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Optimum instrument spacing for cyclone interception in the Australian tropics

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In the design of an instrument network required to measure characteristics near the eye of tropical cyclones in the Australian region, network spacing is obviously an important parameter. This short paper discusses the factors involved in optimising this parameter and, on the basis of an elementary statistical analysis, recommends network spacings to maximise cyclone interception frequencies.

Introduction

Since detailed data on the surface characteristics of tropical cyclones in the vicinity of the cyclone eye are particularly poor for cyclones in the Australian region, it might be thought desirable to design and implement an instrument network specifically to obtain such data. Such a network would probably be quasi-linear, in line with coastline configuration, and an important parameter, from cost/benefit considerations, is the spacing between instruments. Thus, for example, a coastal network in which this spacing is, say, 10 km should intercept all cyclones crossing the coast but would involve prohibitive costs, while a network spacing of 200 km would result in such infrequent interception as to prove ineffective. Some optimisation of network spacing should be possible.

To do this, in effect what is required is a knowledge of the probability that cyclone maximum wind bands will intercept at least one instrument in the network as a function of instrument spacing and location. The following argument to determine such probabilities is similar to that employed by Spillane and Dexter (1976) for deriving design wave heights generated by tropical cyclones in the Australian region.

If a cyclone of radius of maximum wind R passes within a distance D of an arbitrary point P, then the probability that P will experience the maximum wind (assumed circular, though windspeed need not be symmetrical about the cyclone eye) is just R/D (for $R \leqslant D$). Thus if N_D cyclones pass within D of P in Y years, the frequency, per year, of interception of maximum wind bands is

$$F = \frac{R}{D} \cdot \frac{N_D}{Y} \qquad \ldots 1$$

Distribution of cyclone radii

Radius R is interpreted now as being the average, $\bar{R}$, for all cyclones at the latitude of P. Detailed data on the distribution of $\bar{R}$s in the Australian region are very limited. Using dynamical arguments Riehl (1963) showed that $R \propto \sin^2 \varphi$, and the data of Shea and Gray (1973) supported this. By combining what radar data are available for Australian cyclones P. J. Meighen (personal communication) has established the relationship

$$R_{radar} = 0.909 + 135.78 \sin^2 \varphi \text{ (km)} \qquad \ldots 2$$

where R_{radar} is the radius of the cyclone eye as detected by S-band radar and φ is latitude. This is supported by the results of Shea and Gray (1973), and since it may be inferred from that paper also that

$$\bar{R}_{max\ wind} = \bar{R} \simeq 1.39\ \bar{R}_{radar}\ (\bar{R} \leqslant 50 \text{ km}) \qquad \ldots 3$$

$$\text{then } \bar{R} = 1.263 + 188.74 \sin^2 \varphi \text{ (km)} \qquad \ldots 4$$

Eqn 4 is probably valid only for cyclones at latitudes greater than 10°S. A relationship similar to Eqn 4 has been used by Martin and Bubb (1976) in their study of tropical cyclone gust speeds in the Australian region.

While the available R data are barely sufficient to allow determination of $\bar{R}$ by a regression technique, they are totally inadequate to describe the distribution of Rs about $\bar{R}$. For the purposes of the study, some assumption must be made as to the form of this distribution. Such an assumed distribution should be both simple and conservative, while conforming to relevant physical constraints.

One such constraint is that there must be a minimum radius of maximum wind $R_L > 0$. R_L may vary slightly with latitude (as $\bar{R}$), but the simplest (and conservative) assumption would leave it constant. Again, there may be an upper limit, R_U, to the distribution of radii, at which the probability of occurrence of a cyclone with $R = R_U$ is vanishingly small. R_U will probably increase with increasing latitude, and for some assumed distributions it may be simpler to allow $R_U \rightarrow \infty$.

