Density change in an Australian west coast trough

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The roles played by the major components of density change during the development of an Australian west coast trough sequence are investigated by the application of Fleagle's (1948) density change equations. It is concluded that advected heat and surface-based heating are important components of the trough's development.

Introduction

The Australian west coast trough is a quasi-permanent surface trough that forms on the western coast of Australia during summer and contributes significantly to the climatology of the region. Its persistence is associated with prolonged periods of very high temperatures and, in terms of operational forecasting, it poses significant forecasting problems for almost six months of the year.

In a general sense, the existence of the trough may be explained in terms of the relatively low latitude of the western periphery of the Australian land mass, and the location of the southern hemisphere's high pressure region during summer months. But the characteristics of upper airflow in the region, and the trough's internal dynamics, are important determinants of its day-to-day behaviour. As such, the west coast trough is a complex synoptic entity whose formation and propagation must be explained in terms of its interaction with contiguous circulation systems at both surface and upper levels.

In this respect, large-scale numerical models have been successful in representing the diverse elements that contribute to the behaviour of the trough. The ability of the Australian Region Primitive Equation (ARPE) model to predict the behaviour of the Australian west coast trough has recently been enhanced by increased spatial resolution and the inclusion of a nested, lateral boundary condition arrangement, detailed orography, and a surface heat balance scheme (Skinner and Leslie 1982). For example, the force/restore soil temperature prediction scheme employed (Leslie 1980), explicitly incorporates the effects of deep-soil temperature and radiational heating on ground surface temperatures and thereby accounts well for the diurnal development of the trough. The reported refinements to the model thus provide useful insights into the dynamics of the trough.

In the interest of documenting one aspect of the trough's dynamics in more detail, the present paper considers the role of density changes in the development of the trough. The ARPE model incorporates these processes in a method described by McGregor et al. (1978) but it is also possible to investigate density variations using the approximate forms suggested by Fleagle (1948) who determined the contribution to density change made by local compression or expansion, advection, and vertical motion for 132 cases during different synoptic conditions including cyclogenesis and anticyclogenesis.

Density change equations

Fleagle derived a relationship between pressure and density change rates in the troposphere from the hydrostatic equation

$$-\frac{1}{\rho g}\frac{\partial}{\partial z}\left[\frac{\partial p}{\partial t}\right] = \frac{\partial}{\partial p}\left[\frac{\partial p}{\partial t}\right] = \frac{1}{\rho}\frac{\partial \rho}{\partial t} \qquad \ldots 1$$

(where ρ is density, p is pressure, g is acceleration due to gravity, and t is time) and integrated with respect to pressure to give,

$$\left[\frac{\partial p}{\partial t}\right]_0 - \left[\frac{\partial p}{\partial t}\right]_1 = -\int_{p_0}^{p_1}\frac{1}{\rho}\frac{\partial \rho}{\partial t}\,dp \qquad \ldots 2$$

where the subscripts indicate pressure intervals. If a layer-mean density change rate is defined as

$$\overline{\frac{1}{\rho}\frac{\partial \rho}{\partial t}} = \frac{1}{p_1 - p_0}\int_{p_0}^{p_1}\frac{1}{\rho}\frac{\partial \rho}{\partial t}\,dp,$$

Eqn 2 becomes

$$\frac{1}{p_0 - p_1}\left[\left(\frac{\partial p}{\partial t}\right)_0 - \left(\frac{\partial p}{\partial t}\right)_1\right] = \overline{\frac{1}{\rho}\frac{\partial \rho}{\partial t}}. \qquad \ldots 3$$

The atmosphere was then subdivided into 100-mb layers so that the rate of pressure change could be described as

$$\frac{1}{100}\left[\left(\frac{\partial p}{\partial t}\right)_0 - \left(\frac{\partial p}{\partial t}\right)_1\right] = \overline{\frac{1}{\rho}\frac{\partial \rho}{\partial t}} \qquad \ldots 4$$

which essentially means that the rate of pressure change in a 100 mb layer is brought about by the rate of density change in that layer. (Although the units of Eqn 4 are s^{-1} it is possible to conceive of this rate of change in units of millibars per 100 mb layer per 12 hours (mb(100 mb^{-1})12 hour^{-1}) which is helpful when evaluating the contribution of the terms (once separated) to the total density change in the trough.) Fleagle ascribed the mean density change to the effects of either horizontal and vertical mass divergence or, as we shall concentrate upon here, the combined effects of horizontal advection of heat, local compression or expansion, adiabatic vertical motion, and non-adiabatic processes.

