Recent ideas on roughness, inhomogeneities and mixing in the atmospheric boundary layer*

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The Monin-Obukhov similarity theory forms the basis of the methodology used to calculate surface fluxes of momentum, and sensible and latent heat in numerical models of the atmosphere. However, observed surface properties are often not homogeneous over the subgrid length scales used in models as assumed in the theory. Conditions necessary for homogeneity are discussed. Several simple models are then presented to generalise the concept of roughness to apply in non-homogeneous conditions because of variations in terrain, vegetation, or the presence of buildings or other obstacles. These models are preliminary and indicate the need for a comprehensive theory for heterogeneous flow. Recent results from the HAPEX-MOBILHY, FIFE, BLX83 and other experiments are used to illustrate spatial variability and the problem of averaging over regional scales. Turbulent mixing within the atmospheric boundary layer (ABL) is another important element of ABL parametrisation. Several different approaches including first-order closure, higher-order (second-moment) closure, transient and similarity theory parametrisations are discussed and their performances are compared in homogeneous and non-homogeneous situations.

Introduction

Information about the properties of the underlying surface is transferred to the general atmospheric flow through the region known as the atmospheric boundary layer (ABL). The physics of the transfer involves small-scale turbulence processes and must be parametrised in numerical models of the atmosphere. A methodology to relate surface fluxes of momentum, and sensible and latent heat to the mean gradients of wind speed, potential temperature and specific humidity, including their variation with thermal stability, was developed by Monin and Obukhov (1954) in their landmark paper. This methodology continues to serve as the basis for calculation of these fluxes in numerical models. Over the past forty years most of the field experiments (for example, the Great Plains experiment; the Kerang, Hay and Gurley experiments; the Kansas experiment; the Wagara experiment; the Minnesota experiment; and the Koorin experiment; see Stull (1988, pp. 418-419) and Garratt and Hicks (1990) for partial lists of ABL field experiments) were conducted over relatively flat, homogenous terrain in compliance with the Monin-Obukhov theory.

However, the observed properties of most land surfaces are not homogeneous on the subgrid scales used in models and as assumed in the theory. In this paper we will present some recent ideas for dealing with inhomogeneities and for performing areal averages. We will also examine several different approaches to parametrising ABL mixing and compare their performances. We begin by considering the case of homogeneous roughness.

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Homogeneous roughness

We can subdivide ABL flows, following Wieringa (1993), into four types: smooth (where the obstacles are too small to produce appreciable wakes); semi-smooth (where the obstacles are isolated and are spaced sufficiently far apart so that a region exists where the wake is almost absent); wake-interference (where the wake of one obstacle impinges on the wake of another obstacle); and skimming (where there is such a high density of obstacles that the flow between the obstacles has a separate regime from the flow above). These categories are illustrated in Fig. 1 and depend on the ratio of the distance between obstacles, $x$, to the height of the obstacles, $H$. Smooth and skimming flows are predominantly homogeneous and semi-smooth flow may be homogeneous; wake-flow is not homogeneous.

In Fig. 2 we show a schematic diagram which illustrates the effect of a discontinuity in surface roughness. Above the new (homogeneous) roughness the air-flow adapts to, and comes to equilibrium with, the new surface condition; the depth of the adapted layer gradually increases with fetch $F$. Above the adapted layer and up to the top of the internal boundary layer the air only partially adjusts to the new surface and the flow is not in equilibrium with the surface. The height of the adapted layer is approximately 0.1 the height of the internal boundary layer and the minimum fetch requirement can be estimated, for example, by Miyake's (1965) equation based on diffusion theory:

$$ F \approx 2z_0 \left[ \frac{10z}{z_0} \left( \ln \frac{10z}{z_0} - 1 \right) + 1 \right] \quad \ldots 1 $$

where $z_0$ is the downstream roughness length. This form of the fetch equation gives better agreement with data (Jackson 1976; Walmsley 1989) than the earlier power law formation suggested by Elliott (1958). For further discussion see the reviews by Garratt (1990), Wieringa (1993), and Kaimal and Finnigan (1994, pp. 112–115).

There is also a minimum height requirement, $z_{\text{min}}$. This is the lowest height above the canopy where horizontal location is unimportant for the flow structure. It measures the height of the transition or roughness sublayer that exists immediately above the canopy where irregularities in the canopy disturb the flow. Based on their analysis of a number of wind tunnel studies of flow over regular arrays of cylinders, Raupach et al. (1980) found:

$$ z_{\text{min}} \approx H + 1.5D \quad \ldots 2 $$

where $H$ is the height of the vegetation/obstacles and $D$ the distance between obstacles. Wieringa (1993) examined atmospheric data for skimming flow over various types of forests and found the same relationship held when the results were averaged over the six experimental studies. Other investigators, e.g. Garratt (1980), Jacobs and van Boxel (1988), and Parlange and Brutsaert (1989), relate $z_{\text{min}}$ to the zero-plane displacement height, $d$, but this method has the disadvantage of requiring iteration — the value of $d$ cannot be well determined until the value of $z_{\text{min}}$ is known.

