

# A model of diffuse broadband solar irradiance for a cloudless sky

Warwick Grace

Grace Research Network, Bellevue Heights, South Australia

(Manuscript received November 2005; revised April 2006)

**An analytic model is presented for the broadband diffuse irradiance received at the earth's surface through a cloudless homogeneous atmosphere containing light-absorbing and isotropic-scattering agents. The model is extended to include a contribution from light re-scattered after ground reflection. The model is validated, with some qualification, against a Monte Carlo model based on the same physics, two empirical models and ground-based observations at Adelaide and Alice Springs.**

## Motivation

Sophisticated and comprehensive spectral and broadband radiation-transfer equation models, which require numerical solution, are well established in the study of direct and diffuse light (Mishchenko et al. 2002, 2003; Bird and Riordan 1984). At the other extreme are well-supported and simple empirical broadband models for diffuse irradiance in terms of the direct irradiance (Peterson and Dirmhirn (1981) and Campbell and Norman (1998), hereafter PD and CN respectively). The highly idealised broadband model developed in this paper provides an analytic expression for diffuse irradiance that is comparable in simplicity to the empirical models. This model also allows an *a priori* indication of the empirical constants in the empirical models. Compared to the more sophisticated numerical models, an analytic model of this nature provides an easy way to evaluate the sensitivity of diffuse irradiance to environmental factors of zenith angle, albedo, atmospheric transmittance and the relative proportions of scattering and absorption agents in the atmosphere. As such it is suitable for illustrative or instructional purposes.

## Introduction

At midday on a cloudless day, about a quarter of the sun's radiation is scattered or absorbed as it passes

through the atmosphere. The radiation that reaches the earth's surface, coming from the direction of the sun, is called direct normal-irradiance or beam irradiance. Some of the scattered sunlight returns to space and some of it reaches the surface of the earth. The scattered radiation reaching the earth's surface is called diffuse irradiance. Some radiation is reflected off the earth's surface and then re-scattered by the atmosphere back to the surface. This is also a component of the observed diffuse-irradiance. Other components are the radiation reflected directly from clouds or topography, which are ignored in this study. The total solar radiation on a horizontal surface is called global irradiance and is the sum of incident diffuse-irradiance plus the direct normal-irradiance projected onto the horizontal surface.

Diffuse irradiance in clear, cloud-free skies is typically 10 per cent of the global irradiance. Air molecules scatter light while aerosols may act to either absorb or scatter light; these interactions are wavelength and height dependent. Air molecules cause scattering that is predominantly along or opposite a photon's original direction; aerosols cause scattering that is predominantly transverse to the photon's original direction (Rayleigh and Mie Scattering respectively). Table 1 shows typical amounts of scattering and absorption due to the various atmospheric constituents. Water vapour is responsible for the largest variability in absorption and scattering. Background information on light transmission through the atmosphere is available at the websites of the Australian

---

Corresponding author address: W. Grace, Grace Research Network, 29 Yurilla Drive, Bellevue Heights, SA 5050, Australia.  
Email: graceconsulting@optusnet.com.au

**Table 1. Typical values of the amount of solar radiation absorbed or scattered by various constituents of a cloud-free atmosphere, from Solar Radiation Monitoring Laboratory, University of Oregon (2002).**

Constituent	Per cent absorbed	Per cent scattered
Ozone	2	0
Water vapour	8	4
Dry air	2	7
Upper dust	2	3
Lower dust	0	0
Total	~ 14	~ 14

Bureau of Meteorology and other international agencies. Comprehensive texts include Kondratyev (1969), Iqbal (1983), Goody and Yung (1989), Bird and Riordan (1984), and Schwerdtfeger (1995).

In determining the radiation, knowledge of the variation of the solar constant and the angle of the sun to the vertical, solar zenith angle, are required. Useful formulas of sufficient accuracy are available from many sources such as the Smithsonian Tables (List 1984) and de Wit (2000). Half-hourly ground-based observations of irradiance are available from the Bureau of Meteorology with the convenience that these are already tabulated against local solar time; while the equation of time, the longitude distance to the local time zone meridian and any effect of daylight saving are already accounted for. Figure 1 compares the daily variation of the extraterrestrial, global, direct and diffuse irradiance for a cloud-free day at Adelaide Airport. Figure 2 shows in detail the variation of diffuse irradiance for the same day.