In order to derive these terms individually, Fleagle differentiated logarithmically the equation of state and Poisson's equation such that

$$\frac{1}{p}\frac{\partial p}{\partial t} - \frac{1}{T}\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial \rho}{\partial t} \quad \ldots 5$$

where T is temperature, and

$$-\frac{1}{\theta}\frac{\partial \theta}{\partial t} + \frac{1}{T}\frac{\partial T}{\partial t} = \frac{\kappa}{p}\frac{\partial p}{\partial t} \quad \ldots 6$$

where θ is potential temperature and κ is the gas constant for dry air divided by the specific heat of air at constant pressure. Combining Eqns 5 and 6,

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{1-\kappa}{p}\frac{\partial p}{\partial t} - \frac{1}{\theta}\frac{\partial \theta}{\partial t}\,. \quad \ldots 7$$

The density change equation is finally reduced to

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{1-\kappa}{p}\frac{\partial p}{\partial t} + \frac{1}{\theta}\,\underset{\sim}{V}_h\cdot\nabla_h\theta + wE - \frac{1}{\theta}\frac{d\theta}{dt} \quad \ldots 8$$

where w is the vertical velocity, $\underset{\sim}{V}_h$ is the horizontal velocity vector, $\nabla_h\theta$ is the horizontal gradient of potential temperature, and $E = \theta^{-1}\partial\theta/\partial z$. Fleagle then took 100 mb layer averages of the terms in Eqn 8 to get

$$D = P + A + V \quad \ldots 9$$

where

$D \equiv \overline{\rho^{-1}\,\partial\rho/\partial t}$, is the density term (positive for increasing density),

$P \equiv \overline{(1-\kappa)p^{-1}\,\partial p/\partial t}$, the compression term (positive for increasing pressure),

$A \equiv \overline{\theta^{-1}\,\underset{\sim}{V}_h\cdot\nabla_h\theta}$, the advection term (positive for advection of colder air), and

$V \equiv \overline{wE - \theta^{-1}\,d\theta/dt}$, which Fleagle referred to as the vertical motion term, since he considered non-adiabatic changes as small compared with the effect of vertical motion (positive for upward motion with positive stability).

In order to evaluate the role of each of these terms during the development of a west coast trough, it was necessary to compute them using 12-hourly radiosonde observations at Guildford (WA). Data from additional stations were not used since computations on the basis of 24-hour finite differences proved to be too coarse and Guildford is the only station in Western Australia with upper air soundings every 12 hours. Each of the terms in the density change equation was evaluated for nine levels in the troposphere corresponding to standard pressure surfaces although only the first two levels are used here.

The compression and expansion term, P, was computed from pressures reported for constant height surfaces. In the Australian region, available data were geopotential heights for constant pressure surfaces, so it was necessary to convert the data to constant height surfaces, using an integrated form of the hydrostatic equation

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{-g}{R\bar{T}}(z_2 - z_1) \quad \ldots 10$$

where p_1 and z_1 are known pressure (at a standard level) and observed geopotential height at that level, respectively, and z_2 is an arbitrary constant height. $\bar{T}$ is the mean temperature between z_1 and z_2, where the temperature for z_2 is determined by linear interpolation.

