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Modelling mixed-layer growth in the Koorin Experiment

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The Koorin Experiment offers a unique set of boundary-layer data taken over land at a low latitude (16° 16′S) and over a tree-covered terrain with large roughness ($z_0 \cong$ 0.5 m). These data have been used to predict the rise of the mixed layer using the 'jump' (or 'slab') model formulation recommended by Driedonks and Tennekes (1984). The results show that the closure constant C_F for these data is larger than the popular value of 0.2. For predictions of the mixed-layer height at 1500 h and 1800 h the best value of C_F was found to be 0.6 and at 1200 h the best value was 0.5. Possible reasons for these large values are discussed.

Introduction

The Koorin Experiment (see Clarke and Brook 1979) was a major Australian planetary boundary-layer experiment. It was designed to provide high quality data on boundary-layer dynamics over land at low latitudes (i.e. at small Coriolis parameter f). Boundary-layer observations over land at low latitudes are very rare. Most observations have been taken at mid-latitudes, or if they have been at low latitudes, they have been taken over the sea or on the coast.

The second aspect that makes the Koorin data unique is that the terrain was tree-covered (see Fig. 1) and the roughness length was large ($z_0 \cong$ 0.5 m). Most previous experiments have been conducted over relatively smooth surfaces.

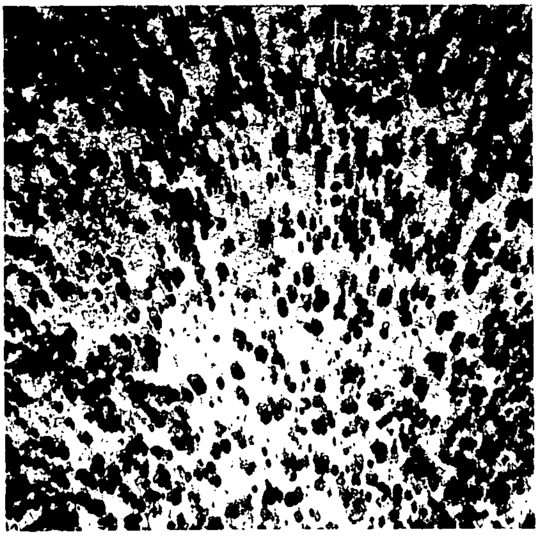
One of the aims of the Koorin Experiment was to advance research in modelling the rise of the mixed layer. This is a subject about which there is considerable literature (see for example the textbooks by Sorbjan (1988, pp. 161–168) and Stull (1988, pp. 441–497)). In this paper we investigate a simple model of the mixed layer, known variously as a 'jump' model, a 'slab' model or an 'integral' model. This model was first proposed by Ball (1960) and has been developed further by Lilly (1968), Deardorff et al. (1969), Betts (1973), Carson (1973), Tennekes (1973), Zilitinkevich (1975), Mahrt and Lenschow (1976), Zeman and Tennekes (1977), Yamada and Berman (1979) and many others. As far as is known, however, the Koorin data have not been used for formulating or testing hypotheses concerning the rise of the mixed layer.

The conservation equations for thermal energy and momentum are used to develop the jump model, but they are not sufficient to provide a set of closed equations. An additional relation for entrainment is required. Most authors follow a method similar to the one described here and parametrise the turbulent kinetic energy equation. In its simplest form this produces a relation that states that the heat flux at the top of the mixed layer is proportional to the heat flux at the surface. An alternative method of closure, recommended by Manins (1982), is based on a critical Froude number. In the Manins model, the mixed layer grows in spurts. There is no growth unless the Froude number approaches unity from below; if the criterion is exceeded then growth occurs until the Froude number is reduced to below unity.

*In this paper Dr Clarke presents his modelling results of the rise of the mixed layer. Dr Clarke also intended to present an analysis of the mixed-layer winds to explain the curious behaviour of the dry season wind minimum after sunrise and maximum about 1100 h. He carried out considerable work on this, but wasn't able to complete it before his untimely death. Unfortunately his data files have since been lost.

