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A systematic description of the spatial variability of geopotential and temperature in the Australian region

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The spatial variation of geopotential and temperature on constant pressure surfaces in the Australian region has been described by fitting analytic functions to observed correlation coefficients. These coefficients were computed for 296 station pairs at six levels in summer and winter for the period 1962 to 1973. Similar computations, based upon a subset of this data base, were performed on correlation coefficients of 24-hour changes. The dependence of the two-point correlation coefficient upon the separation of the points, and upon the orientation of the line joining them, is shown to vary systematically between levels, between seasons, and between tropics and mid-latitudes. Characteristic differences in anisotropy are apparent between Australia, and Europe and North America. It is proposed that correlation coefficient functions specific to particular geographical regions provide an appropriate measure of spatial variability, against which to verify dynamical climate models which aim to simulate regional characteristics. It is also suggested that in the Australian region, both the anisotropy of the coefficients and the contrasts between tropics and mid-latitudes are sufficiently systematic to be taken into account in objective analysis.

Introduction

Most quantitative climatological studies of observed planetary and synoptic-scale mass and motion fields have focused upon the mean properties, and various conventional general circulation measures such as mean and eddy partitioned fluxes and energies. The aspect of spatial variability has been studied most commonly by means of wave number spectra. This approach is well suited to a global or hemispheric domain in which the data, often in gridded form as a result of objective analysis, may be decomposed conveniently into spherical harmonics.

A second and indirect measure of spatial variability is the ratio of pressure variance to wind variance at a fixed location. If geostrophy is assumed, this ratio (appropriately scaled) may be interpreted approximately as a characteristic radius of pressure systems (Buell 1957). However, this index is by its nature a local rather than a domain parameter, and the geostrophic assumption limits its application at low latitudes and near the earth's surface.

The subject of this paper is a third measure of spatial variability, namely the spatial autocovariance function (or equivalently the local variance and spatial autocorrelation function). Although in principle the wave number spectrum and spatial autocorrelation function can be derived from one another, the latter may be a more appropriate measure when considering a regional domain, or differences between regions. Unlike the wave number spectrum, the autocorrelation function is conveniently computed direct from station data by means of correlation coefficients between station pairs within the domain of interest, thereby avoiding uncertainties due to different objective analysis methods and sparse data. When expressed as a function of separation and direction, the autocorrelation function is also useful for revealing anisotropy.

In addition to its intrinsic value as another index of atmospheric behaviour, a systematic description of the spatial autocovariance is useful for the following practical purposes:

(a) For objective analysis of meteorological fields, it is necessary to know how far one can legitimately spread the influence of an observed datum. The increasingly used method of optimum interpolation (Gandin 1963) in fact requires specification of a spatial autocovariance function to compute the weights accorded to each observation during analysis.

(b) For the design and rationalisation of observing networks, a knowledge of the spatial autocovariance function enables a quantitative assessment of the impact of adding or removing specified observing stations (Bessemoulin et al. 1960).

(c) The local variance and spatial autocorrelation coefficient together enable the computation of the expected difference between simultaneous observations at neighbouring stations, thus providing a convenient method for validation of observed data.

In summary, therefore, a systematic description of spatial variability via the autocovariance should be regarded as an essential component of the climatology of a region. Such descriptions are available for many parts of the northern hemisphere (Gandin et al. 1976, provide a comprehensive summary), but the southern hemisphere has been comparatively neglected. This study focuses upon the spatial variability of geopotential and temperature on constant pressure surfaces in the Australian region. It follows similar descriptions of wind variability (Seaman 1975; Seaman and Gauntlett 1980).

The earliest Australian work on the subject of this paper was performed by McRae (1970), who produced tables of two-point correlation coefficients of geopotential and temperature, for 32 pairs of stations at six pressure levels. The coefficients were based upon four years of data (1953–56), and the maximum separation of any pair was 1900 km. Several observing stations have been added to the Australian radiosonde network since 1956, and a much longer period of record has become available. It was therefore considered meaningful to compute coefficients for many more station pairs, encompassing a wider range of separations. This enlarged and more reliable data base would then permit systematic description of the way in which the two-point correlation coefficients of geopotential and temperature depend upon the separation of the points, the orientation of the line joining the points, the level, season, and geographical area. The remainder of the paper consists of a description of the data base and the scope of computations, definitions of the correlation coefficient functions which describe systematic variations of the observed coefficients, and a discussion of the results obtained when the functions are fitted.

