ICE Fallout in Convective Storms

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Abstract

Layer-type 10 cm radar echoes from apparently 'clear air', extending to large distances ahead of convective storm clouds are examined for their origin and spatial dimensions. The echoes are attributed to Rayleigh scattering from ice crystals originating in the tops of Cb and anvil clouds. The quantity of ice in the echo volume is assessed from the measured reflectivity factor using mass-reflectivity relationships deduced from size and concentration of crystals sampled by others in clear air and non-raining clouds. The calculated amount of one to two million tonnes of ice contained in the average echo volume associated with moderate to severe storms, compares favourably with an estimate from considerations of water substance continuity, applying a convective precipitation model due to Kessler (1969).

The crystal concentrations in the layer have not yet been established by in situ sampling in the vicinity of storms, but there is indirect evidence for particles in the diameter range 100 to 500μ to be sufficiently concentrated to play a part in natural seeding of middle level clouds.

Introduction

The kinematic and dynamic implications of high-level vertical windshear for the growth and propagation of convective storms have been the subject of many studies over the last two decades (eg, Newton and Newton, 1959; Browning and Ludlam, 1962; Fankhauser, 1971). One consequence of the shear is the shape and extent of anvil cloud formations. Radar and visual observations of this cloud have helped in modelling the three-dimensional airflow in shear-embedded convective storms.

In its cloudphysical, if not dynamical, aspects the anvil cloud and its canopy tended to be regarded as more or less passive by-products. However, Hitchesfeld (1960), using constant altitude plan-position (CAPPI) radar display, analysed long plumes extending from the overhang of storm clouds at heights from 3 to 9 km. The two-dimensional kinematic patterns of hypothetical particle trajectories indicated ice crystals as the source of the radar backscatter.

Radar observations made by the present author in subtropical Australia indicated frequently occurring 50 to 100 km long down-shear extensions from major storm cells (Berson, 1969). To some extent these 'layer echoes' appeared to resemble the plumes observed by Hitchesfeld, their upper limits rarely exceeding 9 km. Kinematic analysis based on Marshall trajectories and streamlines (Marshall, 1953) showed that the position and shape of the echoes were consistent with particle (ice crystal) precipitation from the anvil cloud or from the upper reaches of cumulo-nimbus clouds.

Braham (1967) and Braham and Spyers-Duran (1967) have demonstrated that fore-runner cirrus can organise chaotic middle-level convection fields into fully grown rainstorms through natural seeding by ice crystals falling into the cloud. However, their investigation did not relate specifically to the descent of ice crystals aloft and ahead of convective storms.
The efficiency of natural seeding by ice crystal precipitation will depend, among other things, on the mass and concentration of the ice particles. Observations of radar echo dimensions and reflectivity in conjunction with basic and empirical relationships for particle backscatter are helpful in determining these factors. In the present paper emphasis will be placed on the total mass of ice contained in the echo volume as it can be assessed by consideration of water substance continuity in convective storms. A relevant storm model developed by Kessler (1969) is used for this purpose.

A list of symbols used is given at the end of this article.

FEATURES OF THE REFLECTING LAYER

Examples of the layer-echo phenomenon are presented from two sets of observations made in Australia at Aspendale (lat. 38°S) and Brisbane (lat. 23°S), respectively.

The Aspendale observations were made with a modified Marconi 277 S-band radar from time to time during the years 1967 to 1970. Layer echoes were seen in front of convective storms and synoptic-scale precipitation systems. Part of an unusually extensive echo observed on 23 January 1967 and monitored for several hours (Berson 1967) is shown in Fig 1. It occurred at about 6 km, well below a cirrus canopy which extended across much of southeastern Australia from a depression near the center of the continent. This depression had evolved from an active tropical cyclone. The existence of the cloud canopy was verified from two successive ESSA satellite photographs. A similar echo was observed on 24 February 1967. It was these two cases that led to a detailed study of layer echoes associated with convective storms.

The Brisbane observations were taken during a two-week period of intense thunderstorm activity in the early summer of 1968. They were carried out mainly in conjunction with airport radar watch maintained by the Bureau of Meteorology for aeronautical guidance when significant storm activity occurred within a radius of 50 nautical miles.