This paper and a companion (The lower atmosphere over a dry season savanna site. Part II: modelling and deductions, *Aust. Met. Mag, 38*, 235–44) derive from a single manuscript submitted shortly before Dr Clarke's death. Publication of both papers owes very much to the efforts of Dr G.D. Hess of the Bureau of Meteorology Research Centre who made the revisions, with the assistance of Prof. R.K. Smith of the University of Munich.

Fig. 1 Aerial view of the surface at Daly Waters, the site of the Koorin Experiment. The terrain was covered with trees giving a roughness length of ~0.5 m.



Theory for a jump model of the mixed layer

In the discussion below we follow the framework presented by Driedonks and Tennekes (1984) which has been used operationally by the Dutch National Weather Service for many years.

In a horizontally homogeneous boundary layer, where radiation, divergence and change of phase can be neglected, the conservation equations for momentum and thermal energy can be written as

$$\frac{\partial U}{\partial t} = -f(V_g - V) - \frac{\partial \overline{wu}}{\partial z} \qquad \ldots 1$$

$$\frac{\partial V}{\partial t} = f(U_g - U) - \frac{\partial \overline{wv}}{\partial z} \qquad \ldots 2$$

$$\frac{\partial \Theta}{\partial t} = - \frac{\partial \overline{w\theta}}{\partial z} \qquad \ldots 3$$

where upper case indicates the mean variables and lower case the turbulent fluctuations; U, V are the horizontal wind components, U_g, V_g the geostrophic wind components, Θ the potential temperature, w the vertical velocity, z the height and t the time. Equations 1 to 3 are now integrated over the depth of the mixed layer h yielding

$$h\frac{\partial <U>}{\partial t} = \overline{wu_0} - \overline{wu_h} - fh <V_g - V> \qquad \ldots 4$$

$$h\frac{\partial <V>}{\partial t} = \overline{wv_0} - \overline{wv_h} + fh <U_g - U> \qquad \ldots 5$$

$$h\frac{\partial <\Theta>}{\partial t} = \overline{w\theta_0} - \overline{w\theta_h} \qquad \ldots 6$$

where the averages are defined as $<\Phi> \equiv (1/h) \int_0^h \Phi \, dz$ and Φ is any of the mean variables. The rate of rise of the mixed layer dh/dt in the absence of clouds is the sum of the entrainment velocity w_e and large-scale subsidence w_L. Estimates of subsidence (e.g. Driedonks (1982) and Driedonks and Tennekes (1984)) have found that $w_L \ll w_e$ and we shall ignore w_L and write

$$dh/dt = w_e \qquad \ldots 7$$

At the height z = h we assume a zero order discontinuity or jump in the value of the mean variable (see Fig. 2). The balance equation for a jump in a scalar mean quantity (like Θ) is

$$\frac{\partial \Delta\Phi}{\partial t} = \gamma_\Phi w_e - \frac{\partial <\Phi>}{\partial t} \qquad \ldots 8$$

where γ_Φ is the slope of the variable Φ immediately above the mixed layer. The analogous equations for the mean wind component jumps are

$$\frac{\partial \Delta U}{\partial t} = \gamma_u w_e - (\overline{uw_0} - \overline{uw_h}) / h + f\Delta V \qquad \ldots 9$$

$$\frac{\partial \Delta V}{\partial t} = \gamma_v w_e - (\overline{vw_0} - \overline{vw_h}) / h - f\Delta U \qquad \ldots 10$$

The turbulent fluxes at z = h are given by

$$-\overline{\phi w_h} = w_e \, \Delta\Phi \qquad \ldots 11$$

where Φ refers to any of the above mean variables.

