On meridional circulation near the equator

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A two-layer model of the meridional flow near the equator is presented in order to study the relationship between the gross properties of the Hadley circulation and the externally-controlled parameters. The model results are shown to be consistent with the observed mean behaviour of the Hadley circulation. The model also allows solutions corresponding to cross-equatorial flow. It is found that cross-equatorial flow imposes conditions on the relative rates of heating in the summer and winter hemispheres.

Introduction

Penetrative convection in the equatorial zone generally causes the tropopause to be at a greater altitude in the tropics than at higher latitudes (e.g. Das 1986). This deformation causes a meridional hydrostatic pressure gradient with a maximum near the equator at high levels. In addition to this external pressure gradient, the circulation in the tropical troposphere is driven by internal pressure gradients due to diabatic heating and cooling. The resultant flow in the tropics is a balance of these forces with Coriolis and frictional forces.

The classical Hadley circulation has symmetric cells about an intertropical convergence zone (ITCZ) at the equator. In equatorial regions influenced by continents, seasonal variations occur in the large-scale meridional flow. During the summer monsoon the ITCZ moves into the summer hemisphere and the Hadley circulation leads to a flow of air across the equator.

The purpose of the present work is to investigate theoretical constraints that are imposed on the meridional circulation near the equator. A simple two-layer model is used to analyse the two steady-state regimes of monsoon flow: the symmetric flow and the cross-equatorial flow. In particular, the relative roles of the external pressure gradient across the equator and diabatic forces are compared. The analysis neglects zonal gradients and so it does not consider the role of the Walker circulation or equatorial waves, such as those studied by Gill (1980).

Analytical models for the Hadley circulation have been given by Charney (1973) and Gill (1980). Charney demonstrates that a mean meridional temperature gradient leads to a sheared zonal wind through the thermal-wind balance. Ekman layers at the top and bottom boundaries provide the return flow for the general subsidence that occurs over the region away from the ITCZ. The model of Gill assumes that the meridional structure is represented by broad areas of internal heating and cooling. The internal heating produces vertical motion with rising near the equator and sinking towards the poles.

Both Charney (1973) and Gill (1980) assume that the internal heating or cooling associated with clouds and radiation is balanced by vertical advection in the equation for the conservation of heat. This balance is taken to occur over the whole region of the Hadley circulation.

The present analysis is confined to the equatorial region (0 to 10 degrees latitudes) where there is observed to be an increasing poleward meridional heat flux (e.g. Newton 1969). It is assumed that, outside the narrow ITCZ, there is no significant vertical motion. There is a thin two-layer flow with a horizontal scale of 10^4 km and a vertical scale of 10 km. Then the internal low-level heating is balanced by meridional advection, which produces a favourable pressure gradient for the flow into the ITCZ. The vertical mass flux is confined to the ITCZ where convection-driven diffusion produces the over-turning for the Hadley circulation. The present flow is similar to that proposed by Emanuel (1986), who argues that the intensification of tropical cyclones is forced by feedback between the horizontal wind and heating from the surface.

Equations of motion

We consider two-dimensional flow in the Cartesian coordinate system (x, y, z) where z increases vertically upward and the x-axis is the equator. To simplify the analysis in the equatorial zone we approximate the Coriolis parameter f by

\[ f = f_0 \, \text{sgn}(y), \]

where \( f_0 \) is a constant. It is further assumed that there is a
two-layer flow with rigid boundaries at \( z = 0 \) and \( z = H \), as shown in Fig. 1. The layer properties, including the layer depths, can vary across the equator. Each layer has a uniform depth \( h_i \) (\( i = 1, \ldots, 4 \)), where
\[
H = h_m + h_{m+2}, \quad (m = 1, 2).
\]

Neglecting non-linear effects and zonal gradients, we see that the horizontal momentum equations for each layer are given by
\[
\frac{du_i}{dt} - fv_i = -ku_i, \quad \ldots 3
\]
\[
\frac{dv_i}{dt} + \frac{∂p_i}{∂y} + fu_i = -kv_i + K \frac{∂^2v_i}{∂y^2}, \quad \ldots 4
\]
where \((u_i, v_i, w_i)\) are the velocity components in layer \( i \) (\( i = 1, \ldots, 4 \)), \( t \) is time, and \( p_i \) is the kinematic pressure. Friction across each layer is represented by Rayleigh friction with a time constant \( k^{-1} \), which is taken to scale with the magnitude of \( F^{-1} \). Then Eqn 3 implies that for a steady state the slope of the streamlines \( u/v \) is uniform in magnitude. The meridional momentum equation (Eqn 4) includes a horizontal diffusion term with a constant diffusivity \( K \). This diffusion will be seen to represent the effects of convection, which are needed to avoid singular behaviour at the equator.