For details regarding the radar installation (Plessey S-band WF44), the method of measurement and observational evidence reference is made to the above-cited papers (Berson, 1969).

An example of a plume type layer-echo near Brisbane at its strongest and most extensive phase of development is shown in Fig 2a. The parent thunderstorm echo at a some later stage is shown in Fig 2b.

ICE CRYSTAL PRECIPITATION

The nature of the radar backscatter

Regarding the origin of this type of layer echo, a source other than particle backscatter is possible. Backscatter from the upper troposphere well outside cloud containing supercooled water drops might suggest super-refraction in cirrus or cirrus stratus. Thus Wagner and Conant (1963), using a high-powered 5 cm FPQ-4 radar, observed this type of echo from cirrus cloud at heights up to 9 km.

An interpretation of the echoes as particle fall-out presupposes a generating source of ice crystals at temperatures well below freezing level. Millimetric and 5 cm radars can respond to Mie scattering, i.e., from particles having a diameter >0.06 × emitted wavelength, even at relatively large slant range. The power of backscatter can be enhanced owing to focusing on the rear of dry, non-rimed ice particles (of spherical shape), the small reflection coefficient at their front su permitting most of the incident energy to enter the sphere (Atlas and Glover, 1962). The 10 cm radar echoes here considered are most likely to be generated by Rayleigh scattering; for although particles of diameter exceeding 6 mm such a graupel and hailstones may be present outside the updraft even at great heights, they can be safed excluded since they would not produce almost continuous reflection over as extensive regions of the sky outside storm clouds as indicated by the horizontal dimensions of the phenomenon (see Table 1 on page 10).
Fig 1 Example of an 'all-sky' layer echo observed from Aspendale on a 10 cm modified Marconi 277 radar, 17.06 EST, 23 January 1967. Azimuth of RHI scan, 273°. Range markers in nautical miles. (Similar layers extending to 50 nmi range were observed in all scanning azimuths during the following three hours.)

Fig 2 (a) Plume-type layer echo observed from Brisbane Airport with a WFA Plessey 10 cm radar, 19.40 EST, 3 December 1968. Receiver gain reduction, 3 dB. Range and height are in km.

Fig 2 (b) Echo of parent storms with which the layer in Fig 2(a) was associated. 20.20 EST, 3 December 1968. Azimuth of RHI, 173°.
It should be noted that both the Aspendale and Brisbane radars generally failed to give signal returns above noise level from visually located anvil clouds and anvil canopies, although on rare occasions weak echoes were received from 'blown-off' decaying anvil formations. On no occasions were echoes received from common cirrus formations such as cirrus-uncinus, cirro-cumulus and cirro-stratus; or even from alto-stratus except when the latter was situated a mile or two ahead of precipitation reaching the ground. Visually, within the space occupied by a layer echo, a broken alto-cumulus deck or high-based strato-cumulus clouds were sometimes discernible but when there was no cloud the appearance of the sky resembled a milky blue. The radar and visual observations appeared at times difficult to reconcile. However, it is well known that for clouds of 'large' particles and a given water (ice) content the cloud visibility varies as the reciprocal of the particle diameter. Thus a layer of large particles (≥500 μm) will have a much lower visibility than normal water (5 to 20 μm) or ice (30 to 50 μm) clouds of equivalent water content.

Reflectivity

The above mentioned 'all-sky' echoes were observed with an incompletely calibrated radar lacking facility for receiver gain attenuation and reflectivity measurement. Previous deductions (Berson, 1967) regarding the level of reflectivity associated with these echoes are here briefly summarised.

The base of the reflecting layer was assumed to represent the level at which ice particles below certain limits of size and concentration had evaporated. In the case of these echoes as well as the sample of observations at Brisbane used further below, that level was at least 1 km above freezing level. In all these cases then the 1 to 2 km thick reflecting layers had no connection with the well-known 'bright band' which is attributed to the melting of ice. With the foregoing interpretation of the base of the layer it was possible to deduce the initial median particle mass at the top of the layer by applying basic theory for evaporation of ice particles moving through an unsaturated environment (Thorpe and Mason, 1966). In the great majority of observed layer echoes, large saturation deficits in the middle and/or upper troposphere were measured, i.e., dewpoint depressions from 10° to 20°C.

