is mainly data from east of 130°E which lie above the regression line. A 't' test of the significance of the difference between the mean efficiency indices either side of this line revealed that there was a significant difference at the 95 percent level in summer but no other season.

Interpretation of Results

(i) Winter. It has been noted that high efficiency indices and significant correlation coefficients occur throughout the layer between 15,000 ft (4500 m) and the tropopause. It is also true in this layer during winter that the net meridional wind has a small polewards component. A pronounced subsidence inversion occurs at or below 15,000 ft (4550 m).

Mean conditions are consistent with interpretation that subsidence is occurring within a deep layer according to eq 13; subsidence, by damping out small-scale vertical eddies, decreases the value of F_{xy} and hence F_x, thus enabling the efficiency index and the constancy of the wind to assume maxima.

(ii) Summer. The layer of higher efficiency indices and correlation coefficients is between 40,000 ft (12,200 m) and the tropopause. Unlike winter, the net meridional component has a weak equatorial component within this layer and, as was shown earlier, this implies divergence in the upper troposphere. Upper level divergence will be compensated by rising air from the layers below and thus vertical eddies enhanced.

The contribution of F_{xz} will be enhanced which will tend to depress efficiency index and constancy values. The amount of convergence aloft as indicated in eq 13 increases as the efficiency index decreases and thus it will be most marked in summer.

CONCLUSION

Evidence available from the Australian 1961 to 1971 mean wind data supports the contention that the monthly mean zonal wind field in the upper tropical troposphere conforms closely to a model of partial conservation of absolute angular momentum. Linear regression equations developed to test this conformity, when applied to the 40,000 ft (12,200 m) data, showed correlation coefficients exceeding 0.9 with sample sizes between eleven and eighteen.

An examination of the monthly mean indices of conservation of angular momentum efficiency reveals that characteristic values occur during each season and that the winter values are significantly higher than the summer values. The seasonal variations and vertical profiles of efficiency indices can be qualitatively explained as a function of large-scale meridional circulation variations and the resulting small-scale vertical eddies which the broad-scale circulation imposes.

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