To apply Eqn 2, estimates of the inter-obstacle distance are needed. Wieringa (1993), who based his analysis on a number of field studies, suggests the following relationships for forests:
\[ D = 0.09H + 1.3 \pm 0.5m \quad \ldots 3 \]

or

\[ D \approx 0.15H \quad \ldots 4 \]

Equation 2 applies to skimming flow over obstacles where the inter-obstacle distance is well defined. However, for high grasses, grains or crops arranged in rows, \( D \) is not well defined. For these cases Wieringa (1993) recommends neglecting \( D \) and using:

\[ z_{\text{min}} \approx 1.5H \quad \ldots 5 \]

A third length scale, the zero-plane displacement length, \( d \), is needed if the flow close to the canopy is to be modelled. Although \( d \) depends on the size and geometry of the obstacles, and their spacing and distribution, its value depends mainly on H. Garratt (1992, p. 290) summarises the results from a number of field studies. Typical average values for crops (Munro and Oke 1973) and forests (Thom 1971) are:

\[ d \approx 0.64H \text{ for crops} \quad \ldots 6 \]

\[ d \approx 0.75H \text{ for forests} \quad \ldots 7 \]

Deviations from the logarithmic profile are observed as \( z/z_0 \) becomes small. The lower limit of the applicability of the inertial sublayer (logarithmic profile) is given as 10\( z_0 \) to 150\( z_0 \) (Garratt 1992, p. 60) and 20\( z_0 \) (Wieringa 1993) or if the zero-plane displacement length is not negligible, the latter limit becomes:

\[ z_{\text{min}} < z < d + 20z_0 \quad \ldots 8 \]

Wieringa (1993) carefully examined the results from hundreds of experiments and found fifty experiments that met the above criteria for minimum height of observation and minimum fetch (maximum height of observation) and where the profiles were adequately resolved. His results are shown in Table 1 and compared with several standard references. Except for the summaries of Oke (1978) and Smedman-Högström and Högström (1978), all of the other classifications he examined give \( z_0 \) values that are too large for smooth terrain (probably because the fetches were too short). For semi-smooth flow and skimming flow the classifications of Smedman-Högström and Högström (1978), Cook (1985), Troen et al. (1987) and others underestimate the roughness by a factor of two.

### Table 1. Roughness based on classifications \( z_{0(\text{class})} \) and homogeneous values \( z_0 \). The symbol \( \approx \) indicates that the terrain correspondence is only approximate (after Wieringa (1993)).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Flat snow</td>
<td>0.0003</td>
<td>0.0002</td>
<td>–</td>
<td>0.0001</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>Flat land</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.006</td>
<td>0.005</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Fallow ground</td>
<td>0.002</td>
<td>0.001–0.01</td>
<td>0.015</td>
<td>–</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>Smooth, ( z_{0(\text{class})}/z_0 )</td>
<td>1.1</td>
<td>1.1</td>
<td>(1)</td>
<td>8</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>Short grass</td>
<td>0.013</td>
<td>0.003–0.01</td>
<td>0.015</td>
<td>0.008</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Long grass</td>
<td>0.034</td>
<td>0.04–0.10</td>
<td>0.04</td>
<td>0.02–0.05</td>
<td>0.01</td>
<td>–</td>
</tr>
<tr>
<td>Cropped farmland</td>
<td>( \approx 0.1 )</td>
<td>0.04–0.20</td>
<td>0.11</td>
<td>0.05–0.10</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Semi-smooth, ( z_{0(\text{class})}/z_0 )</td>
<td>1.1</td>
<td>1.1</td>
<td>0.7</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Pine forest</td>
<td>1.2</td>
<td>1.0–6.0</td>
<td>0.8</td>
<td>0.4</td>
<td>0.3</td>
<td>0.30</td>
</tr>
<tr>
<td>Low suburb</td>
<td>0.6</td>
<td>–</td>
<td>( \approx 1.3 )</td>
<td>0.4–0.6</td>
<td>0.3</td>
<td>0.40</td>
</tr>
<tr>
<td>Regular town</td>
<td>1.1</td>
<td>–</td>
<td>( \approx 1.3 )</td>
<td>0.6–0.9</td>
<td>0.8</td>
<td>–</td>
</tr>
<tr>
<td>Skimming, ( z_{0(\text{class})}/z_0 )</td>
<td>(2)</td>
<td>1.3</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
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</tr>
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**Heterogeneous roughness**

Over flat, homogeneous surfaces we have the Monin-Obukhov similarity theory which serves as the basis of our understanding of flux-gradient relationships. What do we do when the surface is not flat or not homogeneous? As a first guess we try Monin-Obukhov theory. We limit the discussion to horizontal length-scales up to about 10 km. Parlange and Brutsaert (1989) use radiosonde data to derive neutral wind profiles over a heterogeneous area consisting of approximately 65 per cent pine forest with breaks for agricultural lands, logging operations and small villages. The data (not shown) indicate the familiar logarithmic profile found over homogeneous terrain. Grant (1991) also makes measurements over heterogeneous terrain (consisting of wooded areas, built-up urban areas and agricultural areas) using the Cardington tethered balloon system. He finds good agreement between his nondimensional wind shear measurements between 40 to 70 m, and 70 to 130 m with the Dyer-Businger (Dyer 1974) and Carl et al. (1973) formulations. These results indicate that at some distance above the roughness elements the general properties of the flow are similar to those over a homogeneous surface (there are some minor differences in the spec-
tra). This minimum height is called the blending height and will be discussed later.