The calculated initial median particle mass \( m \) varied from 30 to 50 μgr, in good agreement with values variously quoted for plane and spatigl (hollow) dendrites and columns at the relevant heights and temperature (9 km, -25° to -30°C). The calculated layer-averaged value varied from 4 to 8 μgr. When M was assumed to have a representative value 0.3 gr m\(^{-3}\), the resulting particle concentration \( N \) was 40 to 80 lit\(^{-1}\). These deduced physical properties of the ice particles give a layer-averaged reflectivity factor \( Z \) lying within the limits \( 3 \times 10^2 \) and \( 5.10^2 \) mm\(^{2}\) m\(^{-3}\). The corresponding crystal precipitation rate is 1 to 5 mm hr\(^{-1}\) (water equivalent).

Heymsfield and Knollenberg (1972) have since carried out direct sampling of particle properties in generating cirrus-uncinus clouds from which they deduced reflectivity factors of at least an order of magnitude below the above given values.

The Brisbane observations of plume-type layer echoes associated with convective storms included the measurement of a maximum reflectivity factor, \( Z^*\). (The actually measured, nominal factor was corrected for the difference between the refractive indices of ice and water.) It was to be expected that \( Z^* >> \frac{Z}{\pi} \). In fact, \( Z^* \) values ranged from \( 3.10^2 \) to \( 10^6 \) mm\(^{2}\) m\(^{-3}\) (see also Fig 3).

With a view to assessing the ice content in the echo volume, a relationship for \( Z^* \) as a function of \( M \) is required. Let \( m \) be the median mass of relatively large ice particles or crystal clusters with diamet\( \bar{E} \) DaDe and concentration \( N \times 10^3 \). We define a reflectivity factor \( Z_p \) such that the Bartoloff-Atlas equation (Atlas, 1964) takes the form

\[
Z_p = D_p N_p T(p) = (6/\pi p)^2 m_p^2 N_p T(p)
\]
Fig. 3 The calculated reflectivity factor $Z_p$ as function of the ice content parameter $M_p$. Measured reflectivities $Z^*$ are also shown (see the insert).
where $T(p)$ is the appropriate size spectrum statistic. The spectrum being sliced at
the particle diameter $D_p$, the statistic $T(p)$ may be set equal to unity, to a first
approximation (see also P. Atlas, 1964, p. 364). $D_p$ is now supposed to represent the
lower limit of particle size which the 10 cm radar will 'see' providing the particles
occur at a concentration equalling or exceeding the value $N_p$.

To calculate $Z_p$ from relevant observational data, one should in the first
instance get some idea regarding the approximate magnitude of $D_p$. Weickman (1948)
sampled ice crystals in Cb anvil clouds and found that both plate and prismatic
crystals were present in relatively large concentrations. The size of the larger
prisms and clusters varied from 100 to 500 $\mu$m equivalent diameter. Weickman's
observation referred to heights of about 6 km at temperatures of $-15^\circ$ to $-20^\circ$C while
the layer echoes extended typically from 5 to 8 km with temperatures from $-10^\circ$ to $-30^\circ$C.
A value of $D_p = 500$ would seem to be indicated. That such relatively large ice
particles are being carried away in the anvil outflow in any significant concentration,
is borne out to some extent by the following considerations based on measurements by
others.

The concentration-size spectrum of solid water-insoluble particles (freezing
nuclei) sampled in rainwater and hail during thunderstorms (Rosinski, 1967) reveals
marked gaps in the 40 to 70 and 70 to 100 $\mu$m intervals while in the inflow region
under the updraft of storms the concentration of soil particles with diameter exceeding
50 $\mu$m is consistently large (Rosinski et al., 1971). Since these particles are excel-
 lent freezing nuclei, the mentioned spectral gaps suggest that a high proportion of
ice particles which have grown on these nuclei are being carried away in the anvil out-
flow.