A uniform distribution in the range R_L to R_U, in which the probability distribution function is

$$p_R = \frac{1}{R_U - R_L} \qquad \ldots 5$$

is the simplest. (It is also the most conservative, since it assigns relatively large probabilities to cyclone radii that differ significantly from $\bar{R}$) Now from Eqn 1 the probability that cyclones with radii in the range R to R + dR will be intercepted by the point P is $p_R.\frac{R}{D}.dR$, and

$$Q = \int_{R_L}^{R_U} p_R.\frac{R}{D}.dR \quad (R_U \leq D) \qquad \ldots 6$$

is the probability that any storm's maximum wind band will be intercepted by P. In this case, substitution for p_R from Eqn 5 in Eqn 6 gives

$$Q = \frac{\bar{R}}{D} \qquad \ldots 7$$

where $\bar{R}$ is the centre of the possible uniform range of Rs in the range R_L to R_U.

While the Gaussian distribution

$$P_R = \frac{1}{\sigma_R \sqrt{2\pi}} \exp\left(-1/2\,(R - \bar{R})^2 / \sigma_R{}^2\right) \quad \ldots 8$$

with mean $\bar{R}$ and variance $\sigma_R{}^2$, is well known and simple to apply, application in this case is not immediately obvious since the distribution is theoretically unbounded at both ends, with the possibility of negative Rs. However, if it is assumed that probability of occurrence becomes vanishingly small for $R = R_L$ (and R_U), then choice of a suitable σ_R and $\bar{R}$ such that

$$p_R\,(R_L) = p_R\,(R_U) \approx 0 \qquad \ldots 9$$

allows application in this range. Since probability is less than 0.3 per cent for R greater than R_U and less than R_L where

$$\bar{R} - R_L = R_U - \bar{R} = 3\sigma_R, \qquad \ldots 10$$

these may be regarded as reasonable limits. Eqn 10 also allows computation of σ_R, given a knowledge of $\bar{R}$ and R_L. Substitution of Eqn 8 into Eqn 6 and integrating under the constraints of Eqn 9 gives once again

$$Q = \frac{\bar{R}}{D}$$

Note that if the correct Gaussian distribution limits $-\infty$ to $+\infty$ are applied, the relationship obtained is $Q = \frac{1}{D}\,(R + \sqrt{\frac{2}{\pi}}\,\sigma_R)$.

It is expected that the quasi-Gaussian distribution defined by Eqns 8, 9 and 10 provides a reasonable description of the distribution of cyclone radii at the latitudes where the bulk of Australian region cyclones occur (i.e. 15°S to 25°S (Lourensz 1977)).

At lower latitudes, one might anticipate the distribution to be skewed towards R values greater than $\bar{R}$ where a distribution function such as the log-normal would be more appropriate, with

$$p_R\,dR = \frac{1}{\sigma_R \sqrt{2\pi}} \exp\left(-(\log R - \log \xi)^2 / 2\sigma^2\right) \; d \log R \qquad \ldots 11$$

Log ξ is defined as being the mean of log R, so that ξ is the median in the distribution of R (Hald 1952). In this case, if Eqn 11 is substituted into Eqn 6, the defining equation for the mean, $\bar{R}$, is obtained (with axis translation) (Hald 1952), and again

$$Q = \frac{\bar{R}}{D} \qquad \ldots 12$$

In addition, from the definition

$$\bar{R} = \xi . 10^{\left(\frac{\sigma^2 (\log R)}{2 \log e}\right)} . \qquad \ldots 13$$

Estimation of $\xi\sigma$ (log R) in this case is difficult since it is related to the skewness, or $\bar{R}/\xi$. However, Hald (1952) points out that, for practical purposes, both log R and R can be regarded as normally distributed provided

$$\frac{\sigma_R}{\bar{R}} < \frac{1}{3}.$$

If the constraints of Eqn 10 are accepted for these low latitude cyclones also then this condition is satisfied provided $R_L > 0$. Thus there is justification in assuming R to be normally distributed for all cyclones below about 25°S, with Eqns 8 to 10 applying. It is worth noting here that such an assumption is also conservative, since if the real distribution is skewed towards larger R values, this will result in larger cyclone interception frequencies than those computed for normally distributed R, as discussed later.

