Dynamical theories of tropical convection

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(Manuscript received April 1988; revised August 1988)

Recent observations show that, in contrast to the middle-latitude atmosphere, the tropical atmosphere is very nearly neutral to reversible parcel ascent from the sub-cloud layer. This realisation, together with the failure of classical convection theory and Conditional Instability of the Second Kind (CISK) to explain many observed features of tropical convection, has led to a renewed interest in evaporatively driven downdrafts as the defining dynamical characteristic of mesoscale convective systems. After briefly reviewing the case against classical theories we summarise recent developments that focus on the role of downdrafts in organising and maintaining tropical convective systems.

Introduction

The sudden outrush of cold air that accompanies passage of tropical squalls is perhaps the most conspicuous feature of deep convective systems. Detailed time series of temperature and wind from ships in tropical waters lend quantitative support to the old sailor’s adage that tropical voyages entail hours and hours of abject boredom interrupted by moments of sheer terror. But despite the prominence nature affords convective downdrafts, they were, until quite recently, studiously avoided by meteorologists seeking to understand the individual and ensemble dynamics of tropical convection. The renewed interest in the dynamical significance of convective downdrafts will be the main focus of this review paper.

Why are dynamical theories of tropical convection still being developed? If convection occurred as randomly spaced cells moving with the mean wind we would hardly expect tropical convection to be a worthy subject of a review paper. The particular aspects of moist convection that continue to present a scientific challenge include the following:

1. Precipitating convection is often organised in clusters and squall lines, features generally not observed in laboratory experiments on dry convection;
2. Squall lines and sometimes individual clouds propagate at significant velocities with respect to the mean wind;
3. Squall lines are often aligned orthogonal to the vector mean wind shear, in striking contrast to dry convection; and
4. The vertically averaged heating due to moist convection depends on the precipitation efficiency, which in turn is sensitive to the details of the cloud scale and mesoscale structure of the convection and to cloud microphysics. This is a crucial point since large-scale circulations are sensitive to this heating. There is no analogue of this in dry convection.

We begin by reviewing the concept of conditional instability in light of contemporary observations and then proceed to review classical theories based on conditional instability. Theoretical investigations that focus on the dynamical role of downdrafts are then highlighted, and finally an outlook for the future is presented.

A new look at conditional instability

The first investigations of conditional instability were conducted in the 1830s by James Pollard Espy using early experimental results on the latent heat of vaporisation and the then budding science of thermodynamics (see Kutzbach 1979). It was even then realised that the observed temperature lapse rate of the atmosphere is close to the moist adiabatic lapse rate, though it was nevertheless assumed that the latent heat of condensation provides the energy for both tropical and extratropical cyclones. The possible role of downdrafts produced by the evaporation of rain in individual convective systems was not investigated until the late nineteenth century (Ludlam 1963).
It is widely believed today that much of the tropical atmosphere exists in a state of conditional instability that is episodically released in tropical convective systems. The degree of instability is quantified by the amount of Convective Available Potential Energy (CAPE), which is proportional to the area enclosed on a tephigram by the environmental virtual temperature sounding and the virtual temperature of a parcel lifted pseudo-adiabatically from the sub-cloud layer. An example of a conditionally unstable sounding is shown in Fig. 1 in which we have plotted Jordan’s (1958) composite sounding from the Caribbean in later summer. The amount of CAPE in the sounding is sufficient to produce a maximum ascent rate of 63 m s⁻¹ in an undilute updraft.

Several approximations are applied in the classical evaluation of CAPE. For the purpose of calculating the parcel virtual temperature it is assumed that all condensate falls out and that the heat of fusion is negligible. Also, the buoyancy of the parcel is taken to be a function of temperature and water vapor content alone; all effects of condensate loading are neglected. Finally, no mixing with the environment is permitted; such mixing invariably reduces the buoyancy of the parcel.