Data base

Magnetic tapes of all available radiosonde flights from the Australian network, for the period 1962 to 1973, were obtained from the Australian Bureau of Meteorology. These data had already been checked by the Bureau for transcription errors. An additional check for gross errors was performed before correlation coefficients were computed. A total of 296 station pairs were considered. These were obtained by pairing all of the 32 stations shown in Fig. 1, with each of the parent stations so denoted. In addition, all possible pairings were made of stations equatorward of 23 degrees.

For each pair of stations, i and j, standard product-moment correlation coefficients (r_{ij}) of geopotential and temperature were computed from simultaneous observations (±1 hour), for six levels (850, 700, 500, 300, 200 and 100 mb) in summer and winter. The coefficients, which will be referred to as 'climatological' were based upon deviations of simultaneous daily values p_i, p_j from the 12-year seasonal means P_i, P_j. In mathematical form, therefore,

$$r_{ij} = [\Sigma(p_i - P_i)(p_j - P_j)] / [\Sigma(p_i - P_i)^2 \, \Sigma(p_j - P_j)^2]^{1/2} \quad \ldots 1$$

where the summation extends over all available simultaneous observations in the appropriate season of each of the 12 years. The total of some 7000 coefficients is tabulated in Seaman (1981). Local means and variances at all the stations, for the period 1957–1975, are available in Maher and Lee (1977). In the following discussion sufficient of the correlation coefficients will be displayed in the figures to indicate the systematic quantitative features of their behaviour.

For all pairings with Laverton (Fig. 1), and for all tropical pairs, two-point correlation coefficients of 24-hour tendencies were also computed. These may be viewed as correlation coefficients of deviations from 24-hour persistence. A comparison of mid-latitude northern hemisphere coefficients presented by Schlatter (1975) and Bengtsson and Gustavsson (1971) suggests that such 'persistence' coefficients may be more appropriate than climatological coefficients for use in an objective analysis-forecast cycle.

Correlation coefficient functions

A preliminary analysis of the variability of correlation coefficient (r) with separation (d) and orientation (θ), was performed by plotting r versus d, and manually drawing contours of r in (d, θ) coordinates. This analysis suggested that the coefficients should be stratified according to element, level, season and latitude, and that a function which produced elliptical contours of r might appropriately describe the data. These first impressions were then objectively quantified by fitting correlation coefficient functions $r(d, \theta)$ to the various stratifications. Specific functions which fitted the data about equally well were generalisations of two suggested by Julian and Thiebaux (1975), namely,

$$r(d, \theta) = (b_1 \cos(b_2 d_*) + Z - b_1) \exp(-b_3 d_*) \quad \ldots 2$$

Fig. 1 The observing stations used to compute the correlation coefficients. Parent stations are denoted by circles.