The set of Eqns 4 to 11 is not yet complete. To close the system of equations we introduce the parametrisation of the turbulent energy equation suggested by Driedonks and Tennekes (1984)

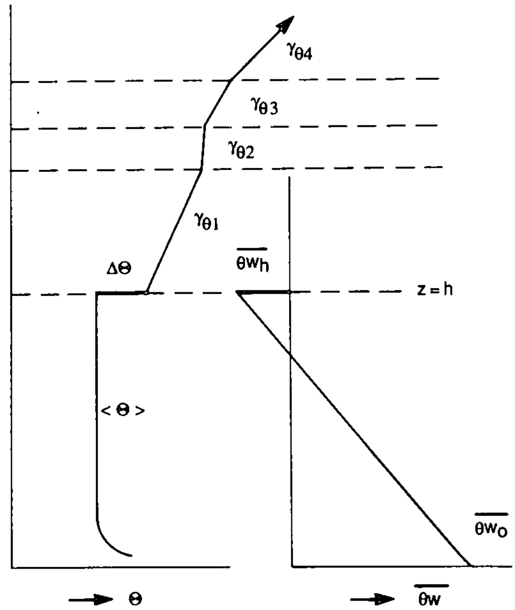
$$C_t \frac{w_m^2}{h} w_e = \frac{g}{T} \overline{\theta w_h} + C_F \frac{w_m^3}{h} \qquad \ldots 12$$

where the mixed-layer velocity scale w_m is defined by

$$w_m^3 \equiv w_*^3 + (A / C_F) \, u_*^3 \qquad \ldots 13$$

where u_* is the surface friction velocity ($u_*^2 \equiv \tau_0/\rho = [(-\overline{uw_0})^2 + (-\overline{vw_0})^2]^{1/2}$), τ_0 the surface stress and ρ the air density. w_* is the convective

Fig. 2 Schematic profiles of Θ and $\overline{\theta w}$ in a jump model of the mixed layer.



velocity ($w_*^3 \equiv u_*^3\,(-h/kL)$), where k is the von Kármán constant ($k \cong 0.4$). L is the Obukhov length ($L \equiv -u_*^3/(kg/T)(H_0/\rho C_P)$), where g is the acceleration due to gravity, T the air temperature, H_0 the surface heat flux and C_P the specific heat of air at constant pressure. C_t, C_F and A are constants.

The first term in Eqn 12 represents the temporal rate of change of turbulent energy, the second term the buoyant production and the third term the flux divergence and dissipation. The shear production terms in the energy equation are not modelled explicitly, because explicit parametrisation of these terms can lead to situations where the entrainment velocity w_e becomes negative, invalidating the model (Manins 1982; Driedonks 1982). Driedonks and Tennekes (1984) assume that the effect of the shear term at h can be included implicitly in the interpolated mixed-layer velocity scale w_m and the empirical constant A which determines the relative importance of the surface friction.

Solving Eqn 12 for w_e we obtain (for $\Delta\Theta \neq 0$)

$$w_e = \left[C_F H_0/(\rho C_P \Delta\Theta) + C_A u_*^3 T/(gh\Delta\Theta)\right] \Big/ \left[1 + C_t w_m^2 T/(gh\Delta\Theta)\right] \quad \ldots 14$$

When $\Delta\Theta = 0$, Eqn 11 shows that the heat flux at $z = h$ is zero. The entrainment velocity in this case is given by

$$w_e = C_F w_m / C_t \quad \ldots 15$$

The rise of the mixed-layer height is found by solving Eqns 7 and 14 or 15. Equation 14 contains $\Delta\Theta$ which in general varies in time. We can integrate Eqn 8 with $\Phi = \Theta$, using Eqns 6 and 7. This procedure yields

$$\Delta\Theta_1 = (h_0\Delta\Theta_0 + \gamma_\theta(h_1^2 - h_0^2)/2 - \int (H_0/\rho C_P)\,dt) / h_1 \quad \ldots 16$$