How can $z_0$ be estimated? There is much literature on estimating roughness (see Oke (1978); Pielke (1984); Wieringa (1993), for example). The simplest idea is due to Paeschke (1938):

$$z_0 = CH$$

where $C$ is a constant. This relation is still used with $C$ often chosen to be about 1/7, but this relationship is not reliable (Garratt 1977); it turns out that $z_0$ depends on other variables in addition to $H$.

Lettau (1969) suggested that the effect of form drag could be estimated by considering the geometry of major roughness elements (also see Businger (1974); Segner (1974) and Kondo and Yamazawa (1986)):

$$z_0 \approx CHS/A \approx CH\lambda$$

where $C$ is a constant, $S$ the silhouette area and $A$ the surface area which on the average contains one surface roughness element, and $\lambda$ the ratio of the areas. However, this formula is not complete because it does not account for the drag due to small-scale surface features. Extensions of these ideas will be discussed below.

Flow over topography

At some height above the tops of ridges, which were approximately two-dimensional and 250 m in amplitude, Grant and Mason (1990) also found evidence of logarithmic wind profiles in near-neutral conditions. They developed a simple model for the effective roughness valid at large slopes by expanding Lettau’s idea. They partitioned the total drag as the sum of the form drag and the surface shear stress (skin friction). This gave:

$$\ln^2(H/2z_{0\text{eff}}) = \frac{k^2}{0.5C_R\lambda + \frac{k^2}{\ln^2(H/2z_0)}}$$

While this assumption is valid for flow over isolated obstacles, Mason found that the equation gave good agreement with the predictions from a second-order closure model for flow over sinusoidal terrain when $C_R = 0.3$.

Another simple expression for the effective roughness length was derived by Taylor et al. (1989) based on studies using six different numerical models and the assumption that sufficiently far away from the topography the flow is independent of horizontal position. For neutrally stratified flow over small-amplitude, two-dimensional sinusoidal terrain (limited to maximum slopes $a \leq 0.38$, where $a$ is the amplitude and $\kappa$ the wave number) they found:

$$\ln(z_{\text{off}/z_0}) = 3.5(\alpha\kappa)^2 \ln(L/z_0)$$

where $L$ is the horizontal wavelength.

A more general expression for the effective roughness length for momentum, not restricted to large or small values of slope, was given by Wood and Mason (1993). In this case the square of the effective value of the friction velocity is parametrised by an undisturbed (stream) friction velocity plus a drag coefficient term evaluated at the pressure scale-height, $z_m$, where $z_m = h_m$ for $H \leq h_m$, $z_m = H$ for $H > h_m$ and $h_m \sim (1/4) \ln^{-1/2}(h_m/z_0)$. Substituting the neutral logarithmic profile relation for the square of the effective friction velocity gives:

$$\frac{1}{\ln^2\left(\frac{z_m}{z_{\text{off}}}ight)} = \frac{C_R}{k^2} + \frac{1}{\ln^2\left(\frac{z_m}{z_0}\right)}$$

Figure 3 shows a comparison of predictions of the effective roughness length from these simple expressions with experimental and numerical model results. The agreement of the theory (in the range of its validity) with the observations and model results is generally good, although a conclusive comparison with observations would require measurement of the local roughness lengths, $z_0$, which so far has not been done.

Flow over obstacles

Scaling arguments for sheltering areas and volumes behind roughness elements and the assumption of random superposition of roughness elements allowed Raupach (1992) to develop expressions for the total drag, partitioning of the drag, the effective roughness length and the displacement height (also see Emeis (1990), and Hanssen-Bauer and Gjessing (1988)).

Raupach finds that the square root of the total drag coefficient is given by:

$$U_H/u_* = (C_S + \lambda C_R)^{-1/2} \exp(c\lambda U_H/2u_*)$$

where $C_S$ is the drag coefficient for the substrate surface, $C_R$ the drag for an isolated, surface-mounted roughness element and $c$ is a constant. Although the assumptions of the theory are valid only for small $\lambda$, the total drag seems to saturate at high values of $\lambda$ and this empirically fixes the value of $c$ ($c \approx 0.37$).

