ANALYSIS OF BOUNDARY LAYER DATA FROM THE LAVERTON SERIAL SOUN丁NG EXPERIMENT

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ABSTRACT

Rossby-number similarity functions for the planetary boundary layer are evaluated from surface observations and wind and temperature profiles up to 2 km gathered during the Laverton Serial Sounding Experiment (Bureau of Meteorology 1968). This site is in inhomogeneous terrain, permitting a comparison with results from the more usual 'flat' sites. Surface stress and turbulent heat flux are derived from analysis of balloon trajectories using an adaption of the 'velocity deficit' method, and near-surface flux-profile relationships. The dependence of similarity functions A, B and C upon stability parameter μ and non-dimensional thermal wind shear components is examined and compared with previous results. Surface stress derived from analysis of balloon trajectories is found to exceed that derived from near-surface flux-profile relationships, and corresponds to an effective roughness length of 0.75 m. The implications of this are discussed.

INTRODUCTION

The Rossby-number similarity theory developed by Zilitinkevich, Laikhtman and Monin (1967) and Gill (1968) predicts universal relationships between physical quantities within the planetary boundary layer. These relationships have been explored empirically and have practical application in numerical weather prediction models, e.g. Clarke (1970a). Data used to verify the theory and determine the universal functions have come from flat and uniform sites as the theory does not allow for horizontal inhomogeneity. These sites are not typical of the real world, hence the question arises as to the extent to which the similarity functions derived from these special data might be more generally applicable. The present study is directed to this question. Boundary layer data from an inhomogeneous site at Laverton, Victoria (37°52'S, 144°46'E) are analysed in terms of Rossby-number similarity theory. The data are part of the Laverton Serial Sounding Experiment (LSSE) data set (Bureau of Meteorology 1968) and consist of near-surface and radiosonde observations of temperature, pressure and wind velocity, made at intervals of three hours over a period of one month. The Rossby-number similarity theory leads to relationships between the vertical fluxes of momentum (τ), heat (H) and water vapour near the surface, and other variables such as the components of the surface geostrophic wind relative to the surface stress direction, (u₀ and v₀), the roughness
length \( z \), the Coriolis parameter \( f \), and vertical change of potential temperature \( \Theta \) and mixing ratio measured through the boundary layer. The LSSE data are adequate to treat the vertical fluxes of momentum and heat, but not the vertical flux of water vapour. A further theoretical development due to Hess (1973) allows for the effect of horizontal variation in temperature, i.e. thermal wind shear, and this is examined in the present analysis.

The Rossby-number similarity approach to parameterisation of the boundary layer is certainly not the only method available. It does, however, derive directly from an examination of the basic equations of motion for this region. An analysis of these results in a vertical height scale of \( u_*/f \). Other approaches have been suggested based on a more descriptive vertical scaling (e.g. see Yamada 1976) such as the height of the capping temperature inversion. Such approaches are particularly attractive at low latitudes when the Rossby-number approach fails, as \( f \) approaches zero. These other schemes do often prove equal or superior to the present approach. But they are often difficult to apply to a numerical model since the height of the inversion is not usually carried explicitly (except in very fine-mesh models, where this parameterisation is not appropriate). In the present data set it was not possible to consistently identify an inversion, so the Rossby-number approach was the only one examined.

METHOD

The non-dimensional Rossby-number similarity functions for a planetary boundary layer may be expressed (e.g. Clarke 1970b) as follows:

\[
A = -k \frac{u_0}{u_*} \ln \left( \frac{z_0}{u_*} \frac{|f|}{u_*} \right); \quad \ldots \quad 1
\]

\[
B = -k \frac{\nu_0}{u_*} \text{Sign} f; \quad \text{and} \quad \ldots \quad 2
\]

\[
C = k \Delta \Theta \rho C_p \frac{u_*}{H_0} \ln \left( \frac{z_0}{u_*} \frac{|f|}{u_*} \right) \quad \ldots \quad 3
\]

where \( k \) is the von Karman constant, here assumed to have a value of 0.4, \( u_* \) is the friction velocity defined by \( \tau = \rho u_*^2 \), \( \rho \) is the air density, \( f \) has a value of \( -8.929 \times 10^{-5} \text{ s}^{-1} \) at the latitude of Laverton, and \( C_p \) is the specific heat of air at constant pressure.

According to Rossby-number similarity theory for a baroclinic boundary layer, \( A \), \( B \) and \( C \) are functions of stability parameter \( \mu \) defined by

\[
\mu = -k^2 \frac{g H_0}{|f| \rho C_p \Theta u_*^2} \quad \ldots \quad 4
\]

and the dimensionless thermal wind shear through the boundary layer defined by

\[
\hat{S}_x = \frac{k^2}{|f|} \frac{\partial u}{\partial z} = -\frac{k^2}{|f|} \frac{g}{T} \frac{\partial T}{\partial y} \quad \text{and} \quad \ldots \quad 5
\]

\[
\hat{S}_y = \frac{k^2}{|f|} \frac{\partial v}{\partial z} = \frac{k^2}{|f|} \frac{g}{T} \frac{\partial T}{\partial x} \quad \ldots \quad 6
\]
where $g$ is gravitational acceleration and $T$ is the mean temperature of the layer.