At latitudes above 25°S the position is even less clear. However, Eqns 4 and 10 would provide both large $\bar{R}$ and large σ_R at these latitudes, which is in accord with available data. Thus in the absence of other information and to preserve simplicity the modified Gaussian distribution has been assumed to apply here also. In any event, the results presented later show that such an assumption will not be critical for cyclone interception frequency. In addition, any symmetrical, bounded distribution will produce similar results, provided the constraints of Eqn 10 are satisfied.

Probability of interception

Now if the 'hit' is defined as the passage of a cyclone maximum wind band across an instrument site, then the frequency of hits per year, in a given length of coastline L centred on specified point P, for an instrument network spacing 2D, is just (from Eqn 1)

$$F_H = \frac{\bar{R}}{D} \cdot \frac{N_D}{Y} \cdot \frac{L}{2D} \qquad \ldots 14$$

Eqn 14 will hold provided $\bar{R} \leq D$, with the $\bar{R}$ being evaluated from Eqn 4 as discussed above.

There is one further piece of information, which allows extrapolation of hit frequencies beyond the apparent limit $\bar{R} = D$ of Eqn 14. This is that as the instrument spacing 2D approaches zero all cyclones crossing (or passing within $\bar{R}$ of) the coast will be intercepted. Thus F_H must asymptotically approach F_L, the number of cyclones crossing (or just grazing) a length of coastline L, per year. (Strictly speaking, an extrapolation would be unnecessary if all cyclones had radius $\bar{R}$, since F_L would be approached asymptotically in this case at $2D = 2\bar{R}$, and the limit of Eqn 14 would not be violated. Radii in practice extend down to a R_L close to zero, hence the requirement for extrapolation. Indeed, Eqn 14 is still applicable, but with $\bar{R}$ replaced by actual R.)

Cyclone data

The basic data source for cyclone occurrence frequencies is Lourensz (1977), and this is currently being updated (Lourensz 1981). Y is set at 30 years, 1950 to 1979, since there can be some confidence that all near-coast cyclone occurrences (i.e. coast crossings or coast grazings) in that time have been counted. Likewise L has been set at 300 km, since the precision with which coast crossing frequencies and positions, or the variations in $\bar{R}$ as given by Eqn 4, are known does not justify any finer distinction. Table 1 contains details of cyclone occurrence frequencies within various distances of the specified points in the 30 years 1950 to 1979.

Table 1. Cyclone occurrence frequencies, 1950-1979

Station	*Within 50 km*	*Within 100 km*	*Within 150 km*
Carnarvon	5	12	19
Onslow	10	14	23
Roebourne	4	13	19
Port Hedland	3	8	15
Broome	7	18	31
Wyndham	10	12	14
Darwin	4	12	17
Maria Island	3	10	15
Normanton	6	10	18
Weipa	12	12	17
Thursday Island	3	5	7
Cooktown	8	14	16
Townsville	5	15	20
Mackay	1	13	15
Brisbane	4	7	15

Results

Table 2 contains values of $\bar{R}$, σ_R (Eqns 4 and 10) and hit frequency (Eqn 14) for various instrument network spacings, per year, per 300 km of coastline about the location specified in Table 1.