The neglect of condensate loading is often justified on the basis of two observations: that clouds precipitate and that actual liquid water contents are considerably less than those resulting from reversible ascent of sub-cloud air. We need to examine the implications of these observations more carefully. In the first place, it is straightforward to demonstrate that in a nonprecipitating system, the reversibly lifted parcel is always the most buoyant parcel. This is because any mixing that reduces the liquid water content also reduces the temperature more than enough to compensate the reduced water loading in the calculation of buoyancy. This can be demonstrated from the definition of buoyancy:

\[ B = g \left( \frac{\theta_v - \theta_{vo}}{\theta_{vo}} \right) \]  

where \( \theta_v \) is the difference between the virtual potential temperature of the cloud and that of the environment, \( \theta_{vo} \); \( l \) is the liquid water mixing ratio and \( g \) the acceleration of gravity. Differentiating Eqn 1 gives

\[ \delta B = g \left( \frac{\delta \theta_v}{\theta_{vo}} \delta l \right) \]  

A lower bound on the magnitude of \( \delta \theta_v \) is given by assuming that the only temperature change is due to evaporation (i.e., we neglect the direct effect of mixing upon \( \theta_v \) which produces even more cooling). Thus, from the first law of thermodynamics,

\[ \delta \theta_v < \frac{L_v}{C_p T} \delta l \]  

where we have assumed that \( \Delta l \) is negative, \( L_v \) is the latent heat of vaporisation and \( C_p \) the heat capacity at constant pressure. Substituting Eqn 3 into Eqn 2 gives

\[ \delta B < g \delta l \left[ \frac{L_v}{C_p T} \right] (\delta l < 0) \]  

Since \( L_v / C_p T \) is always greater than unity in the atmosphere, the evaporation of water during mixing always reduces the positive buoyancy of a parcel. Thus, the upper bound on CAPE for nonprecipitating clouds is given by the area between the environmental virtual temperature and the parcel virtual temperature including adiabatic water loading.

It is often assumed that precipitating clouds will necessarily have less specific liquid water contents than nonprecipitating clouds. But the water content at cloud base is always superadiabatic since its adiabatic value is zero. In general we would expect undiluted precipitating cloud to have superadiabatic water contents near cloud base and subadiabatic water loading near cloud top. The exact distribution will depend on the distribution of vertical velocity and cloud microphysical processes.
When the effects of mixing and precipitation are taken together, we see that the buoyancy of a reversibly lifted parcel including the effects of adiabatic water loading will always represent an upper bound on buoyancy sufficiently near cloud base, and may underestimate or overestimate buoyancy near cloud top depending on the relative contributions of mixing and precipitation. Moreover, the heat of fusion will generally increase the parcel buoyancy in the upper portions of the clouds. Taken together, these deductions suggest that the virtual adiabat is much better suited to the estimation of CAPE than is the pseudo-adiabat as first recognised by Betts (1982).

The two methods of estimating CAPE produce strikingly different results. The dashed line in Fig. 1 shows the cloud virtual temperature of a parcel lifted reversibly from the sub-cloud layer. The cloud virtual temperature is defined

\[
T_v = T \left[ 1 + \frac{w/0.662}{1 + Q} \right], \quad \ldots 5
\]

where \(w\) is the vapour mixing ratio and \(Q\) is the total water mixing ratio. Note that there is virtually no CAPE.

The striking correspondence of the environmental virtual temperature with a virtual moist adiabat, first revealed by Betts (1982), suggests that the tropical atmosphere is maintained in a nearly neutral condition by moist convection. We have conducted an analysis designed to find the extent to which this is true. The buoyancy of parcels lifted reversibly from the sub-cloud layer was calculated for several hundred soundings taken during the summer at the tropical Pacific island of Truk. Soundings that were clearly stable were discarded. The remaining parcel buoyancies were averaged to produce the mean buoyancy shown in Fig. 2. Note that the mean virtual temperature difference does not exceed 1°C. Moreover, the standard deviation of the buoyancy, given by the dashed line in Fig. 2, is nearly equal to the rawinsonde instrumental error (dotted line) so that the difference between the cloud virtual temperature of air lifted reversibly from the sub-cloud layer and the virtual temperature of the ambient environment may be considered to be a constant, within measurement error. Not only is the tropical atmosphere very nearly neutral to convection, it is systematically so. (The decline of parcel buoyancy above 600 hPa in Fig. 2 may be due to the neglect of precipitation effects in the upper part of the cloud and of the latent heat of fusion.)

