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Operational, short-term prediction of rainfall using a cycled Markov chain method

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The existing operational Markov technique, used by the National Meteorological Centre in Melbourne for the short-term prediction of rainfall at the eight Australian capital cities, is extended to include cycling throughout the day. In place of the current once-daily forecasts issued for the period 0600–1800, 12-hour probability of precipitation forecasts can now be issued around the clock at three-hour intervals. This ensures that forecasts have maximum timeliness and diminishes logistical problems of forecast distribution for the Regional Offices. Cycled forecasting also enables the predictions to incorporate the diurnal variations in rainfall distribution existing in some of the capital cities.

A trial of the cycled system was carried out over the two-year period from January 1988 to December 1989 inclusive. Results of the trial (presented as half-Brier scores and reliability diagrams) confirm the skill displayed by the method in the original non-cycled operational forecasts issued only at 0600.

Introduction

The short-term prediction of rainfall, that is for periods of 12 hours or less, continues to be a difficult task for most Australian cities. Significant mesoscale circulations caused by topography, land-sea contrast, radiational cooling, etc., often combine with larger scale disturbances to produce rain. These mesoscale processes are not well resolved, temporally or spatially, in the operational numerical weather prediction (NWP) model currently used by the Bureau. One promising way to bridge the gap between nowcasting (largely observational) methods such as radar and satellite techniques (which perform well for forecasting periods $\leq$ 3 hours) and NWP (which performs best for periods $\geq$ 12 hours) is a statistical method known as the Markov chain technique (see Hess et al. (1989) and the references contained therein).

In the Markov chain technique developed by Hess et al. (1989) surface observations at a single station at the present time and those recorded three hours previously are used to estimate the probability of rainfall. The transitional probabilities that the weather will change from one state to another are computed by a simple algorithm in which the coefficients have been determined from climatological data. Since there is little computation involved, the predictions can be made as soon as the observations are available.

Comparative trials of the Markov chain technique with probability predictions by duty forecasters of the Victorian Regional Office, and with predictions based on persistence and climatology (Hess et al. 1989; Hess and Guymer 1989), have shown that the Markov technique outperformed the other methods. Encouraged by these results the Markov chain technique for predicting the probability of precipitation (POP) was introduced operationally by the National Meteorological Centre (NMC) in Melbourne in April 1989.

In this paper the present operational system is extended significantly by removing three major limitations. Firstly, the present system only predicts rainfall for the 12-hour period beginning at 0600 LST. This is unsatisfactory because it means that the method does not fully utilise all of the available three-hourly surface observations, which are taken around the clock. In the new system a three-hourly cycling of the algorithm coefficients is employed to take advantage of all of the available data, thereby ensuring that the forecast period can begin at any time throughout the day (that is, at 0000, 0300, 0600, 0900, 1200, 1500, 1800 or 2100 hours) and the coefficients will be current for that starting time. Second, the new prediction scheme incorporates large diurnal variations of rainfall which occur in the three-hourly data at some stations, for example, Darwin (Fraedrich and Leslie 1988). Third, a very important advantage of cycling three-hourly is that it will help to meet logistical problems with the distribution of forecasts by the regional offices. In particular, forecasts made from 0600 observations are not timely in terms of media requirements.

Methodology

Technique

The Markov model computes the probability that rain will occur by approximating the transitional probabilities by performing linear least squares regression on the following 14 meteorological variables: previous state (i), station pressure minus 1000 hPa (P), the change in station pressure in the last three hours (ΔP), dry-bulb temperature minus 18°C (T), the change in dry-bulb temperature in the last three hours (ΔT), the wet-bulb depression ($T\text{-}T_w$), the change in wet-bulb depression in the last three hours [$\Delta(T\text{-}T_w)$], the east-west component of the wind (U), the north-south wind component (V), the change in total wind speed in the last three hours [$\Delta(U^2 + V^2)^{1/2}$], the amount of low cloud cover (LCLD), the change in the amount of low cloud cover in the last three hours (ΔLCLD), the amount of middle cloud cover (MCLD), and the change in the amount of middle cloud cover (ΔMCLD). Two additional covariates, sine and cosine harmonics, are employed to represent the annual cycle. The coefficients for the Markov chain were computed from three-hourly surface climatological data for the period January 1960 to June 1988, obtained from the National Climate Centre (NCC).