The total stress, $\tau$, is partitioned into form drag, $\tau_R$, and the surface shear stress (skin friction), $\tau_S$, by the relations:

$$\frac{\tau_S}{\tau} = \frac{1}{1 + \beta\lambda}, \quad \frac{\tau_R}{\tau} = \frac{\beta\lambda}{1 + \beta\lambda}$$
Fig. 3  Comparison of theoretical predictions of effective roughness length for momentum with observations and model results for two-dimensional hills (after Wood and Mason, 1993). A is the silhouette area, $S_d$ the domain area and h the amplitude of the hills. (a) The dotted line indicates the linear predictions (Eqn 12), the dashed line shows the bluff-body predictions with $C_D = 0.3$ (Eqn 11) and the solid line gives the general relation (Eqn 13). The asterisks indicate the results of the numerical model. (b) The solid line indicates the predictions of (13) with a local roughness length of $z_0 = 0.3$ m and topographical wave length of 1000 m. The dashed line shows the predictions of (13) with $z_0 = 0.5$ m and the wave length = 1000 m. The squares show the field observations summarised by Grant and Mason (1990) and the triangles the data of Hopwood (1991) (also see Hignett and Hopwood, 1994).

where $\beta = C_R/C_S$. When compared with the theories of Wooding et al. (1973) and Arya (1975), and the wind tunnel measurements of Marshall (1971), Raupach's theory gives an excellent fit to the data over the whole range $\lambda$, whereas the other theories are valid only over a restricted range of $\lambda$.

The expression for the roughness length is:

$$z_0 = (H - d) \exp(\Psi_H) \exp(-kU_H/u) \ldots 16$$

where $\exp(\Psi_H)$ is the correction function for flow within the transition (roughness) sublayer; $\Psi_H = \ln(c_w) + 1 - c_w^{-1}$, where $c_w$ is a constant. If $c_w = 1.5$, then $\Psi_H = 0.74$.

The zero-plane displacement height, d, can be written as:

$$d = h \left( \frac{\beta \lambda}{1 + \beta \lambda} \right) \left( 1 - c_d \frac{b}{H\lambda} \right)^{-1} \left( \frac{U_H}{u} \right)^{-1} \ldots 17$$

where $b$ is the width of the roughness elements and $c_d$ is a constant ($c_d \approx 0.6$).

Figure 4 shows a comparison of the theoretical predictions for the square root of the total drag coefficient, the zero-plane displacement height and the effective roughness length with wind tunnel data and measurements over vegetation canopies. The evaluation of the empirical constants is tightly constrained and the agreement is good.

As discussed above, several simple parametrisations for flow over obstacles and simple (idealised) terrain exist, and they give reasonable agreement with the available data in neutral conditions. Most of the atmospheric data over terrain have so far been limited to the region of $\lambda \ll 0.1$. For peak-to-valley heights of about 250 m the observed $z_0$ is about 10 m. For larger $\lambda$ and $H$ values, the simple theories suggest that $z_0$ of tens to hundreds of metres may apply! However $z_0 \gg z_0$ must still be within the surface (constant flux) layer to use Monin-Obukhov theory and this may be violated if $z_0$ becomes too large (i.e. larger than several metres); also, the method of obtaining geostrophic drag coefficients from Rossby number similarity theory (e.g. the assumptions of an inner logarithmic layer to match with the outer layer and that the quantity $U_p/f$ for $z_0 \to \infty$, where $U_p$ is the geostrophic wind and $f$ the Coriolis parameter) may be invalid. At present the observational and modelling (physical and numerical) foundation for understanding these parametrisations is still very incomplete. We do not know how robust the parametrisations are or if they can be applied with confidence in all stab-
Areal averages

In numerical models of atmospheric flow, areal averages of the drag coefficients over the horizontal grid are required in order to parametrise the surface fluxes. How can these areal averages be calculated?

The local drag coefficient for neutral conditions is defined by:

$$C_D = \left( \frac{k}{\ln \frac{z}{z_0}} \right)^2 \quad \ldots 18$$

A simple approach would be to replace the local value of $z_0$ with the areally averaged roughness length, $\langle z_0 \rangle$, in this definition. This turns out to give unacceptably large errors (systematically too high by up to 30 per cent (Claussen 1989; Mahrt 1987)).

Another approach would be to replace $\ln z_0$ by its areally averaged value $\langle \ln z_0 \rangle$. This also gives unacceptably large errors (systematically too low by up to 20 per cent).

An approach that does work is to define a blending height, $l_b$, where the flow is in approximate equilibrium and independent of the horizontal position, i.e. the flow is horizontally homogeneous over the grid (Wieringa 1986; Mason 1988; Claussen 1990). This concept is similar to the idea of $z_{\text{min}}$ for skimming flow discussed earlier.