The observed near-neutrality of the tropical atmosphere coincides with the observed weakness of most maritime tropical convection, and supports the quasi-equilibrium postulate of Arakawa and Schubert (1974). This implies that the strength and form of the convection is largely determined by the large-scale forcing, as also suggested by the recent work of Dudhia et al. (1987). (The near neutrality of the tropical atmosphere also suggests that most numerical simulations of tropical convection start with artificially high values of CAPE.) It is not, however, our intention to discuss the detailed relationship between convection and the large scale. We wish here to point out that while the buoyancy available for parcel ascent is small, the negative buoyancy available from evaporation of rain is potentially large, as indicated by the large upward decrease of moist entropy in the tropical atmosphere. The consequences of this energy source will be examined later. In the meantime we shall review the classical theories of moist convection and their limitations.

**Classical theories based on conditional instability**

The classical theories of moist convection deal with the modes of release of potential energy when potentially buoyant air rises. While precipitation is sometimes assumed to create an inherent asymmetry between upward and downward motion, buoyancy is always assumed to be a function of vertical displacement so that evaporation of rain (which produces density fluctuations not directly related to parcel displacement) is
ignored. The classical theories focus on the effects of a stably stratified environment.

**Quasi-linear moist convection**

Classical ideas on the dynamics of moist convection date back to Bjerknes (1938), who showed that the up-down asymmetry associated with precipitating moist convection favours narrow updrafts and broad downdrafts. These ideas were extended by Lilly's (1960) analysis of the unstable normal modes of a conditionally unstable atmosphere. This showed that in an inviscid fluid the most rapidly growing modes are non-propagating convection cells with extremely narrow updrafts surrounded by very broad descent. Bretherton (1987a) added diffusion to Lilly's problem and showed that the most unstable quasi-linear (linear except for the dependence of the sign of the stability on whether or not the air is saturated) modes have updrafts separated by an indefinitely wide descent region. The updrafts have widths on the order of the depth of the convection layer. Bretherton (1987a) also examined the weakly nonlinear extension of this problem which indicates that updrafts will tend to form with a spacing determined by diffusion or the local deformation radius. Three-dimensional updrafts are favoured over two-dimensional convection. Bretherton (1987b) showed that despite the ability of the stably stratified environment to support internal gravity waves, there are no unstable propagating modes in the quasi-linear problem.

The effect of mean vertical shear upon convection has been the subject of several studies. Kuo (1963) showed that the most unstable modes of linear convection in constant shear always occur as rolls aligned with the shear, while Emanuel (1984) demonstrated that the divergent component of flow in linear convection always gives up energy to the mean flow. When the mean shear varies with height, it is possible to have rapidly growing modes aligned across the shear, but these shear-driven modes are distinct from convective modes (Asai 1970).

According to these various linear stability analyses, moist convection should occur in the form of non-propagating isolated three-dimensional cells. In the presence of shear, the cells should show a tendency to organise in lines parallel to the shear, though the asymmetry between upward and downward motions should continue to favour three-dimensional cellular structure. There is no indication from the classical linear theories that convection should occur in clusters, that it should propagate, or that it should ever occur in the form of lines oriented other than parallel to the shear. We must look either to nonlinearity or to effects of falling precipitation to account for these observed features of precipitating convection.

**Steady two-dimensional nonlinear convective overturning**

In the special case that convection occurs as steady two-dimensional overturning, it is possible to find certain constraints on the form taken by the flow. But to do this, density must be a function of moist entropy ($\theta_m$) and pressure alone. This precludes directly accounting for buoyancy effects due to evaporation of precipitation in sub-saturated air. Solutions can be found for the strictly reversible case that all the condensed water is carried along by the flow, and for the pseudo-adiabatic case in which the condensate is completely removed as soon as it forms. Solutions of this kind have been presented by Moncrieff and Green (1972) and Moncrieff (1978).