According to Fleagle, the vertical velocity can be expressed by

$$w = \frac{(dp/dt)_\theta - \partial p/\partial t}{\partial p/\partial z} + \frac{(dp/dt)_n - \underset{\sim}{V}_h\cdot\nabla_h p}{\partial p/\partial z} \quad \ldots 11$$

where subscripts θ and n refer to adiabatic and non-adiabatic components of the individual pressure change, respectively, z is the coordinate directed toward the zenith, and $\nabla_h p$ is the horizontal gradient of pressure. Fleagle multiplied by E, i.e. ($\theta^{-1}\,\partial\theta/\partial z$), introduced the hydrostatic equation and obtained

$$wE = \frac{1}{\theta}\frac{\partial\theta}{\partial p}\left[\left(\frac{dp}{dt}\right)_\theta - \frac{\partial p}{\partial t}\right] + \frac{1}{\theta}\frac{\partial\theta}{\partial p}\left[\left(\frac{dp}{dt}\right)_n - \underset{\sim}{V}_h\cdot\nabla_h p\right] \quad \ldots 12$$

and used this as a basis for computation of the vertical motion term, V.

Fleagle then neglected $\underset{\sim}{V}_h\cdot\nabla_h p$ (which is reasonable because if the wind is nearly geostrophic then $\underset{\sim}{V}_h$ and $\nabla_h p$ are approximately at right angles so their dot product is almost zero) and evaluated V such that

$$\overline{wE - \frac{1}{\theta}\frac{d\theta}{dt}} \equiv V = \overline{\frac{1}{\theta}\frac{\partial\theta}{\partial p}\left[\left(\frac{dp}{dt}\right)_\theta - \frac{\partial p}{\partial t}\right]}\,. \quad \ldots 13$$

Implicit in this expression is the assumption that

$$\frac{1}{\theta}\frac{\partial\theta}{\partial p}\left(\frac{dp}{dt}\right)_n = \frac{1}{\theta}\frac{d\theta}{dt} \quad \ldots 14$$

which is true only if isentropic surfaces are horizontal or nearly so: if $\nabla_h\theta$ is significant, it is invalid.* One would thus expect that with a significant slope of the isentropic surface $(dp/dt)_\theta$ would become large in proportion to $\partial p/\partial t$ and produce absurdly high values of V. The same result is likely if $(dp/dt)_\theta$ is determined when there are large changes in slope of the isentropic surface as brought about, for example, by the introduction of a different air mass. In testing these equations with data from Guildford, such problems were evident and,

*This was pointed out to the writer by Dr J. Hopwood.

therefore, values of V should be considered to give merely an approximation of the contribution of the vertical motion term to density change.

In order to derive the horizontal advection term, A, Fleagle wrote

$$\overline{\frac{1}{\theta}\frac{\partial\theta}{\partial t}} = \overline{\frac{1}{\theta}\frac{d\theta}{dt}} - \overline{\underset{\sim}{V}_h \cdot \nabla_h \theta} - \overline{\frac{w}{\theta}\frac{\partial\theta}{\partial z}} \qquad \ldots 15$$

which, since $E = \theta^{-1}\partial\theta/\partial z$ and $\overline{V = wE - \theta^{-1}d\theta/dt}$, gives,

$$\overline{\frac{\underset{\sim}{V}_h \cdot \nabla_h \theta}{\theta}} = \overline{\frac{1}{\theta}\frac{d\theta}{dt} - wE} - \overline{\frac{1}{\theta}\frac{\partial\theta}{\partial t}} \qquad \ldots 16$$

so that

$$A = -V - \overline{\frac{1}{\theta}\frac{\partial\theta}{\partial t}}. \qquad \ldots 17$$

It is relatively straightforward to compute A, because the second right-hand term can be determined directly from the mean potential temperature difference between two soundings.