A limited source of information is available on diameters and concentrations
actually measured for ice particles of various size and shape at heights and environ-
mental conditions (temperature and saturation deficit) quite similar to those prevail-
sampled crystals in flight through clear air in the vicinity of non-precipitating
clouds, in particular below cirrus. Mossop, Ono and Heffernan (1967) and Mossop
(1968) sampled crystals in flight through, and above, precipitating stratiform and
cumuliform clouds. Heymsfield and Knollenberg (1972), in addition to cirrus-uncinus,
succeeded in sampling a Cb anvil for the physical properties of ice particles. Median
and maximum diameters, and concentrations, communicated in the first two mentioned
investigations, represented flight-leg averages. The maximum diameters varied generally
between 250 and 1,000 $\mu$m, the maximum concentrations were of the order 1,000 lit$^{-1}$.
As a first attempt it seemed therefore justified to identify, in the calculation of
$Z_p$, the quantities $D_p$ and $N_p$ with these maximum values.

To evaluate $M_p (= N_p \times m_p)$, the particle mass $m$ must be expressed as a function
of diameter $D$. To this end, use was made of the well known formula by Nakaya and
Terada (cf. Mason, 1957), namely:

$$m = \varepsilon D^\eta$$

where $\varepsilon$ and $\eta$ have the values 0.0038 and 2, respectively, for plane dendrites; 0.027
and 2 for rimed plates; and 0.065 and 3 for soft hail (graupel).

A plot of the calculated values of $Z_p$ as a function of the calculated values of
$M_p$ resulted in a very large scatter of points. Replacing however maximum concentration
by measured median concentration $N_p$ which varied from 100 to 500 lit$^{-1}$ - a procedure
that seemed also intuitively more correct - a satisfactory regression resulted. This
is shown in Fig 3 where the legend gives information on the type of ice particles and
cloud (if any) predominating in a particular sample. The data pertaining to Cb sampling
by Heymsfield and Knollenberg were also used to deduce $Z$ as a function of $M$, assuming,
as before, that $M$ is identified with the maximum value of $M$ given there as $0.95$ gr
m$^{-3}$. The maximum reflectivity factor calculated independently by these authors is
shown.
A comparison of calculated $Z_r$ values with maximum reflectivity values $Z^*$ measured by radar in layer echoes shows that the range $10^2 - 2 \times 10^3$ embraces all but two points. In contrast, the reflectivity factors associated with the major storm clouds were well above $10^4 \text{ mm}^6 \text{ m}^{-3}$ (see also the footnote on page 9).

The assessment of the total mass of ice contained in the echo volume requires an estimate of $M$ on the basis of reflectivity factors $Z^*$. This is facilitated by a plot of $M_o (=m_o N_o)$ as a function of $M_p$ (Fig 4). Since $M_p = m_p N_o$ in Fig 3, it follows that $M_o / M_p = m_o / m_p < 1$.

Comparison with hail reflectivity relationships

On the basis of reflectivity relationships it was estimated that hail occurred in about 15% of the storm cells tracked on the Brisbane radar during the project period (Berson, 1971). Most hail stones will fall to ground in or close to the storm's main precipitation downdraft. However, a significant proportion of the smallest stones is likely to be carried away in the sheared outflow aloft. A comparison of reflectivity-mass regressions for hail and for ice crystals will be therefore of considerable interest.

The regressions for hail in the upper part of Fig 5 are due to Douglas and Hitschfeld (1961). They are based on 10 cm radar measurements in hailbearing clouds. In the lower part of Fig 5 the factor $Z_o (=D_o N_o T(n))$ is shown as function of $M_o$ for the data in Fig 4 as well as data from sampling in cumulus clouds over Tasmania carried out by Mossop (1971).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The median ice content $M_o$ as function of $M_p$.}
\end{figure}
Fig 5 The calculated reflectivity factor as function of \( M_0 \). The solid slanted lines are hail-reflectivity regressions due to Douglas and Hitchens (1961)
The distribution of data points in Fig 5 indicates that where solid, prismatic and hexagonal crystals are involved, the reflectivity regresses (logarithmically) with the mass on an extension of the Douglas-Hitchcield reflectivity-hail regressions. Since \( \log Z_o \propto \log M_o = \log (D_o^3 N_o) \), it follows \( \frac{Z_o}{N_o} \propto D_o^3 \). Thus the hail regressions would imply that the median hail diameter is inversely proportional to the cube root of concentration. For particles consisting of a mixture of ice and water, \( \theta \), in the case of strongly rimed columns in cumulus clouds, the points lie well outside that extension, the reflectivity being smaller by nearly an order of magnitude.