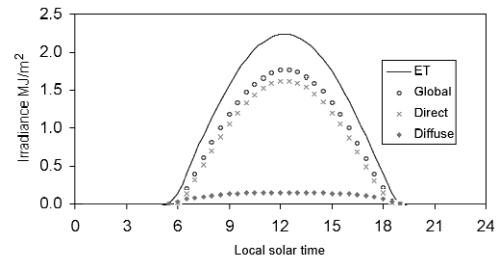
**The model**

The model as depicted at Fig. 3 assumes independence of wavelength and that all scattering is isotropic. Also assumed is an homogeneous atmosphere of finite depth *H*, containing absorbing and scattering agents such that the extinction of the solar beam conforms to Beer’s Law (Eqn 1). In the first instance, it is assumed that there is no ground reflection, that is, surface albedo is zero. The intensity denoted by *q*, which is the radiation energy per unit area on a surface normal to the beam, attenuates exponentially with distance travelled *l* such that

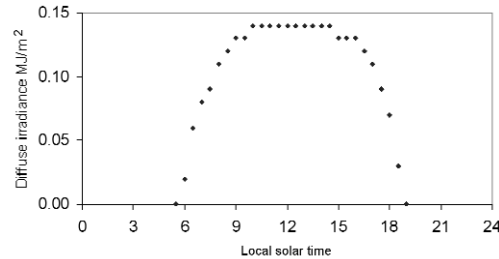
$$dq = -kq dl \quad \dots 1$$

The constant *k* represents the concentration of intercepting agents:  $k = k_a + k_s$ ,  $k_a$  represents absorbers

**Fig. 1 Observed half-hourly exposures of global, direct and diffuse irradiance and calculated extra-terrestrial (ET) irradiance in MJ/m<sup>2</sup> at Adelaide Airport on 17 October 2004.**



**Fig. 2 Typical cloud-free diffuse irradiance. Half-hourly exposures in MJ/m<sup>2</sup> observed at Adelaide Airport on 17 October 2004.**



and  $k_s$  the single and multiple isotropic scatterers. In this paper, a scattering ratio  $\rho$  is defined as  $k_s/k$ . Table 1 shows  $\rho$  is typically 0.5 and some inspection reveals that drier conditions imply  $\rho \sim 0.6$  to  $0.7$  and moister conditions  $\rho \sim 0.3$  to  $0.4$ , all other things being equal. Now let *Q* represent the top-of-atmosphere irradiance, being the solar constant  $Q \sim 1367 \text{ W/m}^2$ ;  $\phi$  the zenith angle of the sun and the vertical coordinate *y*, such that  $y = 0$  at the ground and  $y = H$  at the top of the atmosphere. Thus

$$dq = +k \sec\phi \ q \ dy \quad \dots 1(a)$$

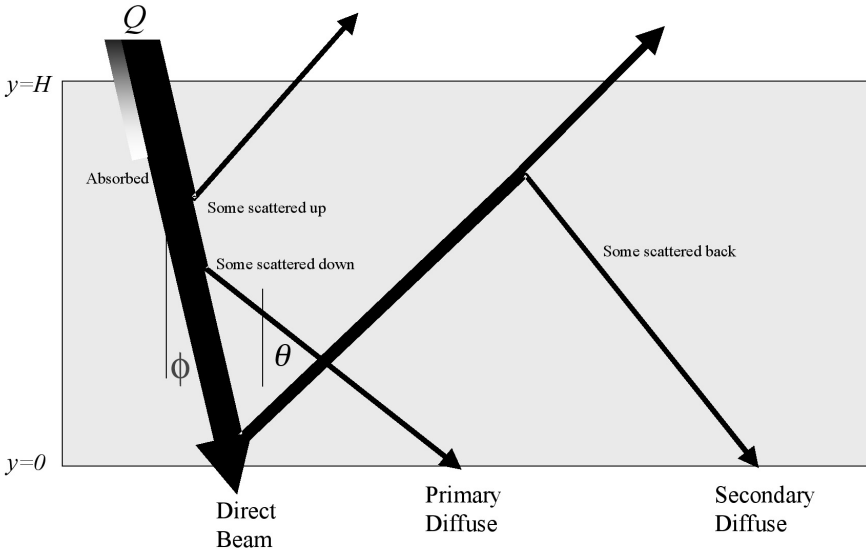
Integrating Eqn 1(a) and noting that  $q = Q$  at  $y = H$ , then