An expression for $l_b$ can be obtained from scaling arguments:

$$\frac{l_b}{L} \left( \ln \frac{l_b}{z_{0e}} \right)^n = Ck^2 \quad \ldots 19$$

where $L$ is the horizontal length scale, $z_{0e}$ the aggregated roughness length for the area, $n$ a constant ($n \approx 1 - 2$) and $C$ another constant. The value of $n$ depends on how the scaling argument is constructed. Jackson and Hunt (1975), for example, use a formal perturbation expansion of the momentum equations and Claussen (1988, 1990, 1991) bases his argument on a representative eddy viscosity and the advective time-scale; both obtain $n = 1$. Taylor et al. (1987), Mason (1988) and Wood and Mason (1991) balance the stress divergence and the horizontal advection to obtain $n = 2$. Wood and Mason (1991) suggest that in neutral conditions the choice of blending height definition with $n = 2$ means that above this height the areally averaged mean flow profile should be logarithmic, while the choice $n = 1$ implies that the local mean flow profile should be logarithmic. Wieringa (1976) simplifies the estimate, based on laboratory studies. He suggests that $l_b = 2H$ and disregards the scale of horizontal variations.

In Fig. 5 the theoretical predictions for $n$ are compared with observations and predictions.
Fig. 5 Comparison of theoretical predictions for the exponent $n = 1$ (line 1) and $n = 2$ (line 2) with numerical model predictions and observations (after Beljaars and Taylor, 1989). The data are from the field experiments summarised by Taylor et al. (1987) and the wind tunnel studies summarised by Gong (1987) and Gong and Ibbetson (1989).

The theory assumes that $l_i$ is located within the surface layer. Although there was some controversy initially, there now seems to be general agreement (e.g. Mason 1988; Taylor et al. 1989; Claassen 1990; Grant 1991) that the effective roughness should be defined to give the correct surface stress (rather than the correct average velocity profile), i.e.

$$\frac{1}{\left( \ln \frac{z}{z_0c} \right)^2} = \sum \frac{f_i}{\left( \ln \frac{z}{z_{0i}} \right)^2} \ldots 20$$

where $f_i$ is the fraction of the area covered by roughness $z_{0i}$ and $z_{0c}$ is the effective roughness based on skin friction alone (the aggregated roughness length).

In areas of complex terrain and/or marked differences in roughness, the overall effective roughness length $z_{0ef}$ must include the form drag contribution as well as the skin friction. Claussen and Klaassen (1992) have developed a simple model of forest and agricultural land. The model calculates the form drag which occurs due to the forest edges. Figure 6 shows the variation of the roughness length based on the skin drag of the two surfaces and the effective roughness length including form drag as the fractional cover of forest changes. They find that the value of the effective roughness can exceed the local value of roughness for the forest because of the effect of form drag.

The regional average momentum flux can be written as (Claussen and Klaassen 1992):

$$\tau_0 = \rho C_D U_1^2 \text{ where } C_D = \left( \frac{k}{\ln \frac{z}{z_{0f}}} \right)^2 \ldots 21$$

where the subscript 1 indicates the lowest model level and $z_{0ef}$ incorporates the effect of form drag and accounts for the blending height. Corrections for thermal stability can be made based on the Dyer-Businger (Dyer 1974) or Louis (1979) formulations (see Claussen (1991) and Claussen and Klaassen (1992)).

Klaassen (1992) has studied the latent heat flux from a two-dimensional region composed of strips of 0.5 km agricultural land and 0.5 km forest. Because the fetch is limited the latent heat flux deviates from the values obtained by assuming that the flow is in equilibrium with the
surfaces. These deviations are mainly due to deviations in friction velocity from the equilibrium values. His multi-level model gives better agreement than his single-level model when compared with the friction velocity data of Gash (1986) and hence the larger deviations of latent heat flux from the equilibrium given by the multi-level model are probably more realistic than those of the single-level model. The multi-level model shows that the latent heat flux is systematically overestimated by the equilibrium theory, particularly over the rougher forest surface.

The regionally averaged sensible and latent heat fluxes can be calculated by determining the fluxes for each land use within the grid area and accounting for the blending height (Claussen (1991); for a different approach see Wood and Mason (1991)). Empirical evidence indicates that these fluxes are relatively unaffected by form drag and hence can be obtained by using a weighted sum of the fluxes for the different surfaces. For example Claussen (1991) suggests the latent heat can be expressed as:

\[
\frac{E_Q}{\rho} = \sum_i \frac{C_{Qi} U_i (Q_i - Q_0)}{Z_i} \quad \ldots 22
\]

where the subscript 1 indicates the lowest model level where the (neutral) bulk transfer coefficient is given by:

\[
C_{Qi} = \frac{k^2}{(\ln \frac{Z_i}{Z_{Q_i}})(\ln \frac{Z_i}{Z_{Q_e}})} \quad \ldots 23
\]

Corrections to the bulk transfer coefficient for thermal stability can be incorporated as described above. A similar equation to Eqn 22 applies for the sensible heat flux.

The commonly accepted ratio of the local roughness length for heat to that for momentum is \( z_{0m}/z_{0f} = 0.1 \) for porous or fibrous roughness elements (for bluff-bodied elements the ratio depends on the roughness Reynolds number and the Prandtl or Schmidt number) (Garratt 1992, pp. 89–93). There is still uncertainty about the value of the ratio of the regionally averaged effective roughness length for heat to that for momentum. For example Claussen (1991) and Wood and Mason (1991) in their model studies obtain values similar to the local ratio. Beljaars and Holtslag (1991) find \( z_{0f}/z_{0eff} = 0.0001 \) \( z_{0eff} \) based on observations at the Cabauw tower in the Netherlands and from aircraft radiometer measurements during the MESOGERS experiment in France. The Cabauw findings are based on infrared surface temperature measurements at two sites, employing two different instruments.