Neither $u_*$ nor $H_o$ was measured during the LSSE and some means of estimating these quantities is necessary. Two distinct methods are used in the present study. The first is based on flux-gradient relationships for the surface or constant-flux layer, which is assumed to extend from $z$ up to a few tens of metres, using observations of temperature in the lowest levels sampled by radiosonde, and wind speed measured at 10 m by anemometer and smoothed by a five point filter described in Appendix A. This method is analogous to the drag coefficient approach often used in engineering type studies. It is somewhat less empirical than such methods in that thermal stratification of the surface layer is accommodated through the well established flux-gradient relationships. However, it does require a knowledge of the momentum and thermal roughness lengths, $z_0$ and $z_T$, of the surface. A value of 0.1 m for $z_0$ was used. This can be estimated from the physical characteristics of the terrain, as described later. The value of $z_T$ is not as simply determined, however. Clarke (1970b), for instance, equates $z_0$ and $z_T$, whereas a recent study by Garratt and Hicks (1973 - see also Garratt and Francey 1978) suggests that $z_T$ is between 5 and 10 times larger than $z_0$. We have chosen to adopt the Clarke criterion to be consistent with his results, and also because the data did not contain extremely unstable stratifications (where the $z_0$, $z_T$ differences have greatest effect upon the estimated temperature difference). Under the range of conditions encountered during the LSSE, we do not expect the equality assumption to introduce significant errors. The direction of the surface stress is assumed to be the same as this smoothed surface wind direction. Appendix B describes this method, referred to as the surface method.

The second method is based on a modified form of the 'velocity deficit' method (e.g. Haltiner and Martin 1957, p. 227) and uses radar-tracked balloon observations of wind. It provides $u_*$ but not $H_o$. However, the $u_*$ obtained can be used together with the near-surface vertical temperature gradient to solve the flux-gradient equations (Appendix B) for $H_o$. A benefit of this method is that estimates of thermal wind shears are produced. It also provides an estimate of the direction of surface stress. Appendix C describes this method, referred to as the planetary boundary layer (PBL) method.

The quantities $\Delta \theta$ and $z_0$ in Eqns 1 to 3 are also required to be estimated. Examination of profiles of potential temperature $\theta$ at Laverton shows that $\theta$ changes most rapidly near the ground and becomes nearly constant in a few hundred metres. Accordingly, as long as the planetary boundary layer extends to the upper limit of, or beyond this region of change, a good approximation to $\Delta \theta$ is simply the increase of $\theta$ from $z_0$ to say 1 km. This approximation is the sum of two components, namely the increase in $\theta$ from $z_0$ to the level of the Stevenson screen, 1.2 m, and the increase from 1.2 m to 1 km. The second is directly available from observations. The first is obtained by integrating the flux-gradient equation for $\Delta \theta/\Delta z$ (Appendix B) from $z_0$ to 1.2 m, using values of $u_*$ and $H_o$ already obtained. As mentioned earlier, $\Delta \theta$ should strictly be measured from $z_0$. But, as pointed out by Garratt and Francey (1978), although measurement from $z_0$ may introduce an error (which for the LSSE data will be rather small) because of compensation in the estimation of $H_o$, the effects on C are negligible.
Laverton is located on a grassy plain 5 km inland from the western edge of Port Phillip Bay. The coast runs approximately southwest-northeast. Grass height is variable from bare earth to 0.6 m. Occasional clumps or lines of trees up to 10 m high occur along the roads or creeks, and farm buildings and fences contribute to the surface roughness. The spacing of these items is about 1 km. Within about 2 km radius of the observation site the southeast quadrant is occupied by a settlement chiefly of single storeyed houses. The roughness length appropriate to the Laverton site was estimated at 0.1 m. This was based on the values given by Plate (1971) for various terrains. The present site was assessed to be somewhere between 'flat and open country' and 'wood land' given in Plate's Fig. 1.13.

During the LSSE a network of barometers was set up around the Laverton site with the aim of obtaining the surface geostrophic wind. However, subsequent analysis of their data has shown them to be inadequate for the present purposes. This is elaborated in Appendix D. The surface geostrophic wind was therefore obtained from the horizontal pressure gradient at Laverton taken from mean sea level synoptic charts prepared at three-hourly intervals by the Central Analysis Office of the Australian Bureau of Meteorology. The wind is decomposed into eastward and northward components, and these are smoothed by the five-point filter described in Appendix A to suppress observational errors and short-duration fluctuations.