Verification of the new operational system

No verification of the performance of the operational Markov system to date is available at present, owing to a coding error discovered by one of the authors in the NMC verification system. A primary motivation of the present work, therefore, is to re-run the system and evaluate its recent performance. The data set used for this verification is described immediately below.

The new operational system was examined in two ways. Firstly, the long-term behaviour of the system was compared with observations over the complete period 1960–1988. This provides a check that no serious programming errors have been introduced into the model. Second, to compensate for the lack of operational verification mentioned above, a data set consisting of two full years of data (January 1988 to December 1989) was prepared. It should be noted that: (a) in order to obtain two complete years of data the verification period slightly overlaps with the period used to compute the Markov coefficients. However, the computed coefficients change insignificantly in such a small overlap; (b) the period of validation is relatively short, so that large differences from climatology are possible.

Measure of skill

The standard method for verifying the skill of the Markov (or any other binary) predictions is by computing the half-Brier score (Brier 1950). This score is defined as

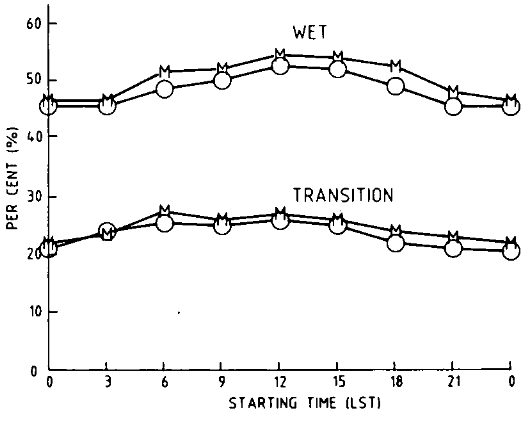
$$Br = (1/N) \sum_{i=1}^{N} (\delta_i - Pr_i)^2,$$

where δ_i is the observed probability for the ith day ($\delta_i = 1$ if rain occurred within the forecast period; otherwise $\delta_i = 0$), Pr_i the forecast probability for the ith day, and N the total number of days. The half-Brier score can be interpreted as the mean square error of the probabilistic forecasts.

Results

Markov models have been developed for the eight capital cities, Adelaide, Brisbane, Canberra, Darwin, Hobart, Melbourne, Perth and Sydney. However, as this article is intended to be brief we will present results only for a tropical station, Darwin, and a mid-latitude station, Melbourne. These stations illustrate in many ways the extremes in rainfall forecasting situations. Darwin's maximum rainfall occurs in summer associated with tropical convection and monsoonal circu-

Fig. 1 **Reliability diagram for Darwin for the climatological data set (1960–1988). Observed relative frequency (O) and predicted Markov probability (M) are shown as a function of time of day and season (wet and transition) for 12-hour forecast periods.**



lation; Melbourne's rainfall maximum occurs in winter associated with frontal passages. In addition, Darwin has a marked diurnal (bi-modal) rainfall distribution in the three-hourly data in the wet and transition seasons. On the other hand, Melbourne has no significant diurnal variation in any season as is shown below.

Climatological check of the new Markov model

In Figs 1 and 2 we show the climatological frequency of observed rainfall of 1 mm or more recorded at the stations, Darwin and Melbourne, as a function of time of day and season. For

Fig. 2 **Reliability diagram for Melbourne for the climatological data set (1960–1988). Observed relative frequency (O) and predicted Markov probability (M) are shown as a function of time of day and season: (a) summer, (b) autumn, (c) winter and (d) spring for 12-hour forecast periods.**