Table 2. Frequency of 'hits' per year in 300 km of coast

				Instrument spacings (km)			
Station	*Lat.*	$\bar{R}$ *(km)*	σ_R (km)	300	200	100	0
Carnarvon	25.0	35.0	11.2	0.15	0.21	0.35	0.40
Onslow	21.7	27.1	8.6	0.14	0.19	0.54	0.70
Roebourne	20.8	25.1	8.0	0.11	0.16	0.20	0.43
Port Hedland	20.4	24.2	7.7	0.08	0.10	0.15	0.33
Broome	18.0	19.3	6.0	0.13	0.17	0.27	0.90
Wyndham	15.4	14.6	4.5	0.05	0.09	0.29	0.43
Darwin	12.5	10.1	3.0	0.04	0.06	0.08	0.53
Maria Island	14.9	13.7	4.2	0.05	0.07	0.08	0.50
Normanton	17.8	18.9	5.9	0.08	0.10	0.23	0.43
Weipa	12.7	10.4	3.1	0.04	0.06	0.25	0.53
Thursday Island	10.7	7.8	2.2	0.01	0.02	0.05	0.20
Cooktown	15.4	14.6	4.5	0.05	0.10	0.23	0.50
Townsville	19.2	21.7	6.8	0.10	0.16	0.22	0.57
Mackay	21.1	25.7	8.2	0.09	0.17	0.05	0.43
Brisbane	27.5	41.5	13.4	0.14	0.15	0.33	0.40

R_L (Eqn 10) is determined from Eqn 4 as being $\bar{R}$ for $\varphi = 0$.

The results are also plotted in Figs 1 (a) to (e), with points joined by smooth curves since it is expected that frequencies should increase continuously and monotonically. For each solid curve, the dotted line represents the frequencies computed for $R = \bar{R} - \sigma_R$. Since $F_H \propto R$, it thus represents an approximate lower (16 per cent) bound for occurrence frequency. Some of these lines have been omitted for clarity of presentation.

Discussion

From Fig. 1 it can be seen that a sharp increase in hit frequency takes place, for most locations, for network spacings between about 150 and 40 km, with little improvement occurring for spacings finer than this except at lower latitudes (e.g. Darwin, Weipa, Thursday Island). This suggests that an optimised linear network should look for instrument spacings in this range. One suggested network would have spacing $2D \approx 50$ km south of 15°S and $2D \approx 30$ km north of there.

An estimate of the relative efficiency of cyclone interception, as a function of location and network spacing, may be obtained from Fig. 2. This has as abscissa Lourensz's (1977) linearised coast, divided into 100 km segments between Perth, WA and Port Macquarie, NSW. Plotted against location is the network spacing for 95 per cent, 90 per cent, 75 per cent and 50 per cent cyclone interception. This is extracted from Fig. 1 for respective locations and thus reflects variations in both cyclone radius with latitude and, to a lesser extent, the distribution of cyclone occurrences in the vicinity of the coast. Interpolation between the listed locations has been made on the basis of Eqn 4 and Lourensz's (1977) coast crossing frequencies.