Within each layer there is a net kinematic heating rate \( Q_i \) (\( i = 1, \ldots, 4 \)), which is locally balanced by the advection of heat. Representing the horizontal advection term by an Oseen approximation we take the conservation equation for heat to be
\[
\frac{∂θ_i}{∂t} + V_i \frac{∂θ_i}{∂y} = \frac{2θ_0 Q_i}{h_g}, \quad (i = 1, \ldots, 4), \quad \ldots 5
\]
where \( θ_i \) is the potential temperature, \( θ_0 \) is a representative temperature, and \( g \) is the gravitational acceleration. The meridional velocity \( V_i \) is the asymptotic value of \( v_i \) away from the equator. The Oseen approximation (Cole 1968) is commonly used in fluid mechanics to replace the full advection velocity by a uniform vector. As is often the case, the Oseen approximation here corresponds to the leading term in the outer expansion of the full solution, away from the equator. It is therefore appropriate to include it in the perturbation equations near the equator to ensure a proper matching between the inner (\( y → 0 \)) and outer (\( y → ∞ \)) expansions.

The pressure field is assumed to be hydrostatic. Then the meridional pressure gradients in Eqn 4 are controlled by two effects: the horizontal temperature gradient caused by heating and the external pressure gradient imposed at the upper boundary by deformation of the tropopause. Thus the mean pressure gradients for each layer are found to be
\[
\frac{∂p_{m+2}}{∂y} = -(\frac{h_{m+2} g}{2θ_o} \frac{∂θ_{m+2}}{∂y} - h'_g, \quad (m = 1, 2), \quad \ldots 6
\]
\[
\frac{∂p_m}{∂y} = -(\frac{h_{m+2} g}{2θ_o} \frac{∂θ_{m+2}}{∂y} - \frac{h_m g}{2θ_o} \frac{∂θ_m}{∂y} - h'_g. \quad \ldots 7
\]

The last term in Eqns 6 and 7 represents an external pressure gradient with an effective pressure-height gradient \( h'_g \). The deformation of the tropopause is caused by the enhanced penetrative convection near the equator, which should be related to the net heating over the whole equatorial region. It follows that the pressure gradient \( h'_g \) may not be completely independent of the net heating \( Q_1 + Q_2 \), and the two parameters could increase together. However, for the present analysis, they are assumed to be independent.

The vertical velocity \( w \) at the top of each layer is governed by the continuity equation. It is therefore seen that
\[
w_m = h_m \frac{∂v_m}{∂y}, \quad (m = 1, 2), \quad \ldots 8
\]
\[
w_{m+2} = w_m - h_{m+2} \frac{∂v_{m+2}}{∂y}.
\]
A consistent solution requires \( w_2 \), and \( w_4 \) to be zero so that there is no flow through the upper boundary \( z = H \).

Equations 1 to 8 describe the behaviour of a two-dimensional meridional flow near the equator \( y = 0 \). We seek a steady state solution of the equations and then Eqns 3 and 4 reduce to
\[
K \frac{∂^2v_i}{∂y^2} - (f^2/k) \alpha^2 v_i = \frac{∂p_i}{∂y}, \quad (i = 1, \ldots, 4), \quad \ldots 9
\]
\[
u_i = (f/k) v_i, \quad \ldots 9
\]
where \( \alpha^2 = 1 + (k/f)^2 \).

It is clear from Eqn 5 to 7 and Eqn 9 that an asymptotic solution exists for \( v_i \) where diffusion is negligible so that
\[
v_i \sim V_i = -k(αf)^2 \frac{∂θ_i}{∂y}, \quad (i = 1, \ldots, 4), \quad \ldots 10
\]
\[
V_i \frac{∂θ_i}{∂y} = \frac{2θ_0 Q_i}{h_g}.
\]
Equations 9 and 10 imply that, through Coriolis forces, the meridional pressure gradient drives a zonal flow, while the meridional flow is caused essentially by friction.