Density change in a west coast trough

It should be stressed that Fleagle's intention was to comment on the mechanism of pressure change generally. Here, an attempt is made to apply the technique to a west coast trough sequence, enabling the contribution of each term to the overall density change to be evaluated. The sequence chosen for this purpose occurred between 26 December 1970 and 2 January 1971 and represents a number of typical features of the west coast trough. Its synoptic development at mean sea level is shown in Fig. 1. The corresponding density change terms at 900 mb and 850 mb are illustrated in Fig. 2. These two levels are used since values for the 900 mb level represent those processes acting between the surface and about one kilometre wherein much of the west coast trough development occurs. The 850 mb level includes the upper limits of the surface trough as described by Watson (1980). The variation of the density change terms will now be described as the trough formed, deepened and dissipated.

Case study

Two days prior to the formation of the trough, the mean sea level synoptic charts in Western Australia were dominated by the passage of a cold front, closely followed by a migrating anticyclone that directed cool, southerly winds onto coastal districts. The resulting density changes during this period at 900 mb and 850 mb levels (Fig. 2) reflect the prevailing synoptic situation. Figure 2 shows that the advection of colder air (positive values) is associated with southerly winds from higher latitudes. The vertical velocity term was of positive sign (upward motion), and its variations on a 12-hour basis suggest that it is substantially affected by diurnal heating. The greater magnitude of the vertical motion term, and the lesser magnitude of the advection term at 850 mb, suggest that density increases were associated primarily with advection nearer the surface. The local compression and expansion term, P, was of small magnitude in the 48 hours prior to the trough's formation and, indeed, throughout the sequence. It is evident in Fig. 2 that this term was mostly of negative sign (indicating expansion): it acted to assist density decreases, but its contribution was small and fairly consistent during the trough sequence. For the present discussion it will be ignored.

Day 1 (26 December 1970). On the first day of the sequence a weak trough formed within the northern periphery of a high pressure ridge that extended across southwestern Australia (Fig. 1(a)). Density changes at Perth (Fig. 2) for the corresponding 12-hour period remained positive (showing density increases) at both 900 mb and 850 mb although were of lower magnitude than during the preceding 48 hours. The advection term showed a shift to negative values (warmer air) at 900 mb and 850 mb, but these were of relatively small magnitude. Wind directions at Perth during the trough sequence are given in Table 1 and it is evident that southeasterlies prevailed at both levels during Day 1.

Day 2 (27 December 1970). The synoptic situation changed little during the first 12 hours. This nocturnal period was characterised by almost no change in the density, advection, and vertical motion terms at 900 mb. The wind direction remained easterly but without the southerly component (Table 1). A small northerly component was evident in the 850 mb easterlies, which may account for the increased negative advection at this level (Fig. 2), although neither at this nor at the 900 mb levels was advection of sufficient magnitude to produce density decreases.

During the following daytime 12 hours the trough intensified slightly although the isobars assumed a broad, fairly zonal orientation (Fig. 1(b)). However, the 900 mb data in this period were such that the basic assumption of a quasi-horizontal θ surface was invalid and terms in Eqn 13 were not calculated.

Day 3 (28 December 1970). During the following nocturnal 12 hours the trough intensified as coastal winds at 900 mb adopted a strong northeasterly component (Table 1). They advected warmer air (Fig. 2) which served to shift the density change from positive to negative (decrease in density). The subsequent daytime 12-hour period saw an increase in magnitude of both the vertical motion term and advection of warm air in association with winds of a strong northerly component (Table 1). The synoptic chart for Day 3 of the sequence (Fig. 1 (c)) shows that the trough deepened, remained offshore, and developed a weak, closed low-pressure centre offshore from the Geraldton/Carnarvon area.

Day 4 (29 December 1970). The following 12-hour nocturnal period saw an interesting development in the trough sequence. This is evident in Fig. 2 where there was a change to advection of cooler air and downward motion at the 900 mb level while at 850 mb there continued the advection of warmer air and upward vertical motion. An explanation for this development lies in the existence of a small, closed low-pressure centre in the trough (Fig. 1 (d)). This weak low directed northwesterly winds onto coastal districts during the evening of Day 3, but the flow

Fig. 1 Mean sea level synoptic charts for the west coast trough sequence, 26 December 1970 to 2 January 1971.