Similar expressions can be obtained from the velocity jumps

$$\Delta U_1 = (h_0\Delta U_0 + \gamma_u(h_1^2 - h_0^2)/2 - \int [u_*^2 U_{50} / (U_{50}^2 + V_{50}^2)^{1/2}]\,dt + f(h_1 + h_0)(\Delta V_1 + \Delta V_0)\Delta t/4) / h_1 \quad \ldots 17$$

$$\Delta V_1 = (h_0\Delta V_0 + \gamma_v(h_1^2 - h_0^2)/2 - \int [u_*^2 V_{50} / (U_{50}^2 + V_{50}^2)^{1/2}]\,dt - f(h_1 + h_0)(\Delta U_1 + \Delta U_0)\Delta t/4) / h_1 \quad \ldots 18$$

where U_{50} and V_{50} are the observed mean wind components at 50 m height. The values of the mixed-layer mean variables are found using Eqns 4 to 6 in the form

$$\langle U_1\rangle = \langle U_0\rangle + 2\int [u_*^2 U_{50} / (U_{50}^2 + V_{50}^2)^{1/2}]\,dt / (h_1 + h_0) + (h_1 - h_0) \times(\Delta U_1 + \Delta U_0) / (h_1 + h_0) - f\Delta t \times(\langle V_{g1} - V_1\rangle + \langle V_{g0} - V_0\rangle) / 2 \quad \ldots 19$$

$$\langle V_1\rangle = \langle V_0\rangle + 2\int [u_*^2 V_{50} / [U_{50}^2 + V_{50}^2]^{1/2}]\,dt / (h_1 + h_0) + (h_1 - h_0) \times (\Delta V_1 + \Delta V_0) / (h_1 + h_0) + f\Delta t \times (\langle U_{g1} - U_1\rangle + \langle U_{g0} - U_0\rangle) / 2 \quad \ldots 20$$

$$\langle\Theta_1\rangle = \langle\Theta_0\rangle + 2\int H_0/(\rho C_P)\,dt / (h_1 + h_0) + (h_1 - h_0)(\Delta\Theta_1 + \Delta\Theta_0) / (h_1 + h_0) \quad \ldots 21$$

Method of solution

Equations 7 and 12 to 21 are solved by a predictor-corrector technique. The stable layer above the mixed layer is subdivided into regions (as many as four for each variable). In each region the mean profile varies linearly and is characterised by $\gamma_{\theta 1}$, $\gamma_{\theta 2}$, $\gamma_{\theta 3}$ and $\gamma_{\theta 4}$, for example. The variables u_*, U_{50}, V_{50}, H_0 and T are interpolated to time $t + \Delta t$ in order to solve the integrals of the surface fluxes in Eqns 17 to 21 and to evaluate the predicted w_e from Eqns 14 or 15.

In the first trial the present value of the entrainment velocity w_{e0} is used to approximate w_e averaged over the time-step Δt. The new (predicted) value of the mixed-layer height h_{1p} is then found by integrating Eqn 7. The next step is to choose the appropriate γs, based on h_{1p}, and calculate the predicted values for the jumps and the entrainment velocity, $\Delta\Theta_{1p}$, ΔU_{1p}, ΔV_{1p} and w_{ep}. Values of ΔU_1 and ΔV_1 on the right-hand side of Eqns 17 and 18 are approximated by ΔU_0 and ΔV_0.

The predicted results are now corrected to produce more accurate solutions. The corrected entrainment velocity w_{ec} is defined as the average of w_{e0} and w_{ep}. The new (corrected) mixed-layer height h_{1c} is found from Eqn 7, using w_{ec}. The rest of the procedure is then repeated, replacing ΔU_1 and ΔV_1 on the right-hand side of Eqns 17 and 18 with ΔU_{1p} and ΔV_{1p}. If the relative error between the predicted and corrected mixed-layer heights is greater than one per cent, the time-step is halved and the whole procedure is repeated until convergence is reached. If convergence is obtained the mixed-layer averaged variables $<\Theta_1>$, $<U_1>$ and $<V_1>$ are computed from Eqns 19 to 21.

It should be noted that under these assumptions the equations for h, w_e, $\Delta\Theta$ and $<\Theta>$ form a complete set by themselves and the evolution of h, w_e, $\Delta\Theta$ and $<\Theta>$ can be computed without solving for ΔU, ΔV, $<U>$ and $<V>$. Most previous authors have considered only this smaller set of equations. In principle, similar equations to Eqns 16 and 21 can be written for the specific humidity variables, ΔQ and $<Q>$, or other passive scalars; however, because the Koorin Experiment was conducted during the dry season, the radiosonde ascents did not include measurements of mixed-layer specific humidity profiles. Also, specific humidity and other scalars tend not to be as well mixed as potential temperature.