RESULTS AND DISCUSSION

Friction velocities

The two approaches to the evaluation of \( u_* \) give markedly different values. Closer examination is warranted since these differences are relevant to the basic problem in hand. Frictional velocities obtained by the planetary boundary layer method (denoted by the subscript PBL) are weakly correlated with those obtained by the surface layer approach (denoted by subscript SFC). The coefficient of correlation \( r = 0.29 \), based on 175 data pairs, is significantly different from zero at a probability level better than 0.01. The linear regression equation is:

\[
(u_*)_{\text{PBL}} = 0.509 + 0.278 (u_*)_{\text{SFC}}
\]

... 9

(if \( u_* \) has units of m/s). It can be seen in Fig. 1 that over the range of \( u_* \) obtained \( (u_*)_{\text{PBL}} \) is generally larger than \( (u_*)_{\text{SFC}} \).

The essential difference between the two approaches is the 'fetch' over which the flow has been sampled. Pasquill (1972) shows how effective fetches are dependent on the height above the surface at which observations are made. In the case of the surface layer method this height is 10 m with an effective fetch of the order of 500 m, whereas for the planetary boundary layer method the height is 2 km with an effective fetch of the order of 100 km. Thus the two approaches measure the stresses on different scales. The surface method will measure stress on a local scale over the flat terrain surrounding the anemometer, whereas the planetary boundary layer method will include stresses due to larger scale topographical features. Since no assumptions have been made about \( z_* \) in deriving \( u_* \) using the latter method, the values of \( (u_*)_{\text{PBL}} \) may be used to compute a roughness length by assuming a logarithmic wind profile of the form
Fig. 1  Individual values of \((u^*_\text{PBL})\) versus \((u^*_\text{SFC})\).
where \( u \) is the mean wind speed observed at \( z = 10 \) m. It may be argued that the use of the wind at 10 m is inappropriate in Eqn 10 because the friction velocity is representative of large scale conditions. However, the 10 m wind will be correlated with the large scale flow, and by using a lower level wind one tends to minimise the effects of thermal stratifications. Certainly the result should be representative of the large scale \( z_0 \). Because of the skewed distribution of \( z \) obtained by this equation a logarithmic mean value has been obtained. This is 0.75 m with a standard error of 0.09 m. It is most pleasing to find that this value lies midway between estimates of effective roughness lengths made by Fiedler and Panofsky (1972) for plains (0.42 m) and low mountains (0.99 m). Their estimates, from low level aircraft data are at essentially the same scales as the present data. The values of \( (u_*)_{PBL} \) are also consistent with recent work by Garratt (1977).

Since the topography varies with fetch direction but the local roughness surrounding the anemometer is practically constant, the ratio \( (u_*)_{PBL}/(u_*)_{SFC} \) might be expected to be dependent on wind direction. A contingency table was set up (Table 1) with the data grouped according to the range of the ratio and wind direction. A chi-square value of 20.7 on six degrees of freedom, which is significant at the 0.005 level, is obtained from this table on the assumption of no directional dependence. In all quadrants the average ratio is greater than unity with a maximum of 2.5 for the southeast quadrant and a minimum of 1.7 for the southwest quadrant. However, it is not clear, from an examination of the topography within 100 km of Laverton, precisely what are the causes of the variations in these values.

<table>
<thead>
<tr>
<th>Range of ((u_<em>)<em>{PBL}/(u</em></em>)_{SFC})</th>
<th>NE</th>
<th>SE</th>
<th>SW</th>
<th>NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 1.19</td>
<td>8 (10.1)</td>
<td>7 (9.5)</td>
<td>24 (18.4)</td>
<td>6 (7.0)</td>
</tr>
<tr>
<td>1.20 to 1.99</td>
<td>8 (9.4)</td>
<td>5 (8.9)</td>
<td>22 (17.1)</td>
<td>7 (6.6)</td>
</tr>
<tr>
<td>2.00 to 2.99</td>
<td>8 (6.5)</td>
<td>7 (6.1)</td>
<td>11 (11.8)</td>
<td>3 (4.5)</td>
</tr>
<tr>
<td>3.00 and over</td>
<td>9 (7.0)</td>
<td>12 (6.5)</td>
<td>3 (12.6)</td>
<td>7 (4.9)</td>
</tr>
</tbody>
</table>

A further criterion indicates that \( (u_*)_{PBL} \) is not locally representative. This is the surface heat flux implied by \( (u_*)_{PBL} \) and observed temperature profiles. The diurnal variation of mean values of heat flux is shown in Fig. 2. It can be seen that when integrated over 24 hours the heat flux implied by \( (u_*)_{PBL} \) gives only a very small net heat input to the atmosphere, amounting to about 0.3 MJ m⁻². In contrast the heat flux obtained by the surface method gives a net heat input of 4.5 MJ m⁻² in 24 hours, which is a more realistic fraction of the mean daily amount of incident solar radiation received at Laverton over the period of observation, namely 17 MJ m⁻².
Functions A, B and C