The basic idea can be understood through the nonlinear equation for vorticity about an axis parallel to the convective line (we shall call this the $y$ axis). This states that the production of vorticity is proportional to the horizontal gradient of density on pressure surfaces. But if density is a function of pressure and moist entropy alone, then it follows that the production of vorticity is proportional to the horizontal gradient of moist entropy along pressure surfaces. Since moist entropy is conserved, it is constant along streamlines and thus its gradient is proportional to the horizontal gradient of streamfunction, which equals the vertical velocity. Thus, following a streamline, the production of vorticity is proportional to the vertical velocity, which implies that the vorticity itself varies only with altitude along a streamline. This can be exactly quantified as follows. The equation for the $y$ component of vorticity can be written

$$\frac{d}{dt}(\alpha \zeta) = -\alpha \nabla \alpha \times \nabla p.$$  \hspace{1cm} (6)

where $\zeta$ is the vorticity, $\alpha$ is the specific volume and $p$ is pressure. Now the crucial step is to assume that $\alpha$ can be expressed as a function of $p$ and moist entropy ($s$) alone, which will only be true if we neglect irreversible buoyancy effects such as those arising from falling precipitation. Then

$$\frac{d}{dt}(\alpha \zeta) = -\alpha \left(\frac{\partial \alpha}{\partial s}\right)_p \nabla s \times \nabla p.$$  \hspace{1cm} (7)

Now the first law of thermodynamics can be used to show that (see Emanuel 1986a): \hspace{1cm} (8)

$$\left(\frac{\partial \alpha}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s,$$

which is a Maxwell relation. In a steady flow, $s$ is constant along streamlines, so \hspace{1cm} (9)

$$\nabla s = \frac{ds}{d\psi} \nabla \psi.$$
where $\psi$ is the streamfunction. Combining Eqn 8 and Eqn 9 with Eqn 7 gives

$$\frac{d}{dt} (a\xi) = -\alpha \left( \frac{\partial T}{\partial p} \right) \frac{ds}{d\psi} \nabla \psi \times \nabla p. \quad \ldots \quad 10$$

Finally, we note that $a \nabla \psi \times \nabla p = V \nabla \psi \times \nabla p = \frac{dp}{dt}$. Then Eqn 10 can be written

$$\frac{d}{dt} (a\xi) = -\alpha \frac{\partial T}{\partial p} \frac{ds}{d\psi} \frac{dp}{dt} = -\frac{ds}{d\psi} \frac{dT}{d\psi}. \quad \ldots \quad 11$$

If this is integrated along a streamline ($s$, $\psi$ constant), then

$$a\xi = a_0\xi_0 + (T_0 - T) \frac{ds}{d\psi}, \quad \ldots \quad 12$$

where $a_0$, $\xi_0$ and $T_0$ denote known values of the respective quantities at a point on the streamline. For example, if we follow a streamline along the surface into a convective line, then given knowledge of its initial value of $a_0$, $\xi_0$ and $T_0$ and the variation of moist entropy with height (which together with the relative horizontal velocity gives $ds/d\psi$), we can calculate the vorticity as a function of $a$ and $T$. If these can be related (hydrostatically or otherwise) to height $z$, then the shapes of the streamlines can be determined. Lilly (1979) gives an excellent review of this procedure.

Two important features of solutions using Eqn 12 are of concern to us here. First, it is possible to obtain convective lines which propagate at speeds proportional to $\sqrt{CAPE}$ or to $\sqrt{DCAPE}$, where DCAPE is the available energy for downdrafts (Moncrieff and Miller 1976). Second, in a constant shear flow solutions always tilt downshear, giving up energy to the mean flow (Moncrieff 1978). This suggests that the linear models' tendency to produce shear-parallel lines is carried over to the nonlinear regime. The prediction of propagation speed has been supported by observations presented by Betts et al. (1976), but more recent work (e.g. Barnes and Sieckman 1984) shows little or even negative correlation between propagation velocity and CAPE. If near neutrality is assumed in the tropics, as suggested in the Introduction, then extratropical squall lines (which also have larger DCAPE) should propagate much faster than the tropical variety, but this is not uniformly so. The prediction of downshear tilt is also inconsistent with numerous observational studies, dating back at least as far as Ludlam (1963). The basic model appears to be deficient, and I shall argue in the following section that buoyancy effects of falling precipitation are the crucial missing ingredient. Thorpe, Miller and Moncrieff (1982) were able to account indirectly for buoyancy effects from evaporating rain by including a net pressure difference across their domain. If this difference is large enough, the convection can assume an upshear slope and extract energy from the mean shear.