For near-neutral conditions, local equilibrium, and assuming sparsely distributed obstacles, Beljaars and Holtslag show that:

\[
\ln \frac{Z_{Q_e}}{Z_0} = \ln \frac{z_{0f}}{Z_0} \quad \ldots 24
\]

which for typical values of blending height and local roughnesses gives values of the ratio of effective roughness lengths supporting their observations. Figure 7 gives a schematic diagram to illustrate this. The sparse roughness elements cause a first-order discontinuity in the velocity profile (evidence of an internal boundary layer) so that the stress and friction velocity are not constant with height. This produces an effective roughness length for momentum that is larger than the local roughness length of the dominant cover.

The sensible and latent heat fluxes are assumed to be constant with height. However, because the scaling temperature, \( T_s \), and scaling humidity, \( Q_s \), depend on the friction velocity, the potential temperature and specific humidity profiles also exhibit a first-order discontinuity, giving much smaller effective roughness lengths than those of the dominant cover.

Beljaars and Viterbo (1994) find that using \( z_{0eff}/z_{Heff} = 10^3 \) leads to a better ECMWF model simulation of Cabauw winter evaporation than using a ratio of unity or 10. On the other hand Betts and Beljaars (1993) estimate \( z_{0eff}/z_{Heff} = 18 \) (with an error range from 12–26) for aircraft measurements during the FIFE experiment over hilly grassland. Using surface tower data they obtain \( z_{0eff}/z_{Heff} = 16 \) (with a range of 7–35). For flow over bare soil Kohsiek et al. (1993) find a ratio of about 40.

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**Fig. 7** Schematic diagram illustrating the effect of sparse obstacles on the effective roughness lengths for momentum and heat (after Beljaars and Holtslag, 1991). The subscript 1 indicates the local values.
In another study, Stull (1994) examines data for the convective boundary layer over pastureland in Oklahoma (the BLX83 experiment). His data show that the turbulent transport coefficient for heat is significantly smaller than the one for momentum. Beljaars (1994) reanalyses the data and estimates that \( z_{0,h}/z_{h,eff} = 10^4 \). Stull concludes that the difference in transport coefficients for heat and momentum is due to free convection dynamics, while Beljaars in his analysis considers only the high wind-speed regime (wind speeds \( > 5 \) m s\(^{-1}\)). The same qualitative behaviour is found for both free convection and the high wind-speed regime. The extremely large ratio of the effective roughness lengths found by Beljaars (much larger than found elsewhere) raises the question of possible experimental error in the determination of \( z_{h,eff} \).

Garratt et al. (1993) have questioned the large ratio of effective roughness lengths found by Duynerkerke (1992) where the surface temperature for Duynerkerke's Kew dataset was measured by a 'grass thermometer' and for his Cabauw dataset was derived from independent measurements of the net radiation, the incoming and outgoing short wave radiation, and the incoming long wave radiation. Errors in surface temperature measurement of several kelvins can lead to errors in \( z_{h,eff} \) of two to three orders of magnitude.

However, large ratios have been found at several sites. Another example comes from France. Composites based on observations of the momentum and heat fluxes at 100 m above the mean surface height were made for the ten fair-weather days of HAPLEX-MOBIHY, a major field experiment (Mahrt and Ek (1993); also see the discussion by Garratt (1994)). Effective roughness lengths for momentum were calculated, based on local averages (15 km length scales) and regional averages (100 km length scales) for flow over heterogeneous terrain composed by using the integrated profile relationships of Dyer-Businger (Paulson 1970) and Louis (1979). The results based on Paulson indicate an areally averaged \( z_0 \) of nearly 2 m; the results based on the scheme of Louis give smaller values (of approximately 1 m).

Similar calculations were done for the effective roughness lengths for heat, using the Paulson formulation (the Louis scheme assumes that the roughness length for heat is equal to that for momentum). The areally averaged roughness length for momentum is found to be of the order of \( 10^4 \) times that for heat in agreement with the observations of Beljaars and Holtslag (1991).

The above studies indicate that the bulk transfer coefficients should be defined at the blending height (if it exists within the surface layer) and then corrected to the height of the lowest model level. Areas with large \( z_0 \), even small areas, will dominate the momentum transfer. Observed values of areally averaged roughness lengths for momentum are typically 2 to 10 times larger than the smoother \( z_0 \) value. On the other hand, sensible and latent heat transfers are determined by the dominant surface cover and their areally averaged values are smaller than the local values. The wide range of values obtained for the roughness length for heat (0.1 \( z_0 - 0.0000001 \) \( z_0 \) over similar surfaces indicates a breakdown of the theory (assuming experimental error can be eliminated as a cause). A single length scale does not appear to be adequate to describe the transfer process and the geometry.