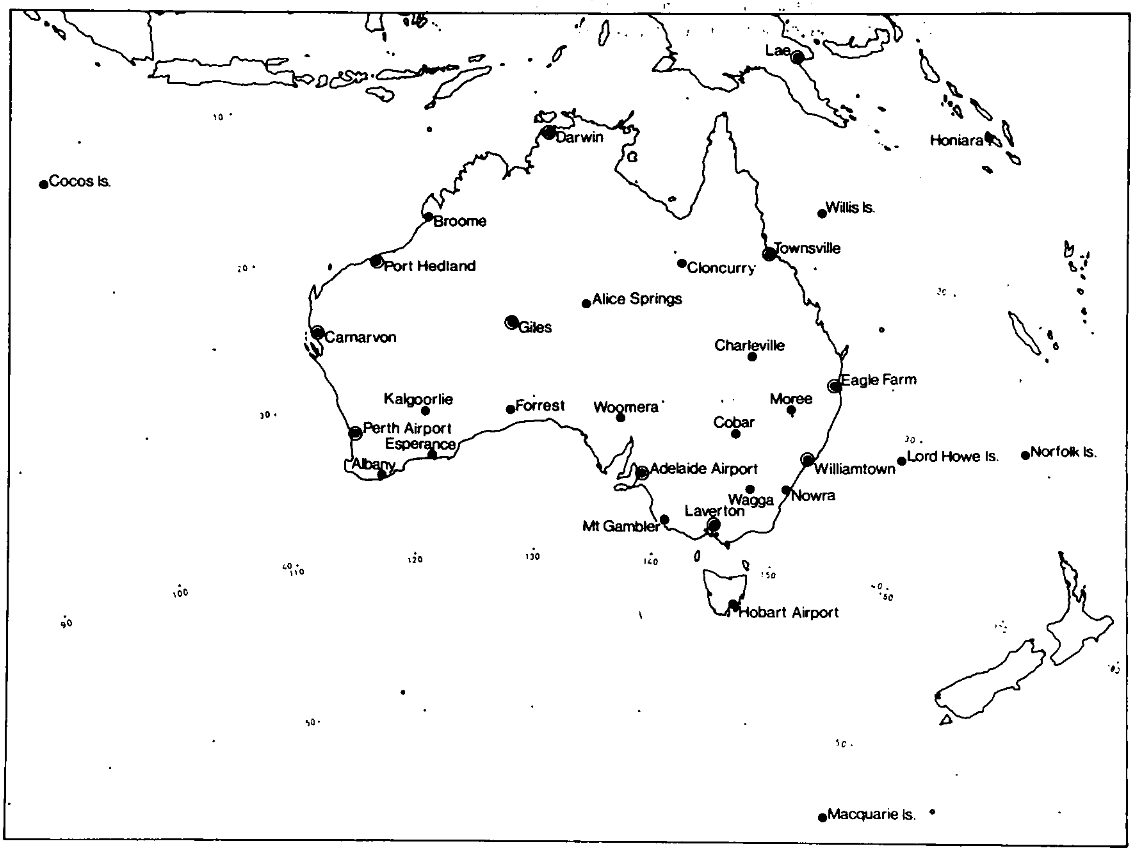


Fig. 2 Left: A contour of correlation coefficient (r), illustrating the constants Z, ϕ and E.
Right: A plot of r versus separation (d) along the major axis OA of the elliptical contours illustrating the constants Z and X. The constant Y is the corresponding separation along the minor axis OB.

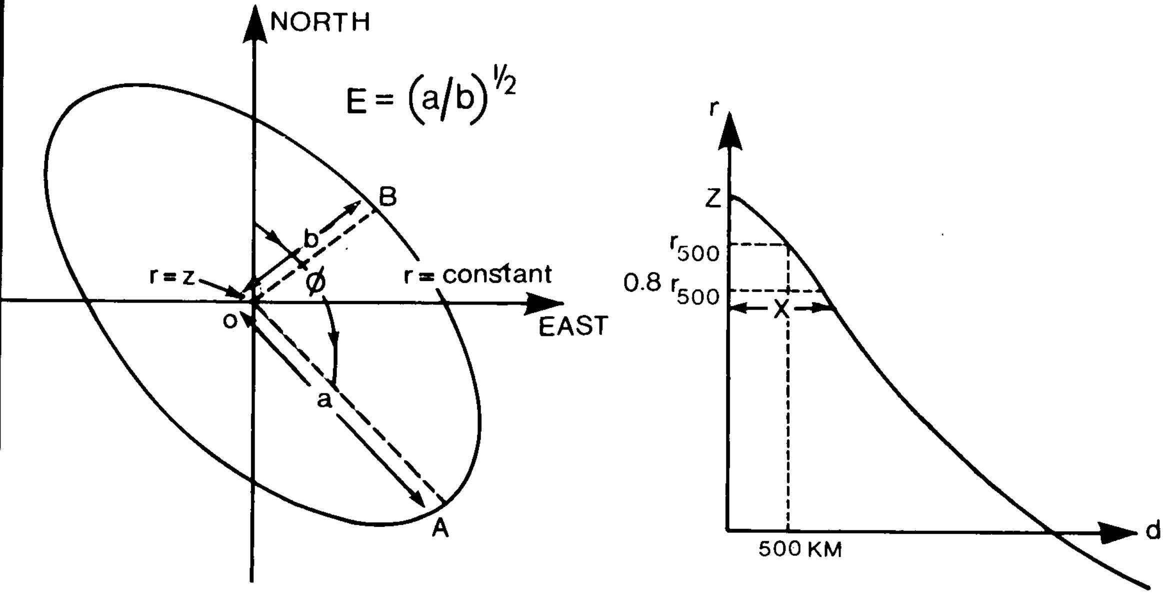


Fig. 3 **Plots of correlation coefficient (preceding decimal point omitted) versus separation (thousands of km) for mid-latitude station pairs. The superimposed curves indicate the variation of correlation coefficient with separation in the directions of slowest and most rapid decay, corresponding to the best fitting function (see text).**