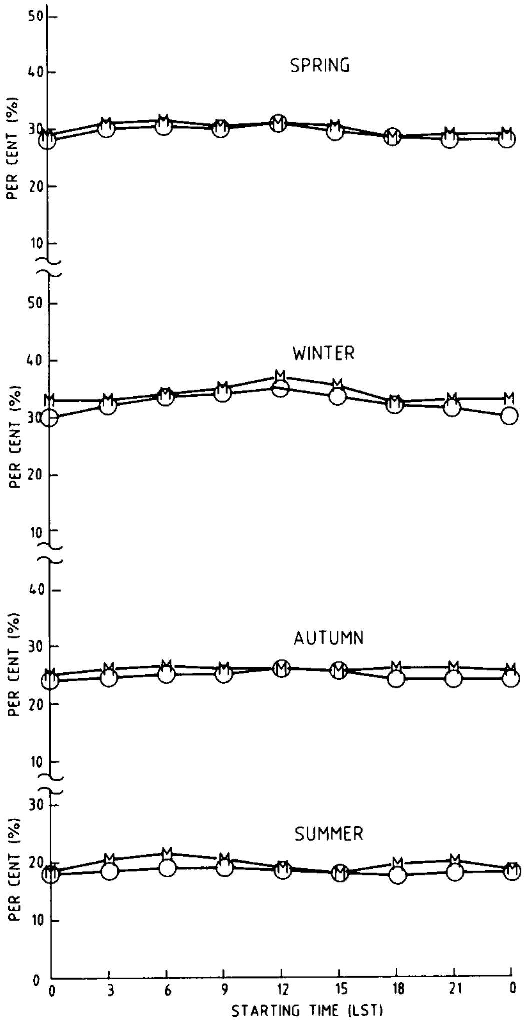


Fig. 3 **Same as Fig. 1, except for the verification data set (1988–1989).**

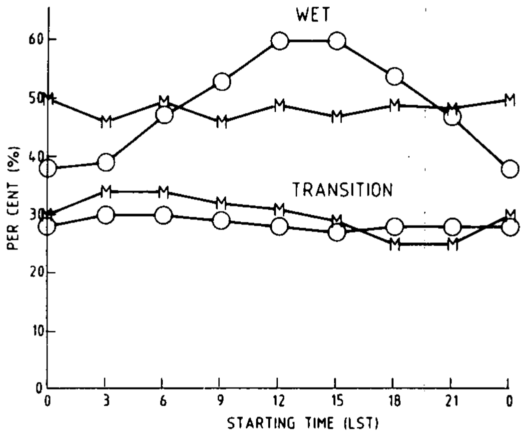
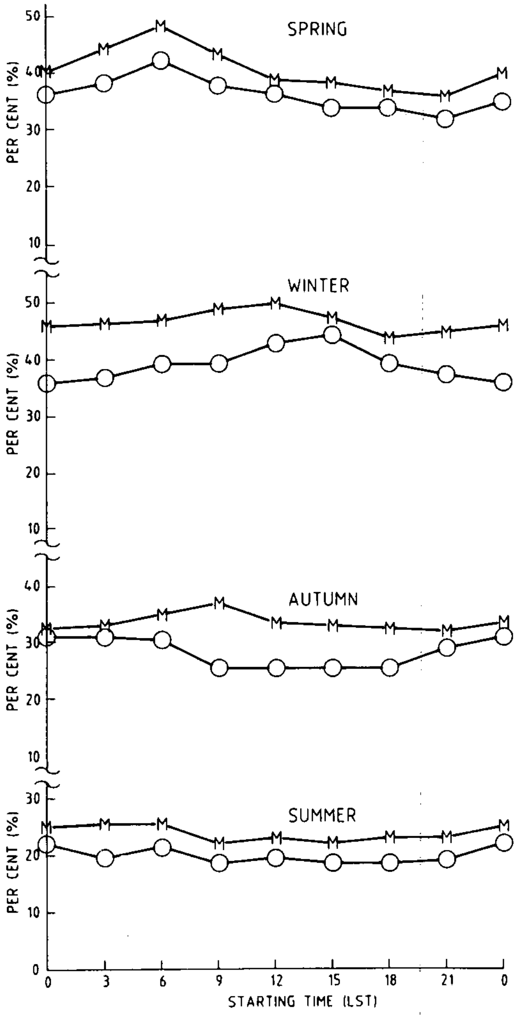


Fig. 4 **Same as Fig. 2, except for the verification data set (1988–1989).**



Darwin, which has a pronounced dry season spanning almost half of the year, we present the results only for the transition season (October to December) and the wet season (January to March). The climatological data cover the same period used to define the Markov coefficients.