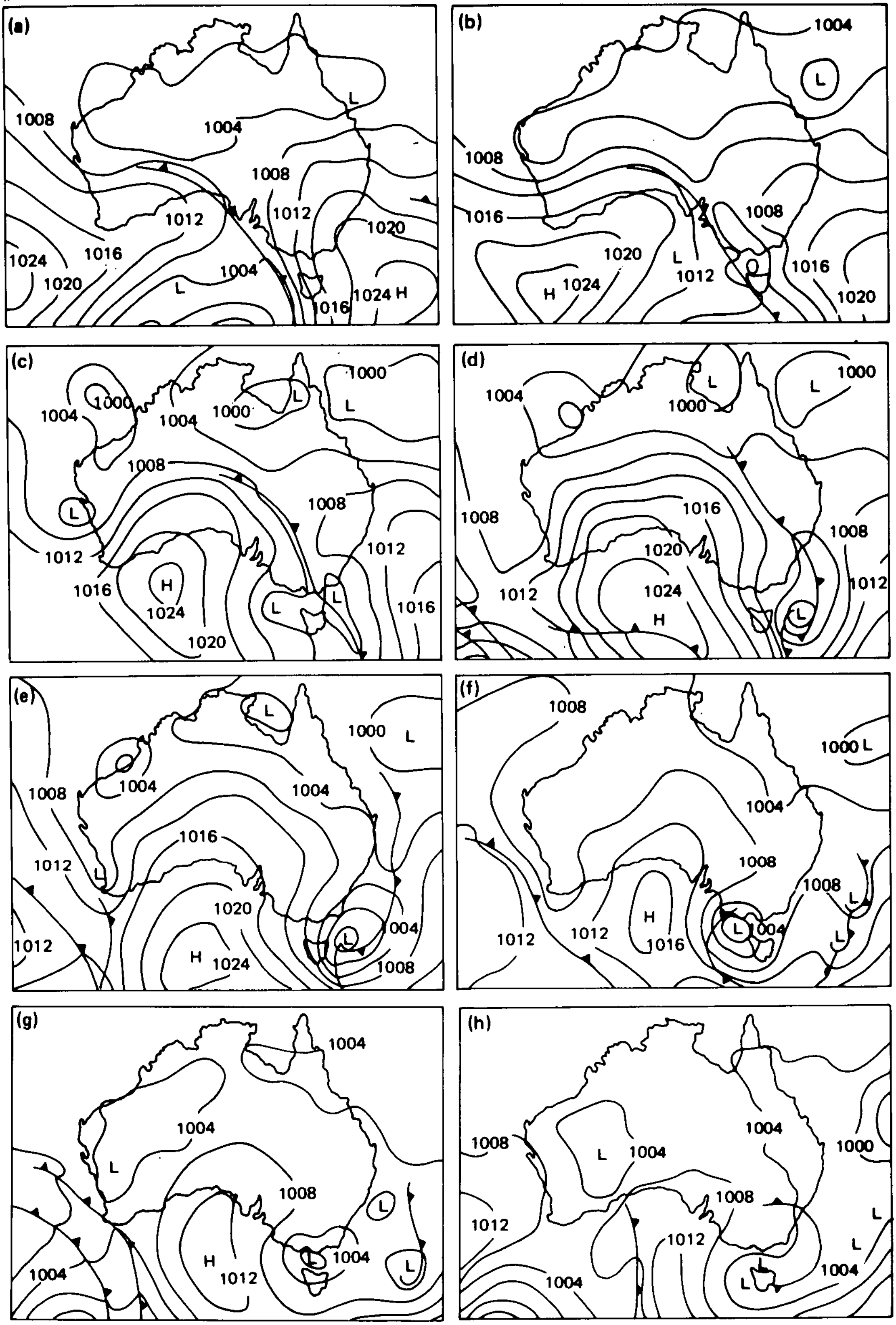

Fig. 2 Density change terms for the trough sequence 26 December 1970 to 2 January 1971 at 900 mb and 850 mb (see the text for a description of the terms). Short time marks on the ordinate show 0700 hours WST; longer ones indicate 1900 hours WST. Day numbers refer to the end of a daytime 12-hour period. Lower case letters indicate the corresponding synoptic charts in Fig. 1.