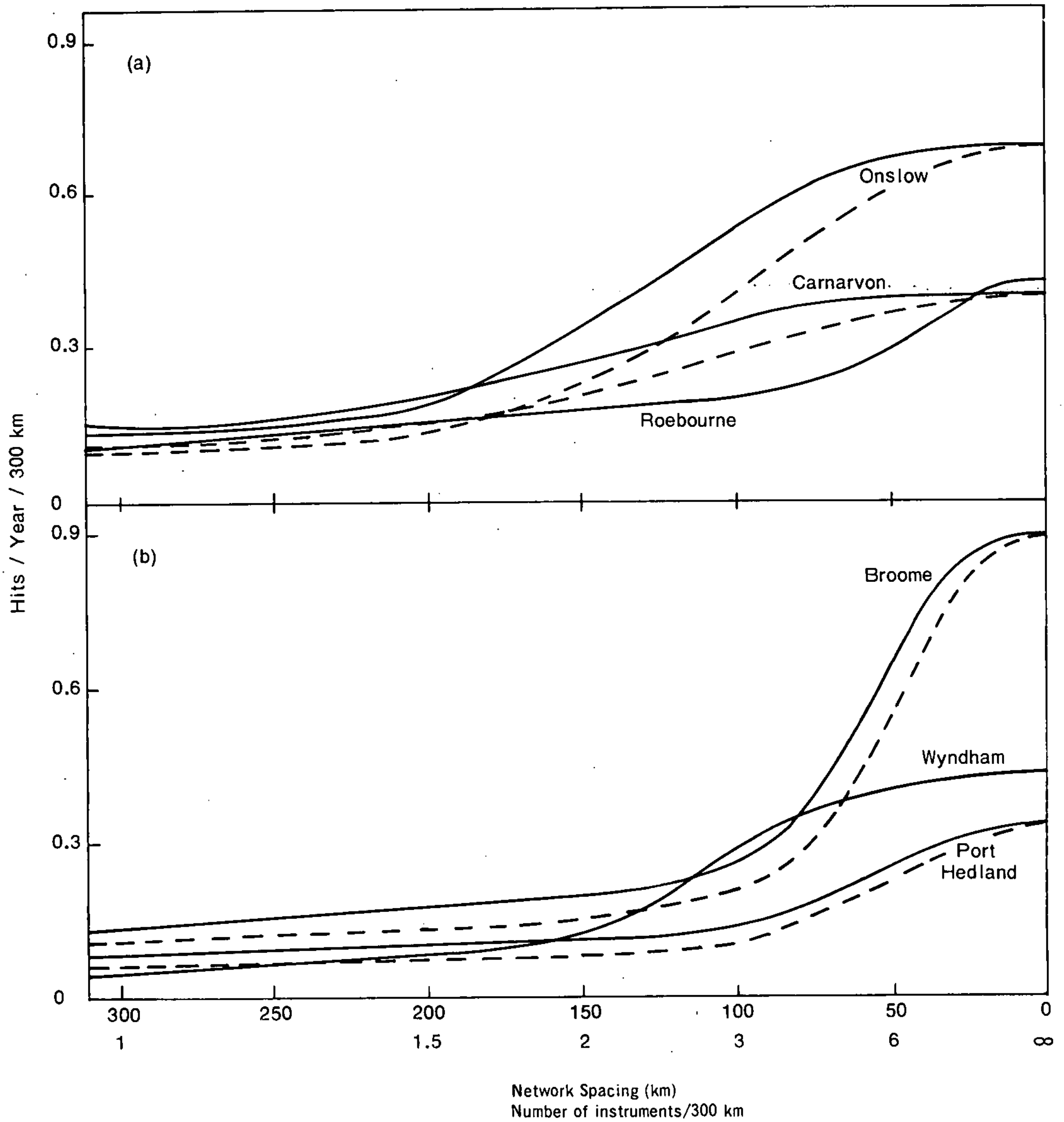
Finally it is instructive to estimate the total number of cyclone interceptions expected each year as a function of total number of instrument sites within Lourensz's (1977) one hundred idealised coast segments. Thus Table 3 contains these results computed from Fig. 1 with extrapolation on the basis that each of the locations of Fig. 1 is representative of a certain number of surrounding coast segments, then scaled to fit Lourensz's data on total numbers of coast crossings plus cyclones passing within a short distance of the coast in the years 1950 to 1975. Scaling is necessary because Fig. 1 may include multiple crossing cyclones, while the extrapolation required is considerable in some areas and may not be accurate. The scaling is such that 300 instrument sites, implying an average instrument spacing of 33 km, should, from Fig. 2, yield an average interception rate of better than 95 per cent of total average annual coast crossing (and grazing) cyclones.

Table 3. Total expected cyclone interceptions per year for various instrument site totals

Cyclones/year	*7.1*	*6.7*	*6.3*	*3.2*	*1.8*	*1.3*
Number of sites	300	234	200	100	50	33

The second column in the table is based on the optimum spacing suggested above. It would give an overall interception rate of approximately 90 per cent.

Fig. 1 (a-e) 'Hit' frequency for various instrument network spacings, per year per 300 km of coastline.