The Darwin data (shown in Fig. 1) show a slight peak in the distributions at 1200 hours for both the wet and transition seasons. The strong bimodal distribution found in the three-hourly data (Fraedrich and Leslie 1989) is smoothed out when the data are averaged, using 12-hour periods with different starting times. The Melbourne data (Fig. 2) show little dependence on time of day, except in winter when there is a slight peak at 1200 hours. The frequency has a maximum in winter and a minimum in summer.

In Figs 1 and 2 we also show the predicted probabilities based on the Markov model. The agreement with the observations is shown for both stations, and clearly indicates that the model fits the climatological data set very well. It is noted that there is a tendency for the predicted values to be systematically slightly higher than the observations, see the Darwin wet season results for example. For a purely linear model the squares of the deviations between the model and observations would sum to zero. However, the statistical model employed here possesses some nonlinearity owing to the inclusion of the sine and cosine functions of time.

Operational verification

In Figs 3 and 4 we present the frequency distributions for the verification data set which covers the period during which the Markov system has been operational. The 1988–1989 data set for Darwin (Fig. 3) shows a strong peak in the wet season between 1200–1500 hours with higher rainfall frequency than usual. The predicted Markov probability curve does not exhibit this peak, but is flatter than the observed curve. The correspondence between the predicted and observed curves for the transition season is closer. The differences shown in the wet season reflect the large variations associated with a relatively short (two-season) sampling period at Darwin, the sequence of weather events and the values of the covariates in this short data set. The Melbourne data (Fig. 4) show systematic overprediction of the observed frequency using the Markov model. As for Darwin, this validation data set was not 'average'.

Table 1. Half-Brier scores for 12-hour forecast periods with various starting times (LST) for the Markov model, persistence and climatology. Lower scores indicate more skill. Climatological data set (1960–1988). % P and % C denote improvement over persistence and climatology, respectively.

Station	*Season*	*Model type*	*0000*	*0300*	*0600*	*0900*	*1200*	*1500*	*1800*	*2100*
Darwin	Wet	Markov	0.179	0.187	0.195	0.205	0.203	0.203	0.196	0.189
		Persist	0.317	0.318	0.343	0.392	0.415	0.393	0.354	0.350
		Climat	0.229	0.229	0.232	0.237	0.234	0.234	0.236	0.233
		% P	43.5	41.2	43.1	47.7	51.1	48.3	44.6	46.0
		% C	21.8	18.3	15.9	13.5	13.2	13.2	16.9	18.9
	Transition	Markov	0.138	0.148	0.155	0.159	0.163	0.153	0.152	0.148
		Persist	0.189	0.213	0.234	0.242	0.243	0.232	0.230	0.203
		Climat	0.157	0.173	0.176	0.175	0.181	0.173	0.166	0.163
		% P	27.0	30.5	33.8	34.3	32.9	34.1	33.9	27.1
		% C	12.1	14.5	11.9	9.1	9.9	11.6	8.4	9.2
Melbourne	Summer	Markov	0.102	0.104	0.110	0.106	0.104	0.098	0.098	0.100
		Persist	0.146	0.143	0.159	0.162	0.160	0.143	0.150	0.143
		Climat	0.138	0.145	0.148	0.146	0.142	0.139	0.136	0.140
		% P	30.1	27.3	30.8	34.6	35.0	31.5	34.7	30.1
		% C	26.1	28.3	25.7	27.4	26.8	29.5	27.9	28.6
	Autumn	Markov	0.126	0.124	0.129	0.123	0.122	0.125	0.131	0.137
		Persist	0.193	0.204	0.204	0.211	0.220	0.205	0.207	0.215
		Climat	0.177	0.181	0.182	0.179	0.179	0.176	0.178	0.179
		% P	34.7	39.2	36.8	41.7	44.5	39.0	36.7	36.3
		% C	28.8	31.5	29.1	31.3	31.8	29.0	26.4	23.5
	Winter	Markov	0.168	0.159	0.165	0.165	0.157	0.157	0.159	0.166
		Persist	0.249	0.248	0.276	0.292	0.285	0.265	0.271	0.250
		Climat	0.201	0.207	0.212	0.215	0.214	0.208	0.204	0.206
		% P	32.5	35.9	40.2	43.5	44.9	40.8	41.3	33.6
		% C	16.4	23.2	22.2	23.3	26.6	24.5	22.1	19.3
	Spring	Markov	0.139	0.137	0.145	0.140	0.136	0.134	0.142	0.143
		Persist	0.228	0.229	0.252	0.255	0.250	0.235	0.237	0.230
		Climat	0.194	0.201	0.205	0.203	0.199	0.194	0.190	0.195
		% P	39.0	40.2	42.5	45.1	45.6	43.8	40.1	37.8
		% C	28.4	31.8	29.3	31.0	31.7	30.9	25.3	26.7