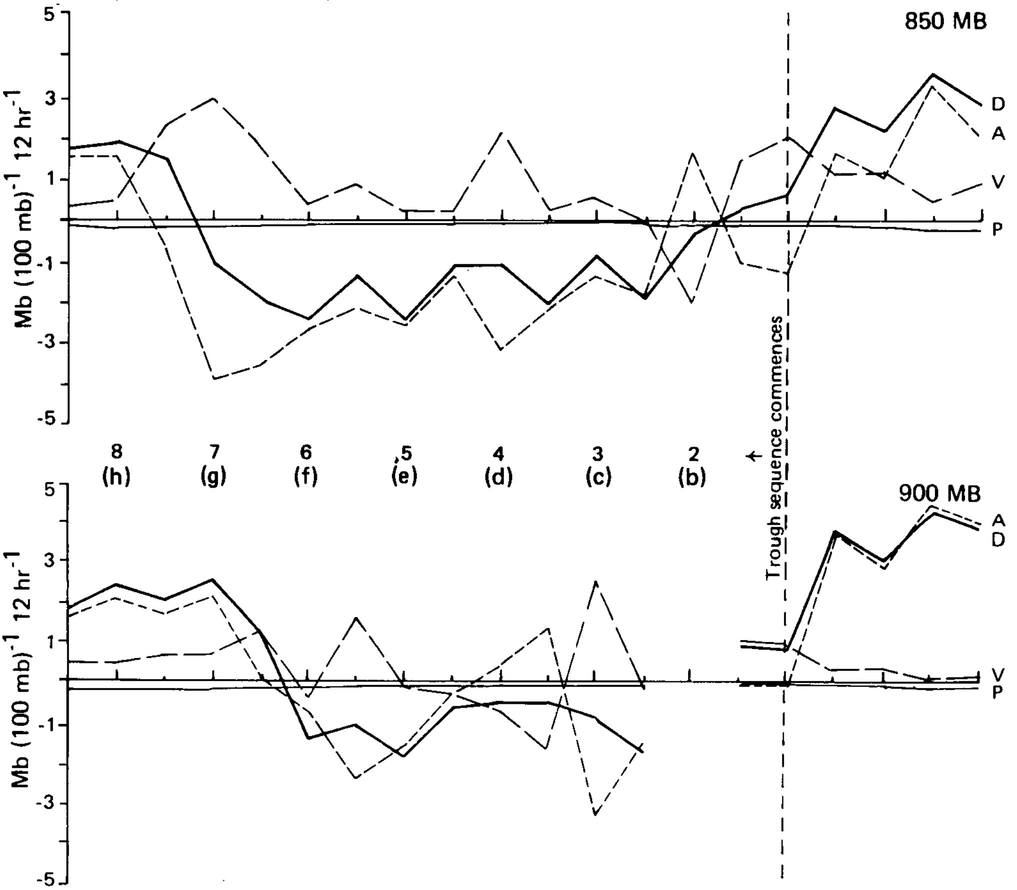

Table 1. Wind speed and direction for Perth at 900 mb and 850 mb throughout the west coast trough sequence, 26 December 1970 to 2 January 1971.