Results and discussion

Equations 7, 12 to 16, and 21 were solved using the values of the constants suggested by Driedonks and Tennekes (1984): $C_t = 1.5$, $C_F = 0.2$ and $A = 5.0$. Predictions of the rise of the inversion from 0900 h to 1200, 1500 and 1800 h, respectively, were made for every available day of the Koorin Experiment except for Day 28, when strong horizontal and vertical advection was present. In all, 27 such 'predictions' were possible for each of three time-spans, with the use of measured initial conditions at 0900 h, and the half-hourly measurements of surface heat flux H_0 and stress τ_0 during the day.

The predicted heights of the mixed layer against the observed heights are plotted in Fig. 3. There is some uncertainty in judging the height of the mixed layer from instantaneous radiosonde measurements because energetic thermals penetrate into the stable layer above, creating elevated cauliflower-shaped boundary-layer heights where the thermals are. The height determined by the radiosonde will depend on whether or not it is in a thermal. The probable error in h is less than 100 m. In all cases the effects of the moisture flux have been neglected, but these effects are known to be minor.

For all three periods in Fig. 3 the rise of the mixed layer is underestimated. To determine whether a larger value of the constant C_F would

Fig. 3 Predicted and observed heights h of the mixed layer at Daly Waters. Predictions are made from data at 0900 h. The closure constant C_F is assumed to be 0.2. The 'perfect prediction' is shown by the solid lines for the observation times of (a) 1200 h, (b) 1500 h and (c) 1800 h.

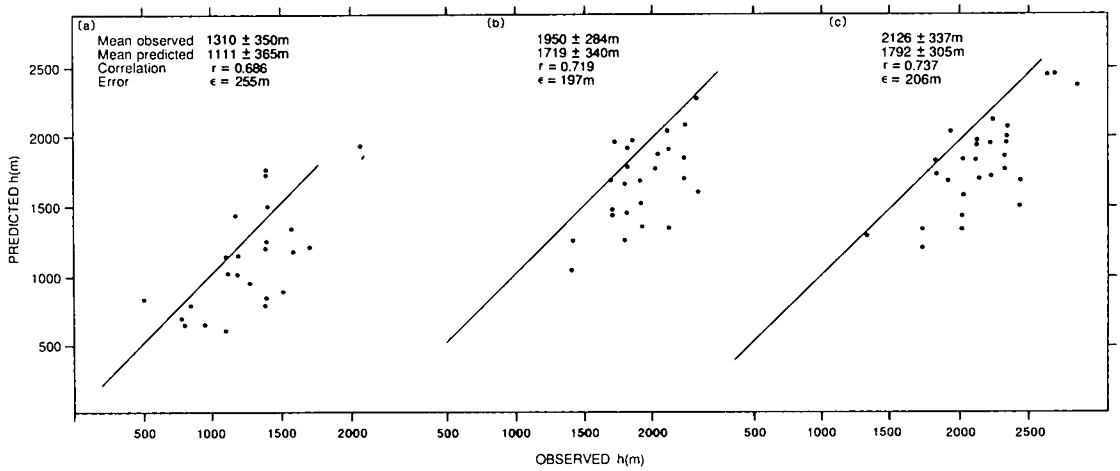


Fig. 4 The same as Fig. 3, except with $C_F = 0.4$.