A problem in determining A, B and C is to decide which value of \( u_* \) to use, that representative of 'local' conditions, or that estimated by the PBL method and representative of the 'large scale'. Probably the 'large scale' values are more appropriate to the non-homogeneous situation. However, several considerations mitigate against their use. Most studies reported in the literature use the equivalent to local fluxes (see Table 2) and thus for comparison purposes their use here is preferred. Also, since \( H \) can only be estimated on a 'local' scale, it would seem appropriate to use the 'local' \( u_* \) to avoid the inconsistencies illustrated in Fig. 2. The analysis presented here is therefore based on 'local' values of \( u_* \) and \( H \). Figures 3 to 5 show mean values of A, B and C for data grouped according to stability parameter \( \mu \). Each data point represents either 19 or 20 observations. All three functions exhibit dependencies on \( \mu \) similar to those found by Clarke (1970b, Figs 9 to 11), except that the magnitude of C becomes very large near \( \mu = 0 \). This is due to the failure of observed values of and computed values of \( H \) in Eqn 3 to approach zero together. Mean values of A, B and C for 67 occasions when \( \mu \) is in the range -10 to +10 (i.e. the near neutral condition) are set out in Table 2.

<table>
<thead>
<tr>
<th>Source</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSSE</td>
<td>3.0 (0.8)</td>
<td>5.1 (0.9)</td>
<td>2.9 (16.3)</td>
</tr>
<tr>
<td>Plate (1971)</td>
<td>1.7</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>Clarke (1970a, b)</td>
<td>1.7</td>
<td>4.3</td>
<td>1.5</td>
</tr>
<tr>
<td>Deacon (1973)</td>
<td>1.9</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>Clarke and Hess (1974)</td>
<td>1.1 (0.5)</td>
<td>4.3 (0.7)</td>
<td></td>
</tr>
<tr>
<td>Arya (1975)</td>
<td>1.0</td>
<td>5.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Yamada (1976)</td>
<td>1.9</td>
<td>3.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Brutsaert and Chan (1978)</td>
<td></td>
<td></td>
<td>5.0</td>
</tr>
<tr>
<td>Garratt and Francey (1978)</td>
<td></td>
<td></td>
<td>5.0</td>
</tr>
</tbody>
</table>

To discover the effect of horizontal temperature gradient upon A, B and C, data have been grouped according to \( \mu \) using the same ranges as Clarke and Hess (1974) and correlation statistics obtained. These are shown in Table 3. The partial derivatives of A, B and C with respect to \( \mu \), \( \hat{S}_x \) and \( \hat{S}_y \) are generally consistent in sign across \( \mu \) categories. No derivatives of A with respect to \( \hat{S}_x \) and \( \hat{S}_y \) are found to be significantly different from zero at the 0.10 level. In the \( \mu \) range -250 to -20, no derivatives of B or C with respect to \( \mu \), \( \hat{S}_x \) and \( \hat{S}_y \) are found to be significantly different from zero at the 0.10 level. Table 3 supports the findings of Clarke and Hess (1974) that the
Fig. 2  Heat flux versus time of day.  Mean values shown. Broken lines indicate surface method, solid lines indicate PBL method.

Fig. 3  Mean values of similarity function A versus mean values of $\mu$. Each data point represents either 19 or 20 observations. Error bars are two standard errors on each side of the mean. Broken line due to Arya (1975).
Fig. 4 Mean values of similarity function B versus mean values of $\mu$. Each data point represents either 19 or 20 observations. Error bars are two standard errors on each side of the mean. Broken line due to Arya (1975).

Fig. 5 Mean values of similarity function C versus mean values of $\mu$. Each data point represents either 19 or 20 observations. Error bars are two standard errors on each side of the mean. Broken line due to Arya (1975).
Table 3  Derived linear dependence of A, B and C on \( \mu \), \( \hat{S}_x \) and \( \hat{S}_y \) in several ranges of \( \mu \). Significance levels shown in parentheses if 0.10 or less