**Mixing**

A wide variety of mixing parametrisations are currently used to model the ABL. Three common classes are: first-order closure in which the eddy coefficients for momentum and heat (\( K_M \) and \( K_h \)) are specified; turbulent kinetic energy (TKE) or one-and-a-half-order closure schemes; and a broad group called non-local closure schemes. The TKE schemes include a turbulent kinetic energy equation and either an equation for the mixing length, \( l \), or for the turbulent energy dissipation, \( \varepsilon \), or for the frequency, \( \omega \) (\( \equiv \varepsilon/\varepsilon \)). For homogeneous turbulence all three of these descriptions are equivalent (Speziale 1991). Non-local schemes include transilience; top-down, bottom-up; convective circulations; similarity theory incorporating effects of large eddies and a prognostic equation for \( h \), the height of the ABL; filtering techniques; spectral diffusivity theory; direct interaction approximation and two-point closures; and integral schemes, among others (see Stull (1993), for discussion and references). In this paper we will compare the performance of a representative sample of these methods.

A major problem in evaluating mixing schemes is what to use as a standard. Holt and Raman (1988) have compared the performance of the first-order schemes of Blackadar (1962), Djolov (1973), O'Brien (1970); the mixing length models of Bodin (1979), Therry and Lacarrere (1983), and Duynerkerke and Driedonks (1987); and the E-s schemes of Beljaars et al. (1987), Duynerkerke and Driedonks (1987), and Detering and Etling (1985) in Fig. 8. They used observations from the Monsoon Experiment (MONEX79) as a standard where aircraft, ship, satellite and tower data were available. However the period studied was dynamically complex because the monsoon circulation had begun to break down; the situation was both non-stationary and non-homogeneous.

They found that some of the first-order schemes (e.g. Blackadar 1962; Djolov 1973) were unable to give good representations of the jet structure and did not properly account for stability or the capping inversion. The remaining schemes predicted the mean flow about equally well.
Fig. 8 Comparison of first-order closure schemes (Blackadar, 1962 (B); Djolov, 1973 (D); modified Djolov, 1973 (M-D); O'Brien, 1970 (O)), TKE mixing length schemes (Bodin, 1979 (B); Therry and Lacarrere, 1983 (T+L); Duynerke and Driedonks, 1987 (D+D)) and TKE dissipation schemes (Beljaars et al., 1987 (B); Duynerke and Driedonks, 1987 (D+D); Detering and Ettling, 1985 (D+E); modified Detering and Ettling, 1985 (M-D+E)) with mean potential temperature data from MONEX79 (after Holt and Raman, 1988).

Fig. 9 Same as Fig. 12, except for latent heat flux (after Holt and Raman, 1988).
Fig. 10  (a) Potential temperature profiles using a first-order closure model. The solid line is at the coast, the short dash line at 12 km, the long dash at 62 km and the dash-dot at 112 km from the coast. (b) Potential temperature profiles at the coast (solid line) and at 112 km; short dashed line LES model, long dashed line transiliens model, dash-dot line top-down, bottom-up model. (c) Heat flux profile at 112 km from coast. Solid line resolved scale LES model, short dashed line subgrid scale LES model, long dashed line transiliens model and dash-dot line top-down, bottom-up model. (d) Wind profiles at the coast (solid lines) and at 66 km; short dashed lines LES model, long dashed lines transiliens model, dash-dot lines top-down, bottom-up model (after Chrobok et al., 1992).
The E-ε models give slightly better predictions of the momentum and heat flux profile compared to the others (in this study an algebraic equation rather than a differential equation was used to close the mixing length models). The results for the latent heat fluxes are shown in Fig. 9. All models generally predict the moisture flux profile poorly. Surprisingly, O'Brien's first-order model performs best. The TKE models overpredicted the height of the ABL and consequently overpredict the convective scaling velocity, $w_c$.

Predicted turbulent variance statistics for E-I and E-ε models have been compared by André (1991) with field data measured under (relatively) homogeneous conditions. Again the mixing length model employed an algebraic equation, rather than a differential equation, for closure. In most cases the two TKE models gave good agreement with each other and the data. The best agreement for the E-ε model was achieved by varying the parametrisation of the production terms for different stabilities, but deficiencies remain in the convective case in the upper half of the boundary layer. Predictions in this case should be improved by including buoyancy effects in the transport terms.

Chrobok et al. (1992) studied the highly convective ABL in a cold air outbreak. They used their large-eddy simulations (LES) as their standard. In Fig. 10 the evolution of the convective boundary layer with increasing fetch is shown. The first-order scheme based on the mixing length of Blackadar (1962) performs poorly, the ABL is not well mixed. The transient model of Stull and Driedonks (1987) and the top-down, bottom-up model of Wyngaard and Brost (1984) and Moeng and Wyngaard (1984) both show a well mixed ABL, but are unable to predict the observed slightly negative potential temperature gradient in the upper part of the ABL due to entrainment of the warmer air above the boundary layer. The LES simulation does capture this effect.