**ECHO DIMENSIONS**

Display modes

In the Brisbane measurements the length \( L \), which was mostly determined by the radial extent on the RHI display, averages 39 km on applying swept gain, but 68 km on applying the basic display mode, \( \theta \), without swept gain (Table 1, row 1). The difference of 29 km equals 1.1 standard deviation (st. dev.) from the mean of 52 measurements, including PPI observations.

Storm cloud and precipitation echoes are much less sensitive to the change in display mode, presumably due to the generally high reflectivity associated with the storm cells. *

The respective values for \( L \) are 36 and 40 km, the difference equal to only 0.18 st. dev. from the mean of a population of 60 observations.

With the assumption of full illumination of the aerial beam, base and ceiling heights \( (H_b, H_c) \) are calculated by replacing the measured elevation angles for echo-limb extinction by \( \alpha'_b = \alpha_b + \theta/2 \) and \( \alpha'_c = \alpha_c - \theta/2 \), respectively. As will be seen from rows 2 and 4 in Table 1, the change in display mode makes on the average a difference to \( H_c \) of only 0.2 km (0.08 st. dev.) whereas \( H_b \) with swept gain is 1.6 km (0.64 st. dev.) higher than with the basic display mode.

**Tangential resolution**

To shed light on this effect in the determination of \( H_b \), the origin of the backscatter must be borne in mind. In ice crystal precipitation the reflectivity will be considerably weaker in the lower part of the echo than in its upper reaches due to continuous evaporation of particles in their descent through the dry environmental air. This poses the question of resolution when the tangential dimension of an echo is measured. When the reflectivity is not much above the receiver noise level and a relatively small fraction of the beam is illuminated, one might subtract \( \theta/2 \) from the measured angle \( \alpha_b \) rather than add it as is customary for high reflectivity levels (Battan, 1959). This procedure yields a lower limit for the base of the echo.

The geometry of the situation is shown in Fig 6. Values of \( H_b \) calculated in this manner are shown in row 3 of Table 1. The difference between swept gain and basic display is now only 0.6 km (0.33 st. dev., a reduction by almost 50%).

* During the Brisbane observational period the thunderstorm cells were displayed with considerable receiver gain reduction allowing the 'bright' and 'blackhole' reflectivity regions to dominate the picture. The minimum detectable signal for the 'bright' display corresponded to a reflectivity factor \( 9 \times 10^5 \text{ mm}^{-3} \). For comparison the range of measured maximum reflectivity factors for the layer echoes may be recalled, namely \( 10^2 - 10^4 \) units.
Table 1  Average values and standard deviations of length, base and ceiling heights for layer echoes. Brisbane observations 1968, Aspendale observations 1967-1970

<table>
<thead>
<tr>
<th></th>
<th>Brisbane</th>
<th>All observations</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Swept gain</td>
<td>Basic display</td>
<td>Standard deviation</td>
</tr>
<tr>
<td></td>
<td>km</td>
<td>km</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>km</td>
<td>km</td>
<td>km</td>
</tr>
<tr>
<td>Length, L</td>
<td>39</td>
<td>68†</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam fully illuminated</td>
<td>7.4</td>
<td>5.8</td>
<td>7.2</td>
</tr>
<tr>
<td>$H_b = R \sin (\alpha_b + \phi/2)^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam partially illuminated</td>
<td>5.3</td>
<td>4.7</td>
<td>4.8</td>
</tr>
<tr>
<td>$H_b = R \sin (\alpha_b + \phi/2)^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceiling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam fully illuminated</td>
<td>8.0</td>
<td>7.8</td>
<td>7.9</td>
</tr>
<tr>
<td>$H_c = R \sin (\alpha_c - \phi/2)^*$</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

* Plus correction for Earth's curvature and average refraction.
† RHI scans only.
Fig 6 Determination of the layer-echo base assuming partial illumination of the aerial beam (see also Table 1, row 3).

Layer thickness and beamfilling

Height determinations were made at the nearest and furthest ends of the echo and corrected for the curvature of the Earth and average atmospheric refraction. The heights in Table 1 and the thicknesses in Table 2 represent the algebraic means of the values at the end points. Table 2 lists thicknesses from two sets of measurements of $R$ and $\alpha$: (i) a 'reflectivity sample' with adequate information on $Z^*$; (ii) a 'volume sample' with adequate information on $L$ and $\delta y$. The first of these samples is further subdivided into two groups according to whether $10 \log Z^* \leq 31$, the median value of the sample population.