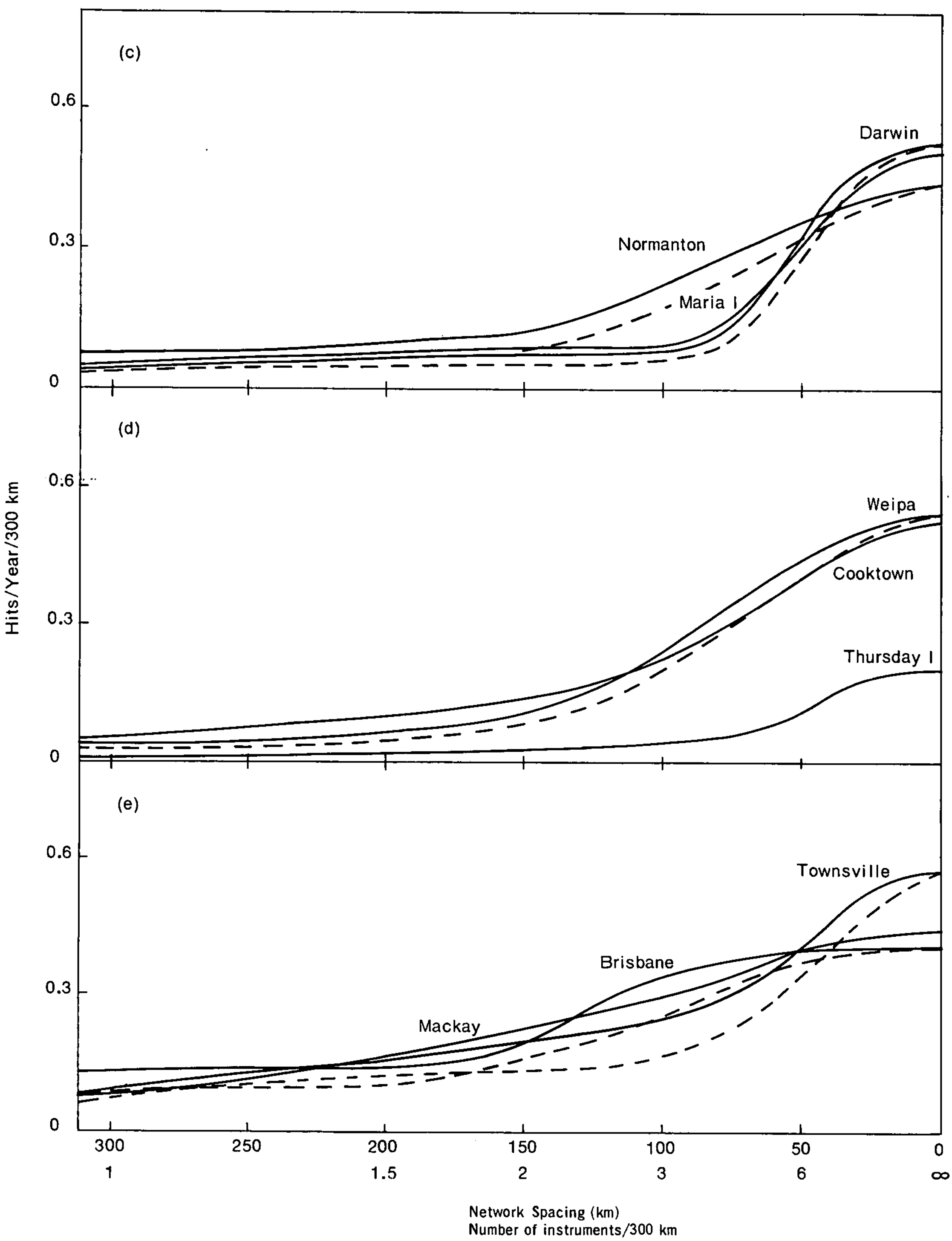
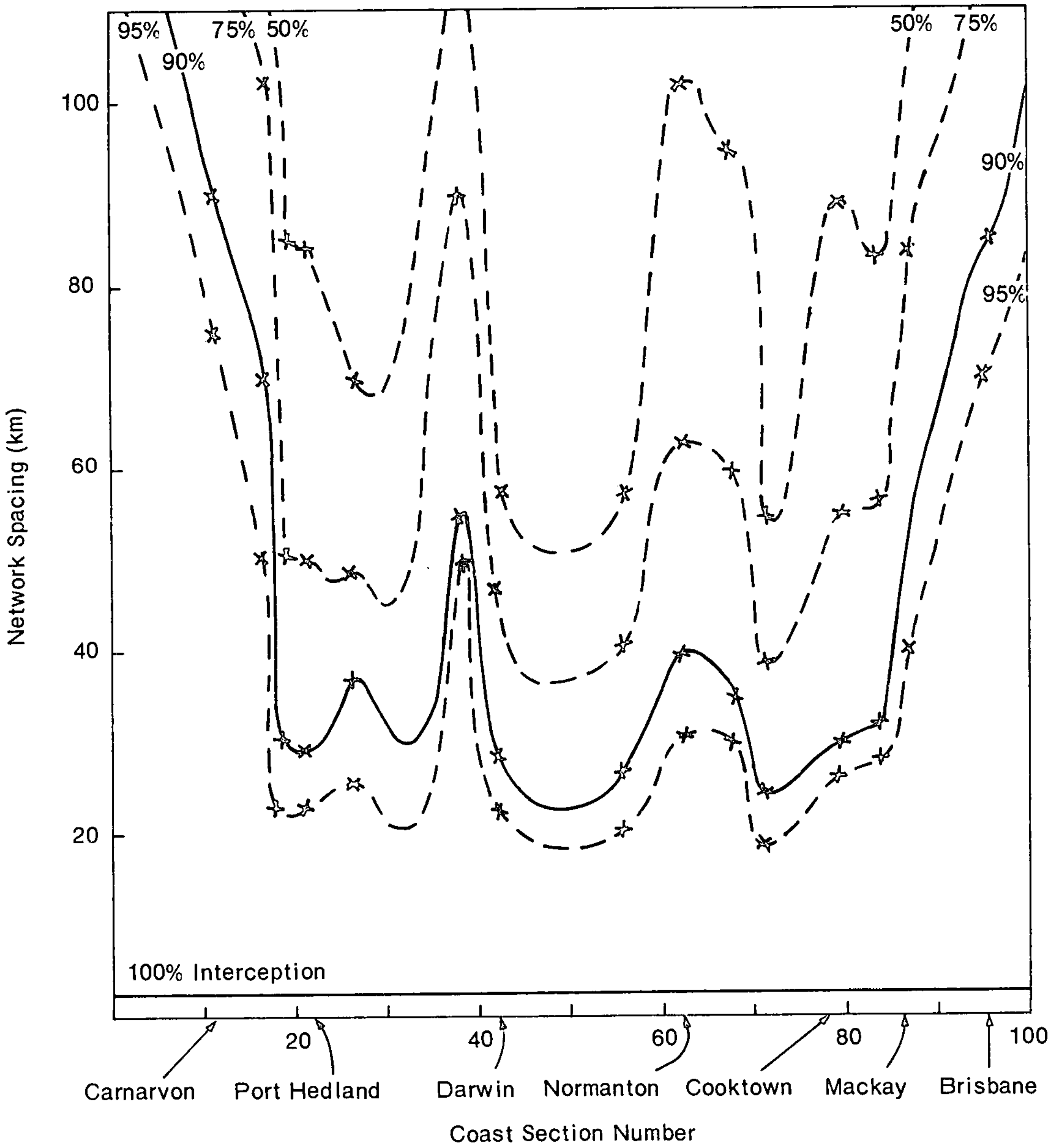

(c)
Darwin
Normanton
Maria I
(d)
Weipa
Cooktown
Thursday I
(e)
Townsville
Brisbane
Mackay
Hits/Year/300 km
0.6
0.3
0
300
250
200
150
100
50
0
1
1.5
2
3
6
∞
Network Spacing (km)
Number of instruments/300 km

Fig. 2 Relative efficiency of cyclone interception as a function of location and network spacing.



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