<table>
<thead>
<tr>
<th>( \theta ) range</th>
<th>-250 to -20</th>
<th>-90 to +90</th>
<th>0 to +100</th>
<th>-20 to +20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>51</td>
<td>149</td>
<td>84</td>
<td>99</td>
</tr>
<tr>
<td>( \partial A / \partial \mu )</td>
<td>-0.013 (0.01)</td>
<td>-0.247 (0.01)</td>
<td>-0.632 (0.01)</td>
<td>-0.116 (0.10)</td>
</tr>
<tr>
<td>( \partial A / \partial \hat{S}_x )</td>
<td>0.326</td>
<td>0.347</td>
<td>0.170</td>
<td>0.081</td>
</tr>
<tr>
<td>( \partial A / \partial \hat{S}_y )</td>
<td>-0.081</td>
<td>-0.034</td>
<td>-0.033</td>
<td>-0.169</td>
</tr>
<tr>
<td>( \partial B / \partial \mu )</td>
<td>0.005</td>
<td>0.099 (0.01)</td>
<td>0.311 (0.01)</td>
<td>0.070</td>
</tr>
<tr>
<td>( \partial B / \partial \hat{S}_x )</td>
<td>0.068</td>
<td>0.169</td>
<td>0.350 (0.05)</td>
<td>0.359 (0.01)</td>
</tr>
<tr>
<td>( \partial B / \partial \hat{S}_y )</td>
<td>-0.031</td>
<td>-0.269 (0.05)</td>
<td>-0.304 (0.10)</td>
<td>-0.260 (0.02)</td>
</tr>
<tr>
<td>( \partial C / \partial \mu )</td>
<td>0.005</td>
<td>1.004 (0.01)</td>
<td>-1.629 (0.05)</td>
<td>-1.847 (0.10)</td>
</tr>
<tr>
<td>( \partial C / \partial \hat{S}_x )</td>
<td>-0.012</td>
<td>-2.422 (0.10)</td>
<td>-3.988 (0.05)</td>
<td>-3.745 (0.05)</td>
</tr>
<tr>
<td>( \partial C / \partial \hat{S}_y )</td>
<td>0.049</td>
<td>2.383 (0.10)</td>
<td>4.748 (0.02)</td>
<td>2.441</td>
</tr>
</tbody>
</table>

Percentage variance of A accounted for by

| \( \mu \), \( \hat{S}_x \), \( \hat{S}_y \) | 16 | 18 | 26 | 7 |
| \( \hat{S}_x \), \( \hat{S}_y \) | 2 | 3 | 3 | 4 |

Percentage variance of B accounted for by

| \( \mu \), \( \hat{S}_x \), \( \hat{S}_y \) | 2 | 10 | 24 | 19 |
| \( \hat{S}_x \), \( \hat{S}_y \) | 1 | 3 | 3 | 4 |

Percentage variance of C accounted for by

| \( \mu \), \( \hat{S}_x \), \( \hat{S}_y \) | 2 | 11 | 16 | 10 |
| \( \hat{S}_x \), \( \hat{S}_y \) | 0 | 3 | 11 | 7 |

The bulk of the variance of A, B and C is not due to \( \mu \), \( \hat{S}_x \) and \( \hat{S}_y \), and that the variance accounted for by \( \hat{S}_x \) and \( \hat{S}_y \) is an order of magnitude smaller than that accounted for by \( \mu \). Table 4 makes a comparison of present and previous determinations of derivatives of A and B with respect to \( \hat{S}_x \) and \( \hat{S}_y \). The confidence interval for \( \partial A / \partial \hat{S}_x \) obtained in the present study includes earlier estimates by Wipperman (1972) and Clarke and Hess (1974), as shown in Table 4.
Table 4  
Linear dependences of A and B on \( \hat{S}_x \) and \( \hat{S}_y \) from sources. Note for \( \partial A/\partial \hat{S}_y \) and \( \partial B/\partial \hat{S}_y \) for Wipperman, and Clarke and Hess the signs of the values have been altered to conform to the right handed system of axes used in the LSSE analysis.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial A/\hat{S}_x )</td>
<td>0.1 to 0.6</td>
<td>0.13</td>
<td>0.2</td>
</tr>
<tr>
<td>( \partial A/\hat{S}_y )</td>
<td>-0.2 to 0.2</td>
<td>-0.11</td>
<td>0.03 to 0.07</td>
</tr>
<tr>
<td>( \partial B/\hat{S}_x )</td>
<td>0.0 to 0.3</td>
<td>-0.11</td>
<td>-0.2 to -0.4</td>
</tr>
<tr>
<td>( \partial B/\hat{S}_y )</td>
<td>-0.1 to -0.4</td>
<td>-0.13</td>
<td>-0.2 to -0.4</td>
</tr>
</tbody>
</table>

* These ranges are one standard error either side of the mean value for derivatives found when \( \mu \) is in the range -90 to +90. This \( \mu \) range contains 86 per cent of all observations.

The confidence interval for \( \partial A/\hat{S}_y \) also includes both earlier estimates, but favours the finding of Clarke and Hess (1974) that \( \partial A/\hat{S}_x \) is positive and an order of magnitude smaller than \( \partial A/\hat{S}_y \). Present results for the derivatives of B agree in magnitude but are contrary in sign in the case of \( \partial B/\hat{S}_x \). It is to be noted that the values presented by Clarke and Hess (1974) are derived from a larger and more comprehensive set of field observations than that used in the present study. Additionally they were made at a near homogeneous site. These facts are reflected in the levels of significance associated with their tabulated partial derivatives, which are mostly 0.001.