Each season in this data set, except for summer, also has greater than the normal rainfall; hence differences between the Markov predictions and observations are expected. As the sampling periods of the validation data sets increase, the differences between the average Markov probabilities and the observed relative frequencies should decrease. In the limit, as the length of the sampling period becomes very large, the Markov results should approach the climatological relative frequencies.

The seasonal half-Brier scores for the 1960–1988 and 1988–1989 data sets are given in Tables 1 and 2, respectively. Lower scores indicate more skill. The scores can range from zero (perfect score) to unity. For comparison we have included the half-Brier scores based on persistence and climatology. It is seen that the Markov chain predictions are better than persistence and climatology for each starting time and each station. The high level of skill indicated by the per cent improvement over persistence and climatology (% P and % C, respectively, in the tables) agrees with results found previously by Hess et al. (1989) and gives confidence in the method.

The significance of the covariates was assessed using the standard Student t-test. The results are presented in Table 3. The most important variables for Melbourne are P, ΔP, $T\text{-}T_w$, U, ΔLCLD, and MCLD. For Darwin they are P, ΔP, $T\text{-}T_w$, $[\Delta(U^2 + V^2)]^{1/2}$, LCLD and MCLD. Some of the other covariates also are significant, but less so.

Discussion and conclusions

In this paper we have extended the Markov technique developed by Hess et al. (1989) by allowing a new forecasting period to begin every three hours around the clock. This makes use of all of the available surface data and permits forecasts to be made with maximum timeliness. It also permits prediction of diurnal variations of rainfall.

Further extensions to the Markov system are planned for the near future. Up to now, the Markov method has relied on single-station surface observations. Stratifying the data by synoptic weather pattern, using vertical and horizontal gradients and the Southern Oscillation Index, and combining the Markov prediction with an independent forecast, e.g. a NWP forecast, should provide further improvements in skill (see for

Table 2. **Half-Brier scores for 12-hour forecast periods with various starting times (LST) for the Markov model, persistence and climatology. Lower scores indicate more skill. Short-term data set (1988–1989). % P and % C denote improvement over persistence and climatology, respectively.**