Day no.	Date	Time (WST)	900 mb Direction	Speed (m s^{-1})	850 mb Direction	Speed (m s^{-1})
		1900	200°	14	190°	11
	25/12/70	0700	180°	11	200°	11
		1900	180°	11	110°	8
1	26/12/70	0700	120°	15	180°	7
		1900	120°	9	120°	10
2	27/12/70	0700	90°	21	70°	9
		1900	100°	10	100°	15
3	28/12/70	0700	40°	14	30°	10
		1900	340°	10	30°	3
4	29/12/70	0700	30°	22	40°	14
		1900	70°	7	—	—
5	30/12/70	0700	350°	7	350°	6
		1900	230°	5	20°	5
6	31/12/70	0700	340°	6	350°	13
		1900	240°	6	330°	8
7	1/ 1/71	0700	220°	4	350°	10
		1900	220°	9	270°	11
8	2/ 1/71	0700	180°	4	330°	1
		1900	230°	8	270°	9

was shallow and extended only to the 900 mb level (Table 1). By the following morning the northwesterly flow had been replaced by strong northeasterlies, but the northerly component dominated and produced cooling at 900 mb. By contrast, winds at the 850 mb level assumed a slightly more easterly component that was sufficient to produce advection of warmer air and density decreases. The advection of colder air at 900 mb was insufficient to cause density increases in the surface layer.

This situation prevailed throughout the daytime hours of Day 4 but rates of cold advection and downward vertical motion decreased at 900 mb, and the magnitudes of warm advection and upward vertical motion increased at 850 mb. Consequently, the rate of density change remained low as trough development ceased. The synoptic chart shows that, while the trough axis remained offshore, zonal pressure gradients in the trough became weak (Fig. 1 (d)) as confirmed by the relatively low 900 mb wind speed at 1900 hours WST on 29 December 1970 (Table 1).

Day 5 (30 December 1970). Density changes in the subsequent 12 hours showed little variation. At both 900 mb and 850 mb density change remained negative, although little change occurred from the previous 12 hours at the lower level. At 900 mb, the advection term reverted to a negative (warmer) value, albeit of small magnitude, and vertical motion remained negative. At 850 mb the advection term also remained negative, but it was of smaller magnitude than during the previous 12 hours.

However, when heating commenced the next day, the system reintensified as northerly winds began to advect warmer air into the trough (Fig. 1(e)). The associated advection terms for 900 mb and 850 mb showed relatively high negative values, and produced density decreases at both levels. The vertical motion term at 850 mb was positive, although of small magnitude, whereas the vertical motion term at 900 mb showed a small negative value as a result, perhaps, of the cooling influence of the late afternoon sea-breeze along coastal regions at low levels. It appears that the trough reintensified primarily as a result of heat advection.

Day 6 (31 December 1970). Warm advection continued throughout the subsequent nocturnal 12 hours, with increased magnitude at 900 mb and slightly decreased magnitude at 850 mb, apparently a function of the direction of airflow: at 0100 hours WST the wind at 900 mb flowed from 360° while at 850 mb it flowed from 330°. By 0700 hours WST, the 900 mb wind direction was 340° while at 850 mb it was 350°. However, vertical motion was positive and the density change was negative in both cases.

During the following 12 daytime hours, the trough was evident on synoptic charts but its pressure gradients became weak (Fig. 1(f)). The advection of warmer air continued at both levels (Fig. 2) but the shift to southwesterlies at the lower level (Table 1) apparently reduced the magnitude of heat advection and suppressed *in situ* heating such that the vertical motion term became negative. At 850 mb the presence of northwesterly flow ensured the continued advection of warmer air. Vertical motion in this layer was upward, and the negative density change increased in magnitude. Despite this, the trough remained in a weakened state (Fig. 1 (f)).

Day 7 (1 January 1971). The decoupling of the surface level from the 850 mb level is evident during the subsequent nocturnal 12 hours. The period saw the intrusion of southwesterlies at 900 mb while northwesterlies prevailed at 850 mb (Table 1). The respective action of these different sources of airflow on density change in the trough is significant. The southwesterly flow at 900 mb served to halt the advection of warm air into the trough, and the advection term showed a small positive value. Although vertical motion remained upward, the density change became positive in this period, thus signifying the beginning of pressure increases at Perth. By contrast, persistent northwesterly flow at 850 mb advected warmer air, and density change remained negative at this level. Vertical motions were of comparatively high magnitude for a nocturnal situation, but it can be seen from Fig. 1(g) that the northwesterlies may have been associated with weak pre-frontal activity.