**CONCLUSIONS**

The results obtained indicate that Rossby-number similarity functions A and B at the inhomogeneous site of Laverton, Victoria, have a dependence on stability parameter \( \mu \) similar to that found at homogeneous sites, e.g. by Arya (1975). Results for function C are too subject to uncertainty to permit meaningful comparison with previous work. The effects of horizontal variation of temperature as expressed by functions \( \hat{S}_x \) and \( \hat{S}_y \) do not completely match those found by Clarke and Hess (1974). Results for \( \partial A/\hat{S}_x \), \( \partial A/\hat{S}_y \) and \( \partial B/\hat{S}_y \) agree in both magnitude and sign with earlier estimates by Clarke and Hess (1974). However, results for \( \partial B/\hat{S}_x \) agree in magnitude but are opposite in sign to estimates by Clarke and Hess (1974).
The PBL method of obtaining $u_*$ by analysis of the path followed by a wind-finding balloon through the planetary boundary layer provides an estimate of the effective roughness length near Laverton. This value, 0.75 m, compares with other estimates for similar terrain, e.g. Fiedler and Panofsky (1972) and Garratt (1977).

The present results provide some justification for using the Rossby-number similarity theory in numerical models requiring boundary layer parameterisation even if the local topography is not homogeneous. As mentioned in the Introduction, other scaling approaches might also be relevant, but they have not been examined in the exercise. However, a problem remains in that such parameterisation still explains only a small proportion of the variance of the surface fluxes of heat and momentum. At the Laverton site the amount of unexplained variance is large and comparable with the 'ideal' sites of other studies. It is, however, very likely that, as with other sites, some of this can be attributed to observational difficulties. Nevertheless, the planetary boundary layer is a most complex region of the atmosphere and simple parameterisations, such as the present, cannot be expected to do more than capture its grosser features.

The difference between shearing stress on local scales and that of scales typically represented in numerical weather prediction models has only been touched here. It can be seen that the stresses are considerably different, and modellers need to give careful consideration to the formulation of their parameterisation when dealing with this quantity.

ACKNOWLEDGMENTS

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REFERENCES


APPENDIX A

Five-point filter

The surface winds and surface geostrophic winds used in this study are obtained from synoptic weather data, and it is unlikely that individual values of these winds are known with accuracies better than ±1 m/s for speed and ±20° for direction. It is therefore considered necessary to smooth the wind data in some way to decrease the variation due to observational errors and to exclude short-term fluctuations not relevant to scales of the present study. The approach adopted is to resolve the winds into orthogonal components and to subject each to a five-point filter of the form

\[ x_i^1 = \sum_{j=0}^{2} a_j (x_{i+j} + x_{i-j}) \]

where \( x_i^1 \) is the filtered value of the \( i \)th observation time, \( x_i \) is the unfiltered value at the \( i \)th observation time and \( a_j \) are the filter coefficients. The filter coefficients chosen are \( a_0 = 0.237 \), \( a_1 = 0.355 \) and \( a_2 = -0.072 \).

The transmission function of this filter is shown in Fig. A1. It can be seen that for periods exceeding 24 hours more than 90 per cent of the variance of sinusoidal components is passed, and this decreases for shorter periods. It is desirable to maintain the variance due to diurnal and synoptic scale components since the theory being examined assumes a steady state boundary layer, and this condition is most likely to be met with respect to these longer period fluctuations. The filter does not cause phase shifts. After filtering the smoothed wind components are recombined.

![Response function graph](image)

**Fig. A1** Response function for five-point filter versus frequency and period of sinusoidal input sampled at 3-hourly intervals.
APPENDIX B

'Surface method' of estimating $u_*$ and $H_o$

Estimation of friction velocity $u_*$ and heat flux $H_o$ is based on the conventional flux-gradient relations

$$\frac{\partial u}{\partial z} = u_* \Phi_M(z/L) / (kz) \quad \ldots \quad \text{B1}$$

and

$$\frac{\partial \theta}{\partial z} = -H_o \Phi_H(z/L) / (\rho C_p u_*^3 \theta / (k g H_o)) \quad \ldots \quad \text{B2}$$

where the non-dimensional functions are assigned the Businger-Dyer form (e.g. Clarke 1970a)

$$\Phi_M = \Phi_H = [1 - 15 (z/L)]^{-0.275}, \quad L \leq 0$$

and

$$\Phi_M = \Phi_H = 1 + 4.7 (z/L), \quad L > 0$$

where $L = -\rho C_p u_*^3 \theta / (k g H_o)$

is the Monin-Obukhov length scale, $u$ is the mean wind speed at height $z$, $k$ is von Karman's constant, $\theta$ is potential temperature at height $z$, $H_o$ is the upward turbulent flux of sensible heat at the ground, $\rho$ is the air density, and $C_p$ is the specific heat of air at constant pressure.