The transient model gives good agreement with the LES heat flux profile over most of the boundary layer, but shows a smaller negative heat flux region and a lower ABL height. The top-down, bottom-up model produces a surface heat flux which is significantly larger than the LES value and also gives a smaller negative heat flux region and lower ABL height.

The predicted wind profiles are well mixed. As before the transient model gives better agreement with the LES simulation than the top-down, bottom-up model, but the ABL height is too low.

In Fig. 11 we present the comparison of simulations for an E-I model, an E-ε model and O'Brien's (1970) first-order model as given by Huang and Raman (1989). Again the mixing length is determined by an algebraic equation. There are no observations or more sophisticated model results to compare with in this case, which

Fig. 11 Comparison of simulations for (a) a TKE mixing length model, (b) a TKE dissipation model, and (c) a first-order closure model for flow from a cold land surface with topography to a warm sea (after Huang and Raman, 1989).

(a) $K_e$ (RUN 1, LST=13 HR)

(b) $K_e$ (RUN 2, LST=13 HR)

(c) $K_e$ (RUN 3, LST=13 HR)
simulates offshore flow over a mountain towards a warm sea. However, the computed structure of 
$K_0$ ($K_H$ in our notation; essentially it is a measure of the turbulent kinetic energy) in the two TKE 
simulations is in good agreement. The boundary-layer height for the first-order simulation is given by 
Deardorff's prognostic equation and is too high. The mountain-wave structure in the first-
order closure model is weakened because of the strong entrainment predicted and the sea-breeze 
structure is not as realistic as that for the TKE models.

Other recent studies of ABL mixing include 
Zhang and Stull (1992) who evaluated two forms of 
the transient model against observations from the Boundary-Layer Experiment (BLX83); 
Holtslag and Boville (1993) who tested a similarity 
scheme with a prognostic equation for $h$ and 
a first-order scheme against radiosonde observ-
vations; Betts et al. (1993) who compared results 
for a similarity scheme and a first-order scheme 
with the FIFE-1987 data; and Ly (1991) who 
applied E-l and E-s models (with an algebraic 
equation for $l$) to a couple atmosphere-ocean 
model.

Summary and conclusions

In this paper we have discussed the criteria for 
homogeneous surface conditions. Based on a 
careful study of fifty experiments that met these 
criteria, Wieringa (1993) found that many of the 
published classifications of the roughness length 
for momentum gave values that were too large 
over smooth terrain (probably because the fetches 
were too short). For semi-smooth and skimming 
flows many of the classifications underestimate 
the roughness by a factor of about two.

Several simple parametrisations of effective 
roughness lengths for flow over obstacles and 
idealised terrain have been presented. These para-
metrisations give good agreement with data, but 
so far the results are only preliminary and mainly 
confined to neutral conditions. For hills and 
mountains these parametrisations indicate rough-
ness lengths of the order of tens to hundreds of 
metres! Such large values violate the conditions 
that $z > d + 20z_0$ (Wieringa 1993), and $z$ must still 
be within the surface (constant flux) layer to apply 
Monin-Obukhov theory. As the steepness and the 
height of the terrain increase, the theoretical basis 
for the parametrisations becomes less rigorous. In 
a practical sense the limits of applicability of these 
parametrisations needs to be established through 
laboratory, field and numerical experiments. 
Especially important are the cases of large $\lambda$ in 
complex (non-idealised) terrain, cases of strong 
flow separation and cases involving non-neutral 
-especially stable- conditions.

Observations of the effective roughness length 
for momentum from the HAPEX-MOBILHY 
experiment indicate that it may be of the order of 
$10^3$ larger than the effective roughness for heat 
(Garratt, 1994, discusses other (older) experimen-
tal results). Other recent field results give ratios 
that range from 18 (FIFE experiment) to $10^7$ 
(BLX83 experiment). This is clearly an area that 
needs more study. The wide variation in the ratios 
due to the variation in $z_H$ may mean that the 
theory is not robust. The concept of a single length 
scale to characterise the roughness of the potential 
temperature profile may not be adequate.

Numerical experiments indicate that fluxes 
should be calculated at the blending height and 
weighted averages should be performed over the 
grid square where the weights are determined by 
the fractional cover of differing surfaces. Such 
changes have not yet been incorporated widely by 
the modelling community. This type of parame-
trisation becomes very important in regions 
where the subgrid scale fluxes have large vari-
ations, for example near Antarctic leads. These 
parametrisations highlight ways models on the 
microscale can be used to develop and test meth-
odologies to be employed in large-scale models.

Finally, many promising turbulent mixing 
models are now available, including TKE and 
non-local schemes. Comparison of first-order 
schemes with data and large eddy simulations 
shows that, generally, first-order schemes yield 
poor performance in convective conditions. One 
area of mixing that has not been adequately 
explored is the formulation of turbulent mixing 
models in the presence of strong separation. 
Under these conditions it may be necessary to 
employ non-linear mixing models, such as 
suggested by Speziale (1991), and these should be 
investigated.

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