The vertical beamfilling requirement $B > 0$ is not met when, as would have occurred in individual instances, low reflectivity is associated with relatively distant echoes. Thus with the assumption of full beam illumination the inequality $B < 0$ predominates, but $B > 0$ in instances of relatively high reflectivity (bracketed figures in column 1, Table 2). In the last mentioned instances the thickness averages about 2 km, but for 'low' reflectivity assuming partial beam illumination, the average thickness amounts to 3 km.

On the basis of these calculations, a sample mean significantly smaller than 2 km and larger than 3 km was regarded as unrepresentative. The relevant value of 3.3 km in the volume sample (Table 2) would thus indicate a slight bias to lower reflectivity.

ICE CONTENT

Of the 15 observations in the volume sample, only 5 were simultaneous with measurements of $Z^*$. As they were within 2 dB of $10 \log Z^* = 31$ (the median value of the reflectivity sample), the average reflectivity factor pertaining to the volume sample may be taken as $Z^* = 10^3 \text{ mm}^6 \text{ m}^{-3}$.
Table 2  Average values of layer echo thickness (H), range (R) and length (L). The beamfilling parameter B is shown in brackets.

<table>
<thead>
<tr>
<th>Reflectivity sample</th>
<th>Beam illumination</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full H km</td>
<td>Partial† H km</td>
</tr>
<tr>
<td>10 log Z* = 33.0 (average)</td>
<td>1.22(-0.82)</td>
<td>3.42(1.08)</td>
</tr>
<tr>
<td>10 log Z* = 36.5 ('high')</td>
<td>2.15(0.32)</td>
<td>3.85(1.98)</td>
</tr>
<tr>
<td>10 log Z* = 29.5 ('low')</td>
<td>0.50(-1.96)</td>
<td>2.95(0.18)</td>
</tr>
<tr>
<td>Volume sample</td>
<td>0.52(-2.25)</td>
<td>3.30(0.52)</td>
</tr>
</tbody>
</table>

† With respect to the lower part of the echo.

On applying the regression of $M_0$ on $M_p$ in Fig 4, and subsequently the regression in Fig 3, one obtains $M = 0.08 \text{ gr m}^{-3}$. The ice content in the echo volume is then given by $U = 0.08 V$, where $U$ is expressed in millions of tonnes and $V$ in $10^3 \text{ km}^3$.

It is not possible to evaluate directly the volume $V$ because the change of $\delta Y$ with respect to $R$ is not known from the available data. However, upper and lower limits of the volume can be determined. They are defined as follows:

$$V_1 = 2 \, R^2 \, \delta H \, \delta Y \, \pi / 180, \quad V_2 = R \, L \, \delta H \, \delta Y \, \pi / 180$$

The calculations were carried out from data for the individual echoes in the volume sample, but for beamfilling cases only. The results are shown in Table 3. The average layer thickness in this set of observations is 2.8 or 3.8 km, depending upon the assumption for beam illumination.

As mentioned before, the average thickness value of 3.3 km pertaining to the complete volume sample is biased toward lower reflectivity. Hence, on the basis of Table 3, the best estimate for the average volume will lie closer to the values listed there for partial than for full beam illumination. With $V_1 = 30 \, 10^3 \text{ km}^3$ and $V_2 = 10^3 \text{ km}^3$ the ice content lies in the range 1.0 to 2.5 million tonnes. This represents an estimate of the average ice content whereas in individual layer echoes it will have varied between much wider limits.

Table 3  Average values of layer-echo volume ($V_1$, $V_2$), total ice content ($U_1$, $U_2$) and echo dimensions - 'volume sample' data (see Table 2), beamfilling cases and Brisbane observations only