Integration of Eqn B1 from $z_o$, at which $u = 0$, to anemometer height 10 m, gives

$$u_{10} = (u_* / k) \sum_{z_o}^{10} (1/z) \Phi_M \, dz \quad \ldots \quad \text{B3}$$

Equations B2 and B3 form a pair of simultaneous equations that can be solved numerically to give $u_*$ and $H_o$ for given values of $u_{10}$ and $\partial \theta / \partial z$ at some height.

The value of $z_o$ has to be given before these equations can be solved. As discussed under 'Method', a value of 0.1 m was used at the Laverton site.

The wind at 10 m is estimated from the Laverton anemometer records. This anemometer is mounted on a 10 m mast and is over 1 km from the nearest obstruction, a hangar 8 m high to the east-southeast. The exposure is uniformly good and the surrounding country is very flat (Whittingham 1964). The wind speed can be estimated to 0.5 m/s and direction to 10° from the anemometer records. To minimise the effect of observational errors and short-duration fluctuations the observed winds are smoothed (Appendix A).

The lapse rate $\partial \theta / \partial z$ is estimated at the arbitrary height of 10 m using radiosonde data. The type of radiosonde used has a baroswitch, and from its calibration data it is possible to estimate the pressure at 'switching points' to better than 0.2 mb. To find $\partial \theta / \partial z$ the temperature $T_1$ and pressure $p_1$ at the first switching point are used together with temperature $T_o$ and pressure $p_o$ observed at screen level. Using the hydrostatic equation and assuming that the lapse rate of temperature varies approximately as the inverse of height (e.g. Priestley 1959, p. 39) it can be shown that at 10 m
\( \frac{\partial \theta}{\partial z} = 0.00976 + \frac{(T_1 - T_0)}{10 \ln [1 - 7(p_1 - p_0)]} \)

for temperature in K, height in m, and pressure mb.

A numerical technique is used to solve Eqns B2 and B3 for \( u^* \) and \( H \) iteratively, given values of \( u_{10} \) and \( \frac{\partial \theta}{\partial z} \) estimated above. Equation B3° is readily integrated numerically by changing the variable of integration to \( \ln z \) and using a ten-point Gaussian quadrature formula.

This method is essentially a drag coefficient approach.
APPENDIX C

"Planetary boundary layer method" of obtaining $u_A$

This is an adaptation of the 'velocity deficit' method (e.g. Haltiner and Martin 1957, p. 227) using as data the displacement of the radar-tracked balloon from the release point. Derivation begins with the usual equations for unaccelerated horizontal flow in the planetary boundary layer (e.g. Plate 1971, p. 11)

$$-\rho \frac{f}{g} (v - v_0) = \frac{\partial \tau_x}{\partial z} \quad \text{ ... C1}$$

$$\rho \frac{f}{g} (u - u_0) = \frac{\partial \tau_y}{\partial z} \quad \text{ ... C2}$$

where $\rho$ is air density, $f$ is the Coriolis parameter, negative in the southern hemisphere, $u$ and $v$ are horizontal wind velocity components in the $x$ and $y$ direction (these directions may be chosen arbitrarily as long as the $y$ direction is perpendicular to and to the left of the $x$ direction), $u_0$ and $v_0$ are geostrophic wind velocity components, $\tau_x$ and $\tau_y$ are eddy stress components, and $z$ is height above the ground. By assuming that, at least over the thickness of the planetary boundary layer, there is a constant horizontal temperature gradient, the thermal wind equations (e.g. Haltiner and Martin 1957, p. 204) give

$$u = u_0 - \frac{g}{f} \frac{\partial \theta}{\partial y} = u_0 + \frac{\frac{f}{g} \frac{\partial \theta}{\partial y}}{k^2} \hat{S}_x \quad \text{ ... C3}$$

$$v = v_0 + \frac{g}{f} \frac{\partial \theta}{\partial x} = v_0 + \frac{\frac{f}{g} \frac{\partial \theta}{\partial x}}{k^2} \hat{S}_y \quad \text{ ... C4}$$

where $u_0$ and $v_0$ are the values of $u$ and $v$ at $z = 0$, and $\hat{S}_x$ and $\hat{S}_y$ are the components of the non-dimensional thermal wind shear defined by Eqns 5 and 6. Manipulation of Eqns C1 to C4 provides expressions for $\frac{\partial \tau_x}{\partial z}$ and $\frac{\partial \tau_y}{\partial z}$, which may be integrated with respect to $z$ from the ground up to some height $\hat{z}$, giving

$$\frac{(\tau_x \hat{z} - \tau_{x0})}{\rho} = \frac{f}{g} v_0 \hat{z} - f Y W + \frac{1}{2} \frac{f}{k^2} \hat{S}_y \hat{z}^2 \quad \text{ ... C5}$$

$$\frac{(\tau_y \hat{z} - \tau_{y0})}{\rho} = -\frac{f}{g} u_0 \hat{z} + f X W - \frac{1}{2} \frac{f}{k^2} \hat{S}_x \hat{z}^2 \quad \text{ ... C6}$$

where subscript $0$ denotes the value of a variable at height $0$, $W$ is the rate of ascent of the wind-finding balloon, and $(X, Y, \hat{z})$ are balloon coordinates with origin at release point. In integrating, $\rho$, $\hat{S}_x$, and $\hat{S}_y$ are assumed to be constant with height.