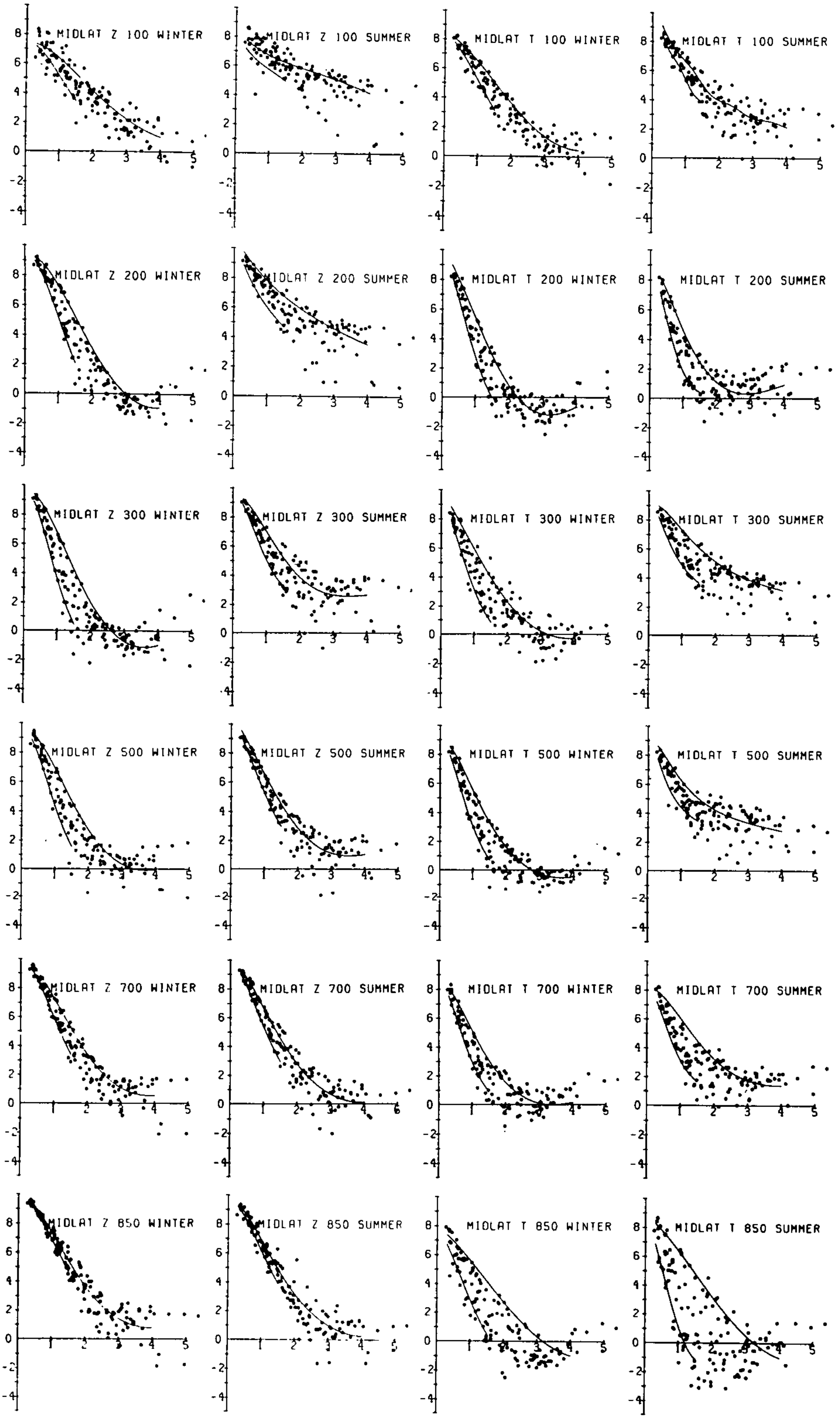
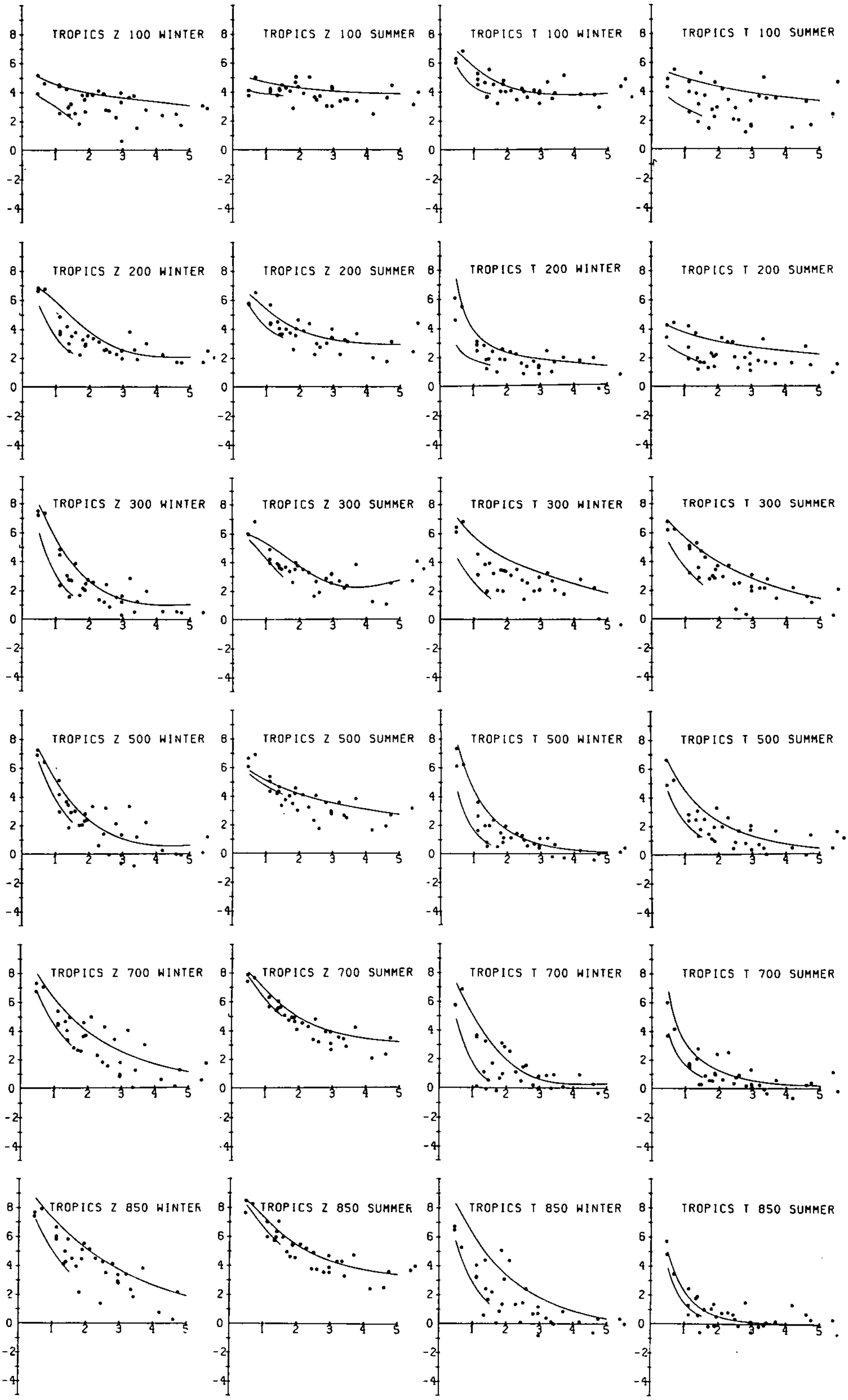