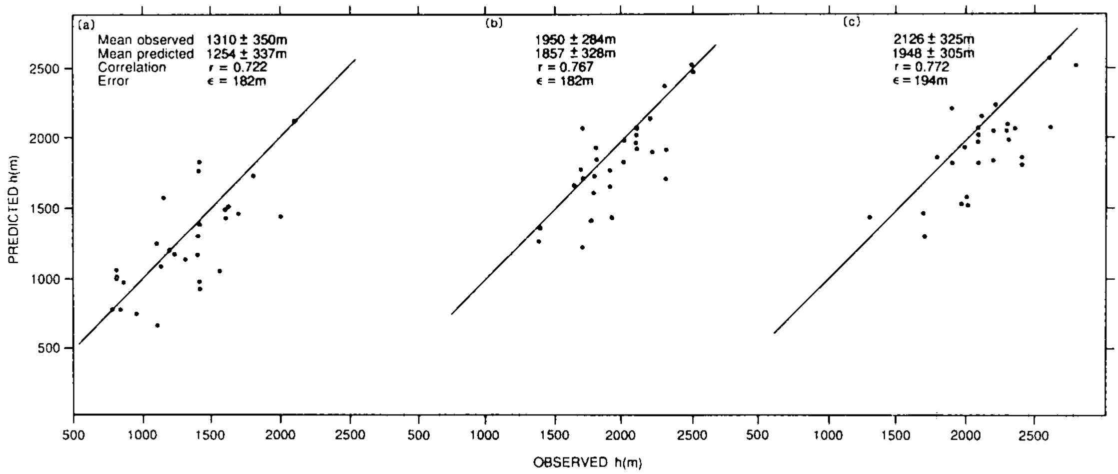
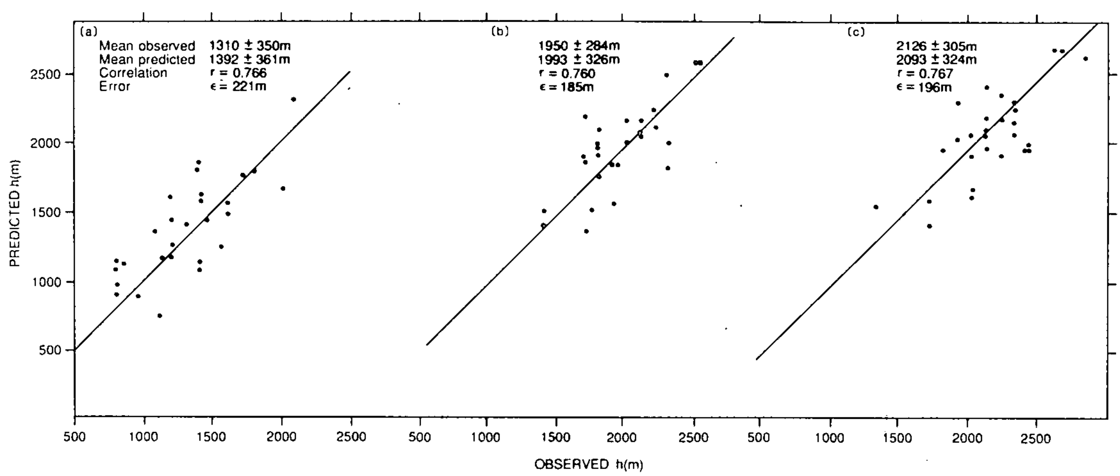


Fig. 5 The same as Fig. 3, except with $C_F = 0.6$.



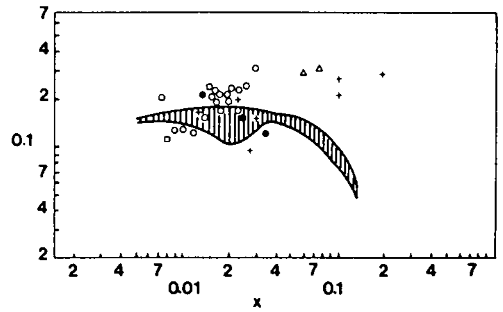
yield better predictions, the predictions were repeated using $C_F = 0.4$ and 0.6 (Figs 4 and 5). Both are seen to yield predictions notably better than with $C_F = 0.2$.