The flow described by Eqn 10 represents a circulating flow and so there must be no net meridional mass flux. The velocity components \( V_i \) must therefore satisfy

\[
V_m h_m + V_{m+2} h_{m+2} = 0, \quad (m = 1, 2).
\]

The full solution of Eqn 9, valid as \(|y| \to \infty\), is

\[
v_i = V_i + A_i \exp(-iy),
\]

where \( \ell = \alpha f/(K\kappa) \) and \( A_i \) is a constant.

Although \( f \) is represented by the simplified form Eqn 1, the behaviour of the solution Eqn 12 is consistent with the more complex Airy function solution, obtained when \( f \) varies linearly with \( y \) near the origin. We consider two solutions of the system Eqs 6 to 12, corresponding to either a Hadley circulation at the equator or the cross-equatorial flow of a summer monsoon.

### Hadley circulation at equator

To solve the equations of motion it is convenient to choose normalised parameters so that

\[
v_i = \alpha f V_i/(kQ)^{1/2},
\]

\[
\Gamma_i = (k/Q)^{1/2} \left( \frac{h_i g}{2\alpha \kappa} \right) \left( \frac{\partial \theta_i}{\partial y} \right),
\]

\[
\gamma_i = Q_i/Q,
\]

\[
\beta = h^2 g (k/Q)^{1/2} / 2\alpha f,
\]

where \( Q \) is a representative value of the kinematic heating rate. The constant parameter \( \beta \) represents an external pressure field that has a maximum at the equator. Thus the pressure-height slope \( h\beta \) is taken to be an odd function of \( y \), proportional to \( f \), and there is a high-level pressure ridge at the equator.

Putting Eqn 13 into Eqs 6, 7 and 10 we find that the meridional velocities and temperature gradients are given by

\[
v_i, \quad \Gamma_i, \quad (i = 1, \ldots, 4), \quad \ldots 14
\]

\[
v_{m+2} = \Gamma_{m+2} + 2\beta, \quad (m = 1, 2), \quad \ldots 15
\]

\[
v_{m} = \Gamma_{m} + 2(\beta + \Gamma_{m+2}). \quad \ldots 16
\]

Equations 14 to 16 are quadratic, and so the appropriate solution must be chosen. To obtain a circulation about the equator we require \( v_{m+2} > 0 \) and \( v_m < 0 \), as shown in Fig. 2. The solution of Eqs 14 to 16 is therefore found to be

\[
v_{m+2} = (\beta^2 + \gamma_{m+2})^{1/2} + \beta, \quad (m = 1, 2),
\]

\[
v_m = (\beta^2 + \gamma_{m+2})^{1/2} - (\beta^2 + \gamma_{m+2} + \gamma_m)^{1/2},
\]

\[
\Gamma_{m+2} = (\beta^2 + \gamma_{m+2})^{1/2} - \beta,
\]

\[
\Gamma_m = -(\beta^2 + \gamma_{m+2})^{1/2} - (\beta^2 + \gamma_{m+2} + \gamma_m)^{1/2} \quad \ldots 17
\]

Equation 17 shows that in the absence of heating in the upper layers \( (\gamma_{m+2} = 0) \), there is no temperature gradient \( (\Gamma_{m+2} = 0) \) and the flow is driven away from the equator by the external pressure gradient. Thus the temperature gradient in the upper layers is generated only by the internal heating \( (\gamma_{m+2}) \), which may even be negative. On the other hand the flow in the lower layers is moving against the external pressure gradient. A net favourable pressure gradient is generated only if there is heating \( (\gamma_m > 0) \), and an equatorward temperature gradient is needed to balance the effects of the external pressure field.
Equation 12 is used to obtain a solution valid for all values of $\gamma$. Then we see that the full solution for the meridional velocity is

$$v_i = (kQ)^i (\alpha f)^{-1} \nu_i [1 - \exp (-\ell y)], \quad (i = 1, \ldots, 4)$$

where $\ell = \alpha f/(Kn)^{1/2}$.