Let functions $G$ and $H$ be defined as follows:

$$G = -\frac{f}{g} (v_0 \hat{z} - Y W) \quad \text{ ... C7}$$

and

$$H = f (u_0 \hat{z} - X W) \quad \text{ ... C8}$$
Successive values of these functions can be computed as the balloon traces its path through the boundary layer. It follows from Eqns C5 and C6 that

\[ G = \frac{\tau_{x_0}}{\rho} + f|f| \frac{\hat{S}_y z^2}{(2 k^2)} - \frac{\tau_{x_0}}{\rho} \quad \ldots \text{C9} \]

and \[ H = \frac{\tau_{y_0}}{\rho} - f|f| \frac{\hat{S}_x z^2}{(2 k^2)} - \frac{\tau_{y_0}}{\rho} \quad \ldots \text{C10} \]

As \( z \) approaches the top of the planetary boundary layer \( \tau_{x_0} \) and \( \tau_{y_0} \) tend to zero, so that \( G \) and \( H \) tend to linear functions of \( z^2 \). The asymptotes of \( G(z^2) \) and \( H(z^2) \) have intercepts \( \tau_{x_0}/\rho \) and \( \tau_{y_0}/\rho \) and slopes \( \frac{f|f|}{2k^2} \hat{S}_x \) and \( \frac{f|f|}{2k^2} \hat{S}_y \) respectively.

The technique used to obtain the asymptotes is to fit a least squares regression line to values of \( G \) and \( H \) versus \( z^2 \), for \( z \) in the range 1 to 2 km, on the assumption that the stress components at these heights are close to zero. The values \( \tau_{x_0} \) and \( \tau_{y_0} \) so obtained define the magnitude and direction of surface stress \( \tau_o \), and \( u_* \) is of course \( (\tau_o/\rho)^{1/2} \).
APPENDIX D

Barometer network method of obtaining geostrophic wind

A network of seven stations within 45 km radius of Laverton was set up for the duration of the Laverton Serial Sounding Experiment to measure surface atmospheric pressure. Observed pressure $p_s$ at station level was used together with temperature $T$ and station elevation $z$ to compute hypothetical pressure $p$ at the level of Laverton, thus

$$ p = p_s \left(1 + g \frac{\Delta z}{R T}\right) \quad \text{... D1} $$

where $\Delta z$ is the height of the observing station above the level of Laverton and $R$ is the gas constant for air. Values of $p$ at each station were subjected to a running-mean filter similar to that described in Appendix A to suppress short-duration fluctuations. Then the seven smoothed values of $p$ were used to obtain regression coefficients for $p$ in terms of $x$ and $y$, as represented in the equation

$$ \hat{p} = a + bx + cy \quad \text{... D2} $$

Since $b$ and $c$ are estimates of horizontal derivatives $\partial p/\partial x$ and $\partial p/\partial y$ in the vicinity of the observing network, it follows from the definition of surface geostrophic wind $(u_{go}, v_{go})$ that

$$ u_{go} = -\frac{R T c}{p f} \quad \text{... D3} $$

and

$$ v_{go} = \frac{R T b}{p f} \quad \text{... D4} $$

where $f$ is the Coriolis parameter, negative in the southern hemisphere.

Geostrophic wind estimates obtained by this method have not been used to compute similarity functions. As evident in Fig. D1 there is a spurious diurnal variation in westerly components due to the height adjustment procedure. Because of the slope of the terrain upwards to the north, at times of maximum air density, i.e. between 0000 and 0600 local time, the northward component of horizontal pressure gradient computed for the level of Laverton reaches a maximum. This produces diurnal maxima in $u_{go}$. The estimation of geostrophic wind is further complicated by the proximity to the coast. Because of the large fluctuations in computed values of $u_{go}$ and $v_{go}$ the mean speed of the geostrophic wind obtained by this method is twice the mean observed wind speed at a height of 1 km, and has three times its standard deviation.
Fig. D1 Westerly components of surface geostrophic wind. Values obtained from the seven-station network indicated by broken lines. Values obtained from synoptic charts and smoothed are indicated by solid lines. Black rectangles indicate local standard time periods 00 to 06 hours, when the seven-station network method gives sharp peaks in the westerly component.