Observations by other investigators have been summarised by Stull (1976a, Table 1) and Sorbjan (1988, pp. 165–168) for example. A sample of measurements of C_F from laboratory studies and field experiments and values obtained from a third-order turbulence closure model are shown in Fig. 6. From inspection of Fig. 6 and the Tables given by Stull and Sorbjan it is seen that the observed values of C_F seem to range between 0 and 2 with a modal peak lying between 0.1 and 0.3. The value favoured by most authors is that recommended by Driedonks and Tennekes (1984), $C_F = 0.2$; an exception to this is the case studied by Stull (1976b). He was able to obtain good results modelling tropical data from north of Puerto Rico using $C_F = 0.7$. However, this was a very complicated situation with clouds, large

Table 1. Mean values of C_F for various time intervals estimated from Eqns 22 and 23.

Time of interval (h)	*No. of days*	*Mean C_F*
0900–1000	15	1.00
1000–1100	15	0.65
1100–1200	16	0.57
1200–1500	15	0.68
1500–1800	15	−0.11
0900–1800	15	0.66

Fig. 6 Measured values of C_F (assuming $C_A = 0$ and $C_t = 0$). In this case $C_F = -\overline{w\theta_h}/\overline{w\theta_0}$. The abscissa x is defined by $x \equiv [(\overline{w\theta_0})^2/(g\gamma_\theta^3 h^4/\Theta)]^{1/3}$. The symbols refer to laboratory measurements by Deardorff et al. (1969) (○) and Willis and Deardorff (1974) (□), field measurements by Cattle and Weston (1975) (△), Yamamoto et al. (1977) (●) and Jouvenaux (1978) (+), and values obtained from a third-order turbulence closure model by André et al. (1978) (shaded region), (Adapted from Artaz and André (1980)).



synoptic-scale subsidence, large wind shear and large gravity wave energy losses present and buoyant energy played only a minor role.

There is some evidence that C_F is not a universal constant, but varies during the day (Carson 1973; Manton 1978; Manins 1982) with very small values occurring during the afternoon. Based on the O'Neill data and the Wangara data, Stull (1976b) concludes that C_F is essentially constant for mixed layers over land between 1000 and 1600 h, but varies strongly before and after these times.

Since there is difficulty in estimating an initial value h_0 of the mixed layer at 0900 h, and of the potential temperature jump $\Delta\Theta_0$, these were varied to find the sensitivity of the predicted heights to variations in them. It was found that varying $\Delta\Theta_0$ from 0 to 0.5° produced changes of -9 ± 19 m at 1200 h, -3 ± 19 m at 1500 h and -4 ± 14 m at 1800 h. Similarly, reducing h_0 by 50 per cent produced changes of 15 ± 37 m at 1200 h, 5 ± 11 m at 1500 h, and 3 ± 12 m at 1800 h; increasing h_0 by 50 per cent resulted in changes of -3 ± 27 m at 1200 h, -5 ± 15 m at 1500 h, and 2 ± 13 m at 1800 h. This list shows that the final predictions are not too sensitive to errors in choosing h_0 and $\Delta\Theta_0$.

To check whether the higher values of C_F found to provide better predictions than $C_F = 0.2$ are consistent with the data, we examined a simpler, more commonly used closure which neglects the effects of time changes of turbulent energy ($C_t = 0$) and mechanical turbulence due to wind shear ($A = 0$) (see Ball 1960; Lilly 1968; Betts 1973; Carson 1973; Tennekes 1973; Mahrt and Lenschow 1976; Yamada and Berman 1978 among others). In this case the closure becomes

$$\overline{\theta w_h} = - C_F \overline{\theta w_0} \qquad \ldots 22$$

and Eqn 21 can be written

$$\langle\Theta_1\rangle = \langle\Theta_0\rangle + 2(1 + C_F) \int (H_0/\rho C_P)\, dt \,/\, (h_1 + h_0) \qquad \ldots 23$$

For this purpose the potential temperature in the mixed layer was determined by averaging Θ from 50 m to the height h of the layer, weighting by density. These values were averaged for the 15 days (or 16 days for the 1100–1200 h time interval) when radiosonde flights were done hourly from 0900 to 1200 h. The integral in Eqn 23 was obtained by integrating the heat flux from site M2*. The quantities $(h_1 + h_0)(\langle\Theta_1\rangle - \langle\Theta_0\rangle)/2$ and $\int (H_0/\rho C_P)\, dt$ were then summed over 15 days (or for the 1100–1200 h time interval, 16 days) to yield the quotient, a mean value of $1 + C_F$, for each of the time intervals. A double summation was necessary to obtain an average C_F over the period 0900 to 1800 h. Table 1 gives the results.