Fig. 4 : As in Fig. 3, for tropical station pairs.



$$r(d,\theta) = (b_1 \cos(b_2 d_*) + Z - b_1)(1 + b_3^2 d_*^2)^{-0.5} \quad \ldots 3$$

where $d_*^2 = d^2 (E^{-2} \cos^2 (\theta-\phi) + E^2 \sin^2 (\theta-\phi))$, and b_1, b_2, b_3, ϕ, E and Z are constants to be fitted by the data. These functional forms result in elliptical contours of r if d and θ are considered as plane geometrical coordinates. The constant ϕ determines the azimuth of the major axis and the constant E determines the ellipticity (Fig. 2, left). Functions 2 and 3 acknowledge the role of random observational error, which results in computed coefficients systematically underestimating the true value. Consequently the constant Z, the fitted value at zero separation, will in general be less than one.

It is useful to define length scale constants X and Y which indicate the decay rate of correlation coefficient with separation in the directions of slowest (X) and most rapid (Y) decay (Fig. 2, right). These correspond to the separations at which correlation coefficient is equal to 0.8 of its value at 500 km. The above definition is arbitrary, its main justification being that X and Y are well defined by the available data in most samples. In particular, the value of the r at 500 km is better determined than the extrapolated value (Z) at zero separation, particularly in the tropics where there are few pairs with separations close to zero.