During the daytime 12 hours on Day 7, the imminence of a cold front off the coast of Western Australia (Fig. 1(g)) brought with it substantial advection of colder air at surface levels (Fig. 2) in association with southwesterly airflow (Table 1). The northwesterlies previously evident at 850 mb shifted to westerlies, but this inhibited neither the rate of change of warm air advection nor vertical motions with the result that density change remained negative. However, the contrasting situation at surface levels marked the end of the trough sequence when it lost its recognisable synoptic structure (Fig. 1(g)) as the cold front moved eastwards. It is clear from Fig. 2 that advection of colder air assumed the prime responsibility for the density increases at surface levels.

Day 8 (2 January 1971). Day 8 saw the dissipation of the trough at both the 900 mb and 850 mb levels. The rate of density change at Perth became positive and of fairly high magnitude as advection of colder air continued at 900 mb and the magnitude of warmer advection was reduced at 850 mb. The synoptic chart (Fig. 1(h)) showed that the trough's dissipation occurred within the context of the eastward movement of a new anticyclonic ridge across the Western Australian coastline. The ridge continued to develop and established itself along the southwest of the continent in the subsequent 24 hours.

Discussion

The foregoing description has shown that individual terms in the density change equations may vary considerably during the development of a west coast trough. The commencement of the sequence was characterised by change to density decreases that persisted until the trough moved eastwards. However, there were variations in the individual terms that highlight some interesting features of the trough. For example, the appearance of a small, closed low-pressure centre off the western coast on Day 3 and the subsequent offshore movement of the trough axis on Day 4 initiated the advection of colder air that reduced the magnitude of density decreases, weakening the trough up to the 900 mb level. Clearly, the enhanced ability of the improved ARPE model to locate such small, low-pressure centres (Skinner and Leslie 1982) gives valuable guidance to forecasting the trough's behaviour in such circumstances: a strongly developed low-pressure centre within the trough could dissipate it by advection of cold air.

Another feature of the trough's development evident in Fig. 2 is the strong diurnal component in the density change terms. This is to be expected, and confirms the need for a surface heat balance scheme as described by Leslie (1980) in an operational numerical model. Also the description of the trough sequence above suggests that *in situ* surface heating does not solely account for density changes. Sustained advection of heat serves to deepen the trough, while advection of cold air causes considerable weakening of its development. However, it should be stressed that heat advection does not initiate the trough. Rather, heat is advected by airflow with an easterly/northeasterly component which eventuates once the trough has formed. Thereafter, heat advection may influence the extent of the trough's development as was illustrated in the sequence described. In this respect it is vital that the position, orientation, and strength of high pressure systems to the east of the trough be properly accounted for.

Conclusion

The application of Fleagle's (1948) density change equations to a case study of an Australian west coast trough suggests that the major components of density change in the trough are vertical motion and heat advection. Both terms may vary in magnitude considerably throughout the development of a trough. These findings confirm the necessity for a surface heat balance scheme in the improved ARPE model and highlight for the forecaster the importance of accurate positioning of neighbouring synoptic systems that determine the extent of heat advection into the trough.

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References

Fleagle, R. G. 1948. Quantitative analysis of factors influencing pressure change. *J. Met., 5,* 281-92.

Leslie, L. M. 1980. Numerical modelling of the summer heat low over Australia. *Jnl appl. Met., 19,* 381-7.

McGregor, J. L., Leslie, L. M. and Gauntlett, D. J. 1978. The ANMRC limited-area model: Consolidated formulation and operational results. *Mon. Weath. Rev., 106,* 427-38.

Skinner, T. C. L. and Leslie, L. M. 1982. Predictability experiments with the Australian west coast summer trough. *Aust. Met. Mag., 30,* 241-9.

Watson, I. D. 1980. A dynamic climatology of the Australian west coast trough. Ph.D. thesis. Department of Geography, University of WA.