**Wave-CISK**

The dynamical models of convection referred to up to now have regarded the organised convective system (e.g. the squall line) and the convective cell as one and the same. But mesoscale convective systems are usually comprised of a large number of individual cumuli and the dynamics of the system may be distinct from those of cells. Unfortunately, in the case of mesoscale systems, the time and space scales of the cell are not well separated from those of the system. Even so, many attempts have been made to calculate the properties of convective systems while regarding the ensemble effects of convective cells as determined by the system-scale flow. Such attempts, starting with Lindzen (1974), essentially treat squall lines as internal waves forced by the ensemble heating of convective clouds. The ensemble heating is usually determined as a function of the system-scale low-level moisture convergence while its vertical distribution is specified.

These 'wave-CISK' models have by now been rather thoroughly discredited. On the observational side, Nehrkorh (1986) has shown that it is impossible for simple wave-CISK models to simultaneously predict the phase speed and orientation of observed squall lines accurately, no matter what the vertical heating profile. While most observed squall lines propagate down-shear, wave-CISK models generally propagate up-shear. And while there is an observed relationship between low-level moisture convergence and convection, there is generally a time lag between the two. Finally, although three-dimensional numerical cloud models with fairly complete physics produce gravity waves, they are not observed to feedback on the convection (Richard Rotunno, personal communication).

The theoretical basis of wave-CISK models has also been called into question. The specification of the vertical heating profile is a rather severe assumption, and there is little observational evidence that this profile is independent of the system itself. This specification is responsible for the positive correlation of heating and temperature perturbation ($Q^T > 0$) in model unstable modes, whereas the observed near-neutrality of the tropical atmosphere suggests that convection tends to dampen temperature perturbations. Raymond (1986) imposed the condition that the upward displacement at the top of the sub-cloud layer must be positive for heating; this eliminates wave-CISK modes unless some form of evaporative cooling by rain is represented. And, as mentioned before, Bretherton (1978b) shows that propagating modes are not possible in a quasi-linear model.
CISK has always been controversial, but still has a following, perhaps because of its mathematical elegance. At present, however, the evidence suggests that it be abandoned as a useful concept, or perhaps redefined as a process involving the co-operation of individual cells and precipitation-induced downdrafts.

The dynamical effects of precipitation

The importance of evaporatively-cooled downdrafts in mesoscale convective systems has been emphasised since at least the work of Ludlam (1963), who postulated that squall line updrafts tilt upshear so that the rain can fall out of them into sloping downdrafts. The detailed observational studies of tropical squall lines by Zipser (1977) and Houze (1977) clearly indicate the prominent role of cumulus and mesoscale downdrafts in the structure of the systems, and also show that many squall lines move relative to the flow at all levels; thus, these lines must be time-dependent and/or three-dimensional. Sanders and Emanuel (1977) showed that an Oklahoma squall line not only exhibited transience on the cell scale, with new cells developing on the gust front every 45 minutes or so, but also underwent a systematic life cycle on the mesoscale.

A plethora of recent investigations focus on the dynamical role of evaporating and/or melting precipitation. The neutrality of the tropical atmosphere to upward displacements, discussed in the Introduction, suggests that the large negative buoyancies that can be produced by evaporating rain into the low entropy air of the tropical middle troposphere constitute a potentially much greater energy source than conventional CAPE; some recent studies address the possibility that the dynamics of mesoscale systems allow them to tap this source.

The central dynamical effect of falling precipitation is to uncouple important buoyancy sinks (due to condensate loading, melting and evaporation) from vertical displacement. When such sinks are operative, the integral theorems forbidding extraction of kinetic energy from shear are no longer valid; nor are the models of Moncrieff and Green (1972) and Moncrieff (1978). Seitter and Kuo (1983) suggest a specific scenario whereby the buoyancy effects of falling precipitation may allow convective lines to slope upshear, as illustrated in Fig. 3. The slope itself allows precipitation to fall out of the updraft, after which the negative buoyancy due to condensate loading and evaporation of rain causes a vorticity source on the downslope side of the updraft. The vorticity produced along the core of the updraft causes it to tilt further in the same direction. Emanuel (1986b) demonstrated that this mechanism works in a linear parallel-plate convection model and that the water loading alone will lead to sloped, propagating convection even in the absence of shear, provided the negative buoyancy due to adiabatic water loading has about the same magnitude as the thermal buoyancy. (The observations discussed in the Introduction suggest that in the tropical atmosphere thermal buoyancy is almost exactly cancelled by buoyancy due to adiabatic water loading.) Bretherton (1987c) has obtained analytic solutions of this model. Propagation speeds are small (a few m s$^{-1}$) and scale with the terminal velocity of raindrops.