Functions 2 and 3 were fitted to observed coefficients by the Marquardt (1963) non-linear least squares algorithm. In the following sections, attention will be focused upon the length scales X and Y, together with anisotropy constants E and $|\phi|$, corresponding to the better-fitting function.

Results and discussion

Climatological coefficients

The essential characteristics of the coefficients, stratified by element, level and season, and by tropics versus mid-latitudes, are summarised in Table 1 (left half), and Figs 3 and 4.

It is apparent that the anisotropy of the coefficients is of a systematic nature. In mid-latitudes the direction (ϕ) of slowest decay is between west-east and northwest-southeast in nearly all the samples, the usual direction being WNW-ESE. In the tropical samples the values of ϕ are somewhat more variable, but the favoured direction is west-east. The values of the constants E, X and Y indicate that the contours of $r(d,\theta)$ are generally quite elongated, and this is confirmed by the superimposed curves in Figs: 3 and 4 which indicate decay rates in the fastest and slowest directions. Plots of individual correlation coefficients in (d,θ) coordinates, as shown typically in Fig. 5(a) for 500 mb winter geopotential in mid-latitudes, strongly suggest that the anisotropy is not due to sampling fluctuations or to any artifact of the functional fitting algorithm. The foregoing results contrast with similar studies of 500 mb geopotential coefficients at North American and European mid-latitudes (Buell 1958; Bertoni and Lund 1963; Gruza and Daznacheyeva 1968) which indicate directions of slowest decay close to south-north (North America) and southwest-northeast (Europe).

The contrast between the tropics and mid-latitudes is another feature of the climatological coefficients. Figures 3 and 4 indicate that particularly above 500 mb, the decay rate of correlation coefficient with separation is less in the tropics than in mid-latitudes. At 100 mb, tropical coefficients of 0.3 to 0.5 are evident at east-west separations of about 5000 km, these values being only slightly lower than coefficients at separations of about 1000 km. Note also that negative coefficients are almost completely absent in the tropical samples. A few high values of correlation coefficient at large separations are found in summer above 500 mb in mid-latitudes, too. But these are predominantly due to east-west pairs between latitudes 23 and 30, which suggests that the arbitrary dividing line between tropics and mid-latitudes should have been further poleward in summer.

It is probable that year-to-year variations in the seasonal means contribute significantly to the positive correlations at high levels in the tropics. This contention was substantiated by some supplementary coefficients presented by Seaman (1981), which were based upon deviations from the seasonal means for individual years, instead of (as here) deviations from the 12-year seasonal mean. The supplementary coefficients were consistently lower than those shown here at high levels in the tropics, but were almost unchanged at lower levels and in mid-latitudes.

Inter-level contrasts in mid-latitudes indicate that the length scale constants X and Y for both elements are in general greater at the lowest (850 mb) and highest (100 mb) levels, than in the middle troposphere. This feature is in agreement with European geopotential data (Bertoni and Lund 1963, and several subsequent investigators), and North American temperature data (Bergman and Gordon 1977). For mid-latitude temperatures, an additional feature is the minimum of X and Y at 200 mb.

The main difference in behaviour between geopotential and temperature is the greater anisotropy of the latter at the lowest four levels. The contrast is particularly marked at 850 mb.

Persistence coefficients

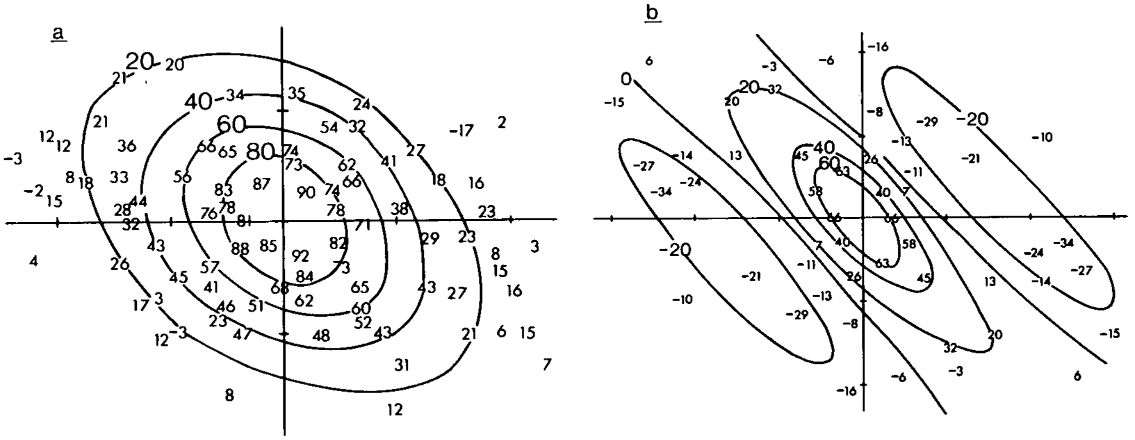
In mid-latitudes, as represented by Laverton, the persistence coefficients differ substantially and systematically from the climatological coefficients. The persistence coefficients are algebraically less, to the extent that the climatological and persistence coefficients are of opposite sign for many pairs in