The vertical velocity field is found from Eqs 8, 11 and 18 to be given by

$$w_{m+2} = 0, \quad (m = 1, 2),$$

$$w_m = (kQ)^{1/2} (\alpha f)^{-1} \nu_{m+2} h_{m+2} \ell \exp (-\ell y).$$

Thus the convective mixing at the equator induces upward motion as shown in Fig. 2. Also, consistent with the initial assumption of a flat upper boundary, the vertical velocity is zero at $z = H$.

Equation 9 implies that the streamlines are straight lines, and so that Hadley circulation has low-level convergence in easterlies (see Fig. 2). The corresponding upper-level flow has divergence in westerlies. The present model is therefore consistent with the classical model of the Hadley circulation.

The constraint Eqn 11 that there must be no net meridional mass flux relates the depths of the layers to the velocities in the layers. In particular, we find from Eqs 13 and 17 that the fractional depth of the lower layer is given by

$$h_m/H = [(\beta^2 + \gamma_{m+2})^2 + \beta^3]/[(\beta^2 + \gamma_{m+2} + \gamma_m)^2 + \beta^3],$$

The mass flux of the circulating flow is given by Eqs 13, 17 and 20 as

$$M = 2\pi R p h_{m+2} V_{m+2}$$

$$= 2\pi R p (\alpha f)^{-1} H (kQ)^{1/2} m,$$

where $m = [(\beta^2 + \gamma_{m+2})^2 + \beta^3]/[(\beta^2 + \gamma_{m+2} + \gamma_m)^2 + (\beta^2 + \gamma_{m+2})^2]/[(\beta^2 + \gamma_{m+2} + \gamma_m)^3 + \beta^3]$.

$R$ is the radius of the earth and $\rho$ is the mean density of the circulating air. Figure 3 shows the variation of $m$ with the normalised heating rate $\gamma_{m+2}$ in the upper layer and with the normalised external pressure gradient $\beta$. It is seen that, for a fixed value of $\beta$, the mass flux is maximised when $\gamma_{m+2} = \gamma_0$, where $\gamma_0$ is a decreasing function of $\beta$. If $\gamma_{m+2}$ is negative (i.e. the upper layer is cooled) then both temperature gradients, $\Gamma_m$ and $\Gamma_{m+2}$, are negative. In that case the flow will be statically stable only if $\Gamma_{m+2} \geq \Gamma_m$. Thus we find from Eqs 17 that valid solutions occur only for $\gamma_{m+2} > -\gamma_m$, that is for $Q_m + Q_{m+2} > 0$.

The net heat flux away from the equator is given by

$$F = 2\pi R p c_p h_{m+2} V_{m+2} (\theta_{m+2} - \theta_m),$$

where $c_p$ is the specific heat of air at constant pressure. From Eqn 10 it follows that

$$F = 2\pi R p c_p (28/\gamma) (Q_m + Q_{m+2}) y,$$

provided $\theta$ is equal in the upper and lower layers at the equator. The condition for a statically stable solution therefore corresponds to the requirement for the circulation to provide a net export of heat from the equatorial region. Comparing Eqs 21 and 22 we see that, while the maximum mass flux of the circulation varies with the external pressure gradient, the heat flux simply increases with the kinematic heating rate. In particular, Fig. 3 shows that the mass flux is maximised by cooling in the upper layer ($Q_{m+2} < 0$) when $\beta$ is greater than about 0.3, but such conditions lead to a reduction in the net heat flux of the circulation away from the equator. We note parenthetically that, although climatologies such as Newell et al. (1969) show a net heating throughout the tropical troposphere, the meridional cross-section of total static energy $(i.e. \theta)$ by Riehl and Simpson (1979) suggests that $\Gamma_{m+2}$ may be negative and so radiative cooling may sometimes dominate convective warming in the upper layers.