Station	*Season*	*Model type*	*0000*	*0300*	*0600*	*0900*	*1200*	*1500*	*1800*	*2100*
Darwin	Wet	Markov	0.209	0.191	0.206	0.243	0.230	0.230	0.226	0.212
		Persist	0.296	0.328	0.367	0.448	0.547	0.503	0.417	0.356
		Climat	0.247	0.251	0.258	0.255	0.251	0.248	0.254	0.253
		% P	29.4	41.8	43.9	45.8	58.0	54.3	45.8	40.4
		% C	15.4	23.9	20.2	4.7	8.4	7.3	11.0	16.2
	Transition	Markov	0.135	0.144	0.166	0.151	0.173	0.152	0.158	0.141
		Persist	0.207	0.219	0.238	0.253	0.271	0.244	0.219	0.217
		Climat	0.172	0.184	0.187	0.194	0.196	0.199	0.209	0.213
		% P	34.8	34.2	30.3	40.3	36.2	37.7	27.9	35.0
		% C	21.5	21.7	11.2	22.2	11.7	23.6	24.4	33.8
Melbourne	Summer	Markov	0.096	0.110	0.112	0.115	0.108	0.087	0.085	0.099
		Persist	0.133	0.126	0.148	0.157	0.155	0.148	0.122	0.137
		Climat	0.130	0.127	0.135	0.122	0.128	0.137	0.137	0.142
		% P	27.8	12.7	24.3	26.8	30.3	41.2	30.3	27.7
		% C	26.2	13.4	17.0	5.7	15.6	36.5	38.0	30.3
	Autumn	Markov	0.165	0.160	0.151	0.155	0.131	0.141	0.153	0.188
		Persist	0.236	0.234	0.255	0.234	0.197	0.225	0.213	0.266
		Climat	0.214	0.211	0.208	0.186	0.186	0.188	0.191	0.203
		% P	30.1	31.6	40.8	33.8	33.5	37.3	28.2	29.3
		% C	22.9	24.2	27.4	16.7	29.6	25.0	19.9	7.4
	Winter	Markov	0.201	0.190	0.180	0.154	0.158	0.171	0.184	0.217
		Persist	0.299	0.321	0.299	0.321	0.337	0.315	0.359	0.326
		Climat	0.233	0.235	0.244	0.242	0.250	0.257	0.244	0.238
		% P	32.8	40.8	39.8	52.0	53.1	45.7	48.7	33.4
		% C	13.7	19.1	26.2	36.4	36.8	33.5	24.6	8.8
	Spring	Markov	0.151	0.158	0.153	0.142	0.138	0.131	0.139	0.158
		Persist	0.244	0.318	0.302	0.317	0.266	0.258	0.247	0.253
		Climat	0.233	0.237	0.229	0.228	0.233	0.216	0.198	0.199
		% P	38.1	50.3	49.3	55.2	48.1	49.2	43.7	37.5
		% C	32.3	33.3	33.2	37.7	40.8	39.4	29.8	20.6

Table 3. **Diurnal variation of Student t-scores of the covariates for predicting rain for a 12-hour forecast period. Absolute value of t scores $\geqslant \sim 2$ indicate significant covariates at the 5% level.**

Station	*Prev. state*	P	ΔP	T	ΔT	$T-T_w$	$\Delta(T-T_w)$	U	V	$\Delta(U^2+V^2)^{1/2}$	*LCLD*	*ΔLCLD*	*MCLD*	*ΔMCLD*
Melbourne														
0000	3.90	−10.46	−7.03	0.49	3.13	−10.40	−6.41	−5.58	−5.02	3.74	−0.56	5.10	6.67	3.18
0900	3.76	−17.97	−4.29	0.27	1.94	−10.47	−4.61	−8.21	1.22	2.51	−1.25	5.33	7.91	3.69
1800	0.77	−11.20	−9.31	2.11	1.54	−12.81	−2.52	−5.94	−2.37	0.37	0.20	8.27	7.65	3.29
Darwin														
0000	3.19	−12.57	8.44	−1.52	−4.03	−7.59	2.50	−3.60	2.87	3.67	7.61	4.26	4.17	2.05
0900	2.46	−10.50	6.73	−0.32	−1.85	−6.34	0.60	−0.10	3.04	1.59	6.15	5.20	4.07	0.41
1800	−2.62	−10.80	6.20	−2.51	1.66	−8.07	0.72	1.03	2.94	5.68	9.17	2.83	3.89	−0.97

example, Woodcock and Southern (1983); Fraedrich and Leslie (1987, 1988)).

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