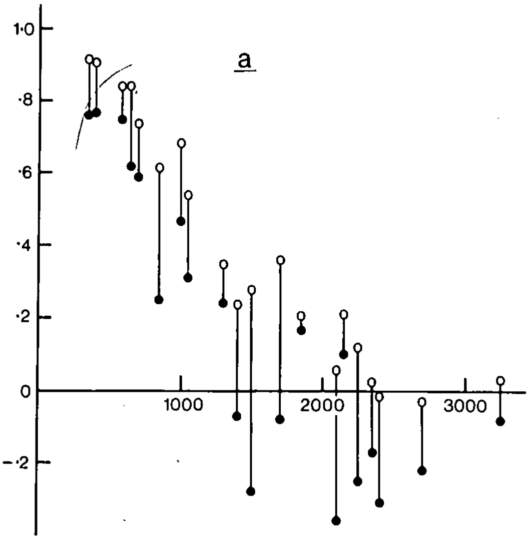
Table 1. **Left: Values of the correlation coefficient function constants $|\phi|$ (degrees), E, X and Y (km), corresponding to the better-fitting function (see text), for mid-latitude and tropical station pairs.**
Right: Values of $|\bar{\phi}$, E, X and Y for climatological (12 year, seasonal mean) and persistence correlation coefficient functions, based on all station pairs with Laverton.

Level (mb)		*All stations*								*Pairs with Laverton*							
		Mid-latitudes				*Tropics*				*Climatological*				*Persistence*			
		Geopotential		*Temperature*		*Geopotential*		*Temperature*		*Geopotential*		*Temperature*		*Geopotential*		*Temperature*	
		Winter	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*	*Winter*	*Summer*
	Ø	134	74	142	131	102	121	118	57	146	53	140	133	167	175	151	136
	E	1.06	1.08	1.39	1.66	1.44	1.14	1.51	1.39	1.09	1.08	1.35	1.75	1.42	1.13	1.51	2.03
850	X	1025	966	1020	1039	1151	1250	906	634	1050	891	1014	1032	956	846	883	1011
	Y	943	863	699	615	814	1040	656	606	914	817	709	597	656	737	610	553
	Ø	130	100	129	118	105	144	95	102	121	87	138	120	156	169	167	144
	E	1.15	1.14	1.23	1.43	1.31	1.13	1.47	1.30	1.14	1.11	1.21	1.46	1.44	1.34	1.37	1.45
700	X	1003	939	834	1012	963	1174	816	618	1026	924	815	888	843	722	714	662
	Y	830	793	676	685	770	1002	622	607	849	808	684	674	615	617	590	558
	Ø	129	111	122	100	91	120	78	98	121	101	131	116	153	151	158	149
	E	1.25	1.15	1.25	1.39	1.18	1.17	1.43	1.35	1.19	1.11	1.21	1.37	1.41	1.23	1.44	1.57
500	X	963	897	838	967	840	1430	698	793	958	903	829	990	822	750	798	733
	Y	727	776	671	763	722	1230	609	657	761	790	692	747	613	633	600	555
	Ø	125	105	102	110	81	88	77	69	120	114	111	116	151	145	143	132
	E	1.27	1.26	1.31	1.47	1.34	1.28	1.93	1.53	1.22	1.21	1.29	1.50	1.42	1.39	1.16	1.40
300	X	924	897	876	1155	825	1373	1056	1110	891	959	867	1272	834	751	659	704
	Y	690	831	678	779	663	925	717	766	703	770	686	800	612	588	610	580
	Ø	128	99	116	116	84	88	75	85	128	106	118	124	148	97	147	150
	E	1.23	1.40	1.23	1.30	1.50	1.34	1.71	2.39	1.22	1.31	1.25	1.26	1.52	1.11	1.29	1.30
200	X	1050	1259	809	762	1200	1140	684	1500	1014	1201	792	740	1016	612	742	612
	Y	784	897	664	618	730	850	633	888	775	868	654	618	653	584	611	549
	Ø	108	87	125	107	89	110	87	82	117	92	135	141	144	145	165	137
	E	1.25	1.46	1.20	1.19	2.28	3.25	1.42	2.48	1.25	1.27	1.16	1.11	1.45	1.24	1.45	1.28
100	X	1276	1760	1107	968	1778	3800	1158	2155	1313	1868	1077	979	1127	687	915	740
	Y	912	1212	860	812	1010	2200	876	963	923	1216	868	851	727	633	645	610