In order to estimate the magnitudes of the parameters in the model circulation, we assume that it extends over about ten degrees of latitude, and so we take $f_0$ in Eqn 1 equal to $1.3 \times 10^{-5}$ s$^{-1}$ corresponding to a latitude of $5^\circ$. The air density $\rho$ in the middle troposphere is of order $0.65$ kg m$^{-3}$, and a representative depth of the layers $h_m$ is 5 km. Newell et al. (1969) find that the net diabatic heating rate in the equatorial troposphere is somewhat less than 1 K day$^{-1}$, and so we take $V_i \delta h/\delta y$ equal to 0.5 K day$^{-1}$. With $\theta_{00} = 300$ K and $g = 9.8$ m s$^{-2}$, Eqn 10 implies that the heating rate in each layer $Q_i$ is of order $5 \times 10^{-6}$ m$^{-2}$ s$^{-1}$. With $R = 6.4 \times 10^3$ km and $C_p = 1005$ J kg$^{-1}$ K$^{-1}$, Eqn 20 yields a heat flux of $1.6 \times 10^{15}$ W out of the boundary $10^7$ from the equator. This flux is consistent with the observations of Riehl and Simpson (1979) $(1.5 \times 10^{15}$ W), although it is only about 50 per cent of the net flux presented by Newton (1969). Riehl and Simpson show that this heat flux is only the residual after radiation and sensible heat fluxes are almost balanced by a condensational heat flux of $9.5 \times 10^{15}$ W. The condensation heating corresponds to a

![Fig. 3 Variation of normalised mass flux $m$ with normalised heating rate $\gamma_{m+2}$ in upper layer for Hadley cell, when $\gamma_m = 1$ in Eqn 13. Numbers refer to the normalised external pressure gradient $\beta$. Dotted line refers to the limiting case where $\gamma_{m+2} = -\beta^2$.](image)
steady precipitation rate of 8 mm day$^{-1}$ over the equatorial region.

Newell et al. (1969) find that the mass flux in the Hadley cell is of order 2 x 10$^{19}$ kg s$^{-1}$. This observation yields from Eqn 21 a representative velocity $V_i$ of 1.5 m s$^{-1}$, which is consistent with the analysis of Riehl and Simpson (1979). To estimate the friction parameter k and the zonal wind $u_z$, we note that the normalised velocity $v_i$ is of order one and substitute for $V_i$ in Eqn 13. Then it is found that $k \sim 8 \times 10^{-7}$ s$^{-1}$ and $u_z \sim 25$ m s$^{-1}$. These values are representative of the upper-level westerlies rather than the low-level easterlies. This result implies that a more sophisticated representative of friction and eddy processes may be required near the surface.

The overturning of the circulation occurs in the ITCZ, which is a region of order 100 km wide. We therefore take the diffusion length scale $L_k^{-1}$ to be 50 km in Eqn 18. The mean vertical velocity in the ITCZ is therefore found from Eqn 19 to be of order 15 cm s$^{-1}$, which is expected to be negligible compared with the turbulent velocity induced by convection in that zone. Using the calculated value for $k$, we find from Eqn 18 that the eddy diffusivity $K$ must be of order $5 \times 10^3$ m$^2$ s$^{-1}$, which corresponds to a turbulent velocity scale of 10 m s$^{-1}$. This velocity is representative of strong cumulus convection, but we note that an increase in the background friction parameter $k$ would yield a corresponding decrease in the turbulent velocity and diffusivity.

Taking the external pressure gradient $\beta$ to be of order one, we now see from Eqn 13 that the height gradient $h_z$ is about $7 \times 10^{-8}$, which corresponds to a deviation of about 70 m over 10$^5$ of latitude. Since $h_z$ is proportional to $\beta$, this result is consistent with the observation of Riehl and Simpson (1979) who find a pressure – height deviation of 40-60 m over 10$^5$ of latitude near the tropopause.

**Cross-equatorial flow**

In addition to the solution discussed in the previous section, the system Eqns 6 to 12, also admits of a solution with cross-equatorial flow, which represents the monsoon regime with low-level flow from the winter to the summer hemisphere. Without loss of generality we take the southern hemisphere ($y < 0$) to be the summer hemisphere. Then the model flow is shown in Fig. 4, and the normalised equations of motion are Eqns 14 to 16. In order to obtain velocities of the correct sign, we find that the appropriate solution is

$$v_3 = (\beta^2 + \gamma_3)^{1/2} + \beta,$$

$$-u_3 = ((\beta^2 + \gamma_3)^{1/2} - \beta),$$

$$v_1 = (\beta^2 + \gamma_3)^{1/2} - (\beta^2 + \gamma_3 + \gamma_1)^{1/2},$$

$$-v_2 = (\beta^2 + \gamma_2)^{1/2} - (\beta^2 + \gamma_2 + \gamma_3)^{1/2}.$$ 23