<table>
<thead>
<tr>
<th>Assumed beam illumination</th>
<th>$V_1$ $10^3\text{km}^3$</th>
<th>$V_2$ $10^6\text{tonnes}$</th>
<th>$U_1$ km</th>
<th>$U_2$ km</th>
<th>$\delta H$ km</th>
<th>$B$ km</th>
<th>$R$ km</th>
<th>$L$ km</th>
<th>$\delta Y$ deg</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>5.1</td>
<td>3.4</td>
<td>0.41</td>
<td>0.27</td>
<td>2.8</td>
<td>1.4</td>
<td>28</td>
<td>20</td>
<td>92</td>
<td>6</td>
</tr>
<tr>
<td>Partial*</td>
<td>39.5</td>
<td>13.7</td>
<td>3.15</td>
<td>1.10</td>
<td>3.8</td>
<td>1.8</td>
<td>40</td>
<td>26</td>
<td>104</td>
<td>12</td>
</tr>
</tbody>
</table>

* With respect to lower part of the echo.
DISCUSSION

In view of the various uncertainties in the reflectivity regressions and the tangential (vertical) resolution of echo measurement it is desirable to assess U independently.

In recent years convective-precipitation models have been developed which incorporate meso-scale kinematics and the micro-physical processes of cloud formation and precipitation. Kessler (1969) developed a model which generates the total liquid water mass from a state of rest and separates cloud water from precipitation water. It consists of a two-dimensional rectilinear circulation with symmetry about a horizontal axis. Vertical windshear in the environment is not allowed for. While such windshear would alter the patterns of the divergent outflow aloft (and therefore also the fallout of ice particles) it is reasonable to assume that the presence of windshear does not alter appreciably the ratio of cloud water to precipitation water.

Continuity of water substance implies that the difference between total liquid water content and precipitation water in the principal generating branch of the circulation, i.e., the cloud water content, will be equal to the evaporated amount of water substance. This can be approximated by the fallout of ice particles since they subsequently evaporate in the descending branch of the circulation. However, quasi-stationarity is here implied, i.e., precipitation water and cloud water should not vary significantly in time. Furthermore, a part of the ice crystal precipitation possibly forming the nuclei in middle level cloud is disregarded as negligible.

Two limiting cases are treated by Kessler, initially saturated air and initially unsaturated air with a specified distribution of the saturation deficit. In turn these conditions were treated separately for maximum vertical velocities of 2.5 and 10.0 m s⁻¹. In the latter case, which is more typical of the moderate to severe storm cells associated with the observed layer echoes, the circulation and the generation of precipitation and cloud water increase rapidly in time to a rather sharp maximum only to decrease almost as rapidly. With an updraft velocity of 2.5 m s⁻¹ there is a quasi-stationary phase of somewhat less than one hour's duration. On balance the latter case is the more appropriate choice for the present purpose.

The horizontal-dimension parameter of the circulation cell is the distance from maximum ascent to the maximum descent, a 'half-wave length' λ/2 (=6 km in Kessler's numerical evaluations).

The cloud water in the ascending branch must form part of the overall circulation of water substance and so also of water vapour. In quasi-stationary conditions, assuming no net losses occur through interchange with adjacent cells, the cloud water will equal the mass of ice falling in the descending branch of the circulation.

The fallout equivalent of Kessler's model to the average horizontal dimensions of the Brisbane storms can now be derived. The length of the layer echo we take to be 26 km (see Table 3). The average large diameter of the storm cloud and precipitation echo* was 39 km, there being on the average a gap of 5 km between the latter and the upwind end of the layer echo (Berson, 1971). Hence the descending branch of the circulation may be set equal to 31 km and λ/2 = 39/2 + 31 = 50 km. The lateral cell dimension is identified with the small diameter of the storm echo and averaged 30 km. In order to deduce the cloud water mass from the values referring to unit width of the circulation (listed in Table 4 in unit 10⁷ gr m⁻¹) one has to multiply by a factor (50/6) 30, or by 2.5 when the mass is expressed in millions of tonnes. The results are shown as bracketed figures in Table 4.

* The extent of this echo is limited by the reflectivity threshold Z = 6.4 10³. Areas of light rain are therefore excluded, which would seem desirable since such regions do not form part of convective-storm cells.
Table 4  Water budget items for a two-dimensional, rectilinear and symmetric model of a convective precipitation cell. The values refer to three instants after onset of the circulation during quasi-stationary phase. (After Kessler, 1969, Fig 35 and 39.) Bracketed values are equivalents of a three-dimensional circulation with average horizontal 'dimensions' of thunderstorms observed in the Brisbane area.