Fig. 5(a) **Correlation coefficients of 500 mb winter geopotentials in mid-latitudes, with subjectively drawn manual contours. A few coefficients have been omitted for clarity, and preceding decimal points are omitted. Axes are oriented north-south and west-east, and graduated in 1000 km intervals.**

(b) **As in Fig. 5(a), for correlation coefficients of 24-hour changes for 850 mb summer temperature between Laverton and other stations.**





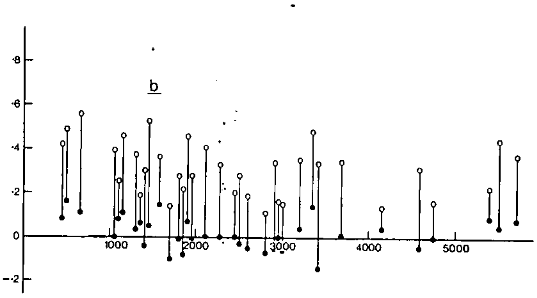


Fig. 6(a) Plots of correlation coefficients versus separation (km) for mid-latitude station pairs with Laverton, 500 mb geopotential winter. Open circles denote coefficients based upon deviations from the 12-year seasonal mean. Solid circles denote coefficients based upon deviations from 24-hour persistence.

(b) As in Fig. 6(a), for tropical station pairs, 100 mb temperature, summer.

the separation range 1000−3000 km (see Fig. 6(a), and the length scale parameters X and Y in the right half of Table 1). This result is in good qualitative agreement with the North American coefficients computed by Schlatter (1975). Particularly at lower levels, the persistence coefficients are more anisotropic than their climatological counterparts, and the direction of slowest decay approximates NNW-SSE. The computed coefficients for 850 mb summer temperatures are shown in Fig. 5(b). The greater anisotropy of the persistence coefficients may be associated with the systematic contribution of phase error to deviations from 24-hour persistence*.

In the tropics, all the persistence coefficients at separations greater than a few hundred kilometres are close to zero (less than 0.2). The climatological coefficients at corresponding separations are still moderately positive, particularly at higher levels (see Fig. 6(b)). The much higher ratio of 'noise' (observational error) to 'signal' (natural variance of 24-hour tendencies) in the tropics, in comparison with mid-latitudes, is probably a contributory factor. Nevertheless, the difference between the observed persistence and climatological coefficients, indicates that the latter are influenced mainly by time scales which are well 'predicted' by 24-hour persistence.

Concluding remarks

This study has described quantitatively some features of spatial variations of geopotential and temperature on constant pressure surfaces in the Australian region. It has been shown that there are systematic differences between levels, between seasons, between the mid-latitudes and tropics, and between the two elements. A characteristic of particular interest is the WNW-ESE elongation of correlation coefficient contours of both elements in mid-latitudes, which contrasts with observed patterns in northern hemisphere studies. It is suggested that the results of this study, together with those of a preceding study of wind variations, should be reproducible by those dynamical climate models which aim to simulate regional characteristics.

For objective analysis applications, at least two of the observed effects are probably sufficiently large and systematic to be usefully taken into account. These are the above-mentioned anisotropy of both geopotential and temperature fields, and the differences in characteristic length scales between the mid-latitudes and tropics. However, the rather large differences between the climatological and persistence coefficients, particularly in the tropics, highlight the need for collection of 'observed minus guess field' statistics appropriate to specific analysis-forecast schemes.

Many aspects of the variability of mass and motion fields in the Australian area remain to be investigated. A systematic study of vertical and temporal variations would be a logical extension of the work already done, and might eventually lead to a coherent four-dimensional statistical description. It should be noted that more than twenty years of observations from the Australian upper air network are readily available on magnetic tapes from the Australian Bureau of Meteorology for use in such studies.

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*This suggestion was made by Mr. N. Davidson of the Australian Numerical Meteorology Research Centre.

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