The velocities $v_1$ and $v_2$ in the winter hemisphere are identical to Eqn 17 with low-level flow towards the equator. However, the alternative roots of the quadratic equations Eqns 14 to 16 are required for $v_2$ and $v_3$ in the summer hemisphere, where the upper-level flow $u_4$ must now work against the external pressure gradient.

It is clear that a consistent solution for cross-equatorial flow requires $V_1 = V_2$ and $V_3 = V_4$. Thus Eqn 23 implies that

$$\gamma_4 = \gamma_3 + 4\beta [\beta + (\beta^2 + \gamma_3)^{1/2}],$$

$$\gamma_2 = \gamma_1 + 4\beta [\beta^2 + (\beta^2 + \gamma_1)^{1/2} - (\beta^2 + \gamma_3)^{1/2}].$$ 24

The heating rates in the summer hemisphere, $\gamma_2$ and $\gamma_4$, must be larger than those in the winter hemisphere to allow for the working against the adverse external pressure gradient in the upper layer. This condition is
clearly consistent with the occurrence of enhanced convection in the summer hemisphere.

Equations 23 and 24 show that cross-equatorial flow can occur even when there is an external high-level pressure ridge at the equator. From Eqn 9 the streamlines for the flow are displayed in Fig. 4; in particular, the low-level easterlies change to westerlies as the air moves from the winter to the summer hemisphere. The solution Eqn 23 is regular at the equator (y = 0) and so the constant A in Eqn 12 is zero. Convective turbulent mixing is not required at the equator to maintain this flow. In the absence of enhanced deep convection, the external high-level pressure ridge is expected to relax. In that case when there is no external pressure gradient, β is zero and condition Eqn 24 simply requires the heating rates in each layer to be continuous across the equator.

It is apparent that, provided condition Eqn 24 is satisfied, both solutions Eqns 17 and 23 are valid. The occurrence or absence of cross-equatorial flow is then determined by the far-field, outside the range of the present analysis. While the flow in the winter hemisphere is independent of cross-equatorial flow, the flow in the summer hemisphere has opposing directions and the depths of the layers is different. Using the continuity condition Eqn 11, we find that the depth of the lower layer for cross-equatorial flow is

$$h_2/H = \frac{(\beta^2 + \gamma_3)^{1/2} + \beta}{(\beta^2 + \gamma_3 + \gamma_1)^{1/2} + \beta}. \quad \ldots 25$$

When there is overturning at the equator, the depth of the lower layer in the winter hemisphere is

$$h_2/H = \frac{(\beta^2 + \gamma_3)^{1/2} + 3\beta}{(\beta^2 + \gamma_3 + \gamma_1)^{1/2} + 3\beta}. \quad \ldots 26$$

Thus the depth of the lower layer is greater when there is overturning at the equator than when there is cross-equatorial flow.

**Conclusions**

The gross features of the Hadley circulation near the equator can be described by the present model, which takes a two-layer flow with convective overturning in the ITCZ at the equator. The mass flux of the circulation is related to the heating rates in the layers and the external pressure field caused by the deformation of the tropopause. Although heating must occur in the lower layer to oppose the external pressure ridge, valid solutions are found when there is cooling in the upper layer of the circulation. The magnitudes of the parameters in the model are found to be consistent with the observed mean circulation near the equator.

The results are consistent with the assertion of Emanuel (1986) that a low-level flow in the tropics does not require conditional instability to drive it. Heating from the surface can provide a favourable pressure gradient, which maintains the basic horizontal flow and the heat flux. Then vertical motion is confined to the ITCZ where convective overturning is dominant.

The model also yields a solution for cross-equatorial flow when convective mixing does not occur at the equator. If there is an external high-level pressure ridge across the equator then the heating rate in the summer hemisphere must be greater that in the winter hemisphere in order to maintain a cross-equatorial flow.

**References**