Evaporation is probably the most important buoyancy effect of falling precipitation, and its most prominent product is the spreading cold pool and associated gust front at the surface. Byers and Braham (1949) ascribe the short life of air mass thunderstorms to the effectiveness of the spreading cold pool in cutting off the supply of potentially buoyant air. Thorpe, Miller and Moncrieff (1982) proposed that low-level vertical shear has the effect of coupling the cloud layer with the gust front, favouring longer-lived convection propagating down-shear. This effect was strongly evident in the linear model of Emanuel (1986b), which contains evaporation below cloud base and a jump in ambient horizontal velocity between the sub-cloud and the cloud layers. The downshear-propagating mode is favoured in this model, and the growth rate is a maximum at a particular value of the shear that depends on the sub-cloud evaporation rate. Figure 4 reproduces the inferred regime diagram of the linear model. Small shear and large evaporation favour two-dimensional shear-parallel lines, as with nonprecipitating convection; somewhat strong shear or smaller evaporation gives rise to two-dimensional shear-perpendicular lines, and yet stronger shear favours lines skewed with respect
to the shear or three-dimensional convection. This has some correspondence with nature. Barnes and Sieckman (1984) showed that tropical squall lines are parallel to the shear if the shear is weak, but orthogonal to it if it is strong and concentrated at low levels. Extratropical squall lines studied by Bluestein and Jain (1985) and Wyss and Emanuel (1988) were systematically orthogonal to strong shear near cloud base.

The nature of the mechanism by which spreading pools of cold air near the surface can interact constructively with ambient vertical shear has been further elucidated by Rotunno, Klemp and Weisman (1988). Their view of the vorticity dynamics of this interaction is illustrated in Fig. 5. In the absence of ambient shear, the solenoidal generation of vorticity at the gust front imparts a decided vorticity to air lifted up at the front, the consequence of which is to reduce the vertical penetration of the air (perhaps to the point of not being able to reach the level of free convection) and to cause the updraft to slope back over the cold air. If ambient low-level shear is present, however, the ambient vorticity partially or completely cancels the solenoidally generated vorticity so that the air entering cloud base is nearly free of vorticity. This favours deep vertical penetration of air forced up by the gust front and erect convective cells.

The vorticity dynamics elucidated by Rotunno, Klemp and Weisman (1988) nicely explain the numerical results of Thorpe, Miller and Moncrieff (1982) and the linear model results of Emanuel (1986b), and is consistent with the observations by Bluestein and Jain (1985), Barnes and Sieckman (1984) and Wyss and Emanuel (1988), all of which show that squall lines are oriented orthogonal to the low-level shear and propagate down the shear vector.

**Summary and outlook**

Ideas about the dynamics of tropical convective systems are currently undergoing a rapid evolution away from classical buoyant convection theories and CISK, and toward a new understanding based on the dynamical effects of precipitation-induced buoyancy. The new theories have the great advantage of qualitatively and quantitatively accounting for the organisation of convection into lines, the direction and speed of their propagation and their orientation with respect to the mean shear. Classical theories have been notably unsuccessful in accounting for these properties. Much progress has been made since the appearance of the review paper on squall lines by Lilly (1979), and it is likely that the present review will become antiquated in a short period of time. Several challenging problems present themselves immediately.
The near neutrality of the tropical atmosphere discussed earlier is a surprising new finding. But most cloud models begin with decidedly unstable soundings. Further work with cloud models starting from nearly neutral conditions may prove illuminating. In view of the importance of evaporating rain, theoretical investigations of the dynamical effects of melting snow and ice are desirable. Although the heat of fusion is only about 12 per cent of that of vaporisation, all the snow melts whereas only part of the precipitation evaporates. While most recent work has focused on squall lines, it is time to start exploring the dynamics of other forms of mesoscale convective systems such as Mesoscale Convective Complexes (MCCs). And, last but not least, the time has come to take precipitation-induced downdrafts seriously in formulating convective parameterisations for large-scale models. Large-scale control of the ensemble convective downdraft fluxes remains a poorly understood and challenging question for the future.

References