<table>
<thead>
<tr>
<th>Time after onset of circulation</th>
<th>1,000 s</th>
<th>2,000 s</th>
<th>3,000 s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10^7 gr m^-1</td>
<td>mill tonnes</td>
<td>10^7 gr m^-1</td>
</tr>
<tr>
<td>Initially saturated:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precipitation water</td>
<td>2.4</td>
<td>(6.0)</td>
<td>2.4</td>
</tr>
<tr>
<td>Cloud water</td>
<td>0.8</td>
<td>(2.0)</td>
<td>0.7</td>
</tr>
<tr>
<td>Initially unsaturated:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precipitation water</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
</tr>
<tr>
<td>Cloud water</td>
<td>-</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Forming averages for elapsed times of 2,000 and 3,000 s after the onset of the circulation and the two initial moisture conditions, one obtains from the bracketed values 1.4 million tonnes. The mean of U1 and U2 in the second row of Table 3 is 2.1 million tonnes.

The order-of-magnitude agreement between independently derived values for ice crystal precipitation associated with convective storms lends considerable support to the evidence for the very large quantities of water substance involved in this precipitation.

It may be of interest to comment further on the total amount of turned-over water substance (assessed from Kessler's model and the spatial dimensions of the relevant sample of thunderstorms in the Brisbane area). It will be seen from Table 4 that the values vary from 5 to 8 million tonnes. This compares favourably with the order of magnitude of turned-over water in large and severe storms in the USA High Plains, namely 10 million tonnes (William C. Swinbank, personal communication).

**CONCLUSIONS**

It appears that the observed extensive 10 cm radar echoes in question are produced by Rayleigh scattering from sufficiently concentrated ice particles originating in the upper reaches of convective storm cells and their anvil formations. Fairly broad inferences only could be made on the crystal size distribution and particle concentration. The observed range of radar reflectivity was shown to be similar to the range derived from the size and concentration of crystals of various shapes sampled in clean air and non-raining cloud at similar temperatures. This source of information suggests that the dominant particle size lies within 100-500μm at concentrations of 50 to 100 per litre. To be representative of the majority of echoes, the ice crystals must have been mainly visible as a cloud. However, a considerable proportion of echoes was observed on the radar in the presence of a broken deck of middle-level cloud or after dusk.

The calculated massive ice fallout corresponding to an average echo-volume content of the order of one million tonnes of ice per storm (in a subtropical region) represents about 15 to 20% of the total turned-over water substance according to the assessment made from a two-dimensional convective circulation model.
With a view to judging the effectiveness of ice crystal precipitation for the
natural seeding mechanism - eg, in regard to the so called 'front-feeder cell' - the
cloud physicist will require detailed information on size spectrum and particle
concentration. A project planned for this purpose jointly with the Cloud Physics
Group of the CSIRO Radio Physics Division included facilities for synchronised radar
observations and in-flight sampling of ice crystals. During a two-week project period
in 1970 only one suitable storm situation developed. The aircraft, which was not
designed for penetration above about 5 km height, sampled crystals beneath the base
of the layer echo, which was at about 6 km. The result was very small concentrations
of order 0.1 l/1 of ice crystals with 250 to 500μm diameter.

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LIST OF SYMBOLS

Measured radar variables

\( \alpha \) aerial elevation angle

\( \phi/2 \) half-power vertical beam width

\( R \) slant range (of echo midpoint)

\( \delta \gamma \) azimuth extent of PPI scope (measured in degrees)

\( Z^* \) maximum reflectivity factor (of an individual echo)

Deduced echo variables

\( b \) subscript denoting base

\( c \) subscript denoting ceiling

\( \alpha' \) adjusted aerial elevation \( (= \alpha \pm \phi/2) \)

\( B \) vertical beam-filling parameter, \( R [\sin \alpha_c' - \sin \alpha_b' - \sin (\phi/2)] > 0 \)

\( H \) height above ground

\( L \) length

\( V \) volume

Ice crystal dimensions and mass

\( D \) particle diameter (of equivalent sphere)

\( \rho \) particle density

\( m \) particle mass

\( N \) particle concentration

\( M \) mass of ice in unit volume \( (= m \times N) \)

\( U \) mass of ice contained in total echo volume \( (= V \times M) \)

\( T \) a size spectrum statistic

\( o \) subscript denoting median value