A disadvantage of this method of determining C_F is that it is very sensitive to the measurement of the mean Θ, and there may well be small systematic radiosonde errors varying with time of day. It is considered likely that systematic baselining errors account for the anomalous values of C_F obtained for 0900–1000 h and for 1500–1800 h. Another important factor for 0900–1000 h interval is that the neglect of the shear terms may not be valid. The other values support the prediction experiments (Figs 3 to 5) that a value of C_F equal to 0.5–0.6 is applicable to the Koorin site.

The reason for the difference in C_F for the Koorin data and the popular value of 0.2 is not clear. The calculations are most sensitive to the heat flux input and the mean temperature slopes in the overlying stable layer. As seen in Fig. 1 the trees were not densely packed, and generally had open spaces between them. It is thought that in convective conditions the heat fluxes measured at site M2 were representative for forcing of mixed-layer dynamics, and not merely reflecting a canopy effect. On two days an independent check of these heat flux measurements was made with heat flux measurements from an instrumented aircraft (Coulman and Warner 1979). The value of the aircraft-measured heat flux on Day 26 extrapolated to the surface was 190 W m^{-2} compared to the surface measurements of 145 and 199 W m^{-2}. On Day 28 the extrapolated heat flux was 150 W m^{-2}

*The heat and momentum fluxes were measured at two sites, M1 and M2. Those at M2 were generally larger than those at M1, and the M2 measurements were used, except for occasions when the system failed there. The use of M1 fluxes would have increased the inferred values of C_F.

compared to the surface values of 132 and 247 W m^{-2}. Some caution must be exercised in interpreting the latter results because Day 28 was a day of strong advection and the flow was not dominated by convection.

The γs were well known initially, but variations in the γs during the day could have affected the value of C_F. However, γ does not appear in Eqn 23, and determinations based on Eqn 23 also showed large values of C_F.

In stable conditions, isolated, large roughness elements can lead to significant horizontal inhomogeneities. However, in convective conditions there is no reason to believe that this is the case. The effect of large values of z_0 on C_F has not yet been determined. One of the few previous studies over a surface with large roughness length is that reported by Lenschow and Johnson (1968). They found $C_F = 0.9$ for flow over a forest with $z_0 = 1.6$ m in northeastern Wisconsin.

Other factors neglected in this analysis (large-scale subsidence, horizontal advection, gravity wave induced mixing at the top of the mixed layer and moisture effects) were not measured but were initially thought to be small. The most likely reason for the large observed values of C_F is the neglect of large-scale subsidence and horizontal advection, but a dependence of C_F on roughness cannot be ruled out.

Conclusions

The Koorin Experiment presents a unique source of high quality data for studying boundary-layer dynamics at low latitudes over a surface with large roughness. In this paper we have examined the prediction of the rise of the mixed-layer height, using a simple jump model. The data show that the best value of the closure constant C_F of the three tested is 0.6 for 1500 h and 1800 h, and 0.5 for 1200 h. These values are larger than the popular value of 0.2 but are within the range of previously observed values.

The reason for the large values needs further investigation. It may be due to the neglect of vertical and horizontal advection. The importance of horizontal advection in Eqn 3 could be estimated as advocated by Manins (1982), using the three-hourly thermal winds determined by Garratt (1985). This would necessarily be a qualitative result, because of the large scatter in the thermal wind data. The qualitative influence of vertical advection could be estimated, assuming that w_L varied linearly with height (Stull 1976b; Manton 1978). The other possibility, that C_F could depend on the roughness length, would require further experiments.

It is hoped that this analysis will provide a stimulus for researchers to continue to study this problem and others, using the rich boundary-layer data set obtained from the Koorin Experiment.

Acknowledgments

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