$$q = Q e^{-k H \sec\phi} + k y \sec\phi \quad \dots 1(b)$$

$$dq = Q k \sec\phi e^{-k H \sec\phi} + k y \sec\phi \ dy \quad \dots 1(c)$$

Direct (beam) irradiance (normal to sun) at the surface  
 $= Q e^{-k H \sec\phi} \quad \dots 1(d)$

**Fig. 3** Schematic of the main features of the model. Incoming solar radiation  $Q$ , at the top of the atmosphere may undergo absorption, scattering within the atmosphere or reflection from the ground. Once scattered or reflected, light may undergo subsequent scattering.



Direct irradiance on horizontal surface  
 $= Q \cos \phi e^{-k H \sec \phi}$  ...1(e)

The product  $kH$  is known as the optical depth of the atmosphere, and the term  $e^{-kH \sec \phi}$  is the transmittance or transmissivity  $T$  of the angled beam. Zenith transmittance  $T_z$  is the transmittance when  $\phi$  is zero and is given by  $e^{-kH}$ . A typical value of  $T_z$  for a clear day is 0.6 to 0.7 with values up to 0.8 for the clearest days (CN, McIlveen 1992).

Integrating Eqn 1(c) through a vertical column of height  $H$  is  $Q \cos \phi (1 - e^{-kH \sec \phi})$ , the amount of radiation energy intercepted in the direct beam on a horizontal unit area. The proportion of this amount that is scattered is  $\rho$  and through symmetry half is scattered downward. Thus, the downward diffuse irradiance  $S_o$  is given by

$$S_o = 1/2 \rho Q \cos \phi (1 - e^{-kH \sec \phi}) \quad \dots 2$$

Assume that this is distributed uniformly within the column, a reasonable assumption for a clear sky, so that the downward scattered irradiance contribution  $dS_o$  from a unit horizontal area element in the column of thickness  $dy$  is therefore

$$dS_o = 1/2 \rho Q \cos \phi (1 - e^{-kH \sec \phi}) (1/H) dy \quad \dots 3$$

This amount may be envisaged as being emitted isotropically from a thin horizontal uniform slab. The

flux density reaching the surface from such a source is known to attenuate as  $w$ , where

$$w = 2 \int_{\theta=0}^{\pi/2} \sin \theta \cos \theta e^{-k_a y \sec \theta} d\theta \quad \dots 4$$

and  $\theta$  is the zenith angle for any scattered beam in question. Now it is apparent that  $w$  is a function not of  $\theta$ , but of the product  $k_a$  and  $y$ , and it turns out that a good parametrisation for  $w$  is given by

$$w \approx e^{-\beta k_a y} \quad \dots 5$$

where  $\beta \approx 1.66$ . Thus Eqn 3 is modified by the attenuation factor  $w$ , to become

$$dS_o = \frac{\rho}{2} \frac{Q w \cos \phi (1 - e^{-kH \sec \phi})}{H} w dy \quad \dots 6$$

The derivation of the expression for  $w$  at Eqn 4 is well established (e.g. Iqbal 1983; Schwerdtfeger 1995 and Coakley 2003) and is not provided here. However, intuitively one might expect that  $w$  would involve exponential attenuation in  $k_a y$ . Substituting Eqn 5 into Eqn 6 and integrating throughout the vertical column gives

$$S_o = \frac{\rho}{2} \frac{Q \cos \phi (1 - e^{-kH \sec \phi})}{H} \int_{y=0}^H e^{-\beta k_a y} dy \quad \dots 7$$

$$\text{and thus } S_o = \frac{\rho}{2} \frac{Q \cos\phi (1 - e^{-kH \sec\phi})}{\beta k_a H} (1 - e^{-\beta k_a H}) \dots 8$$

Anticipating that  $\beta k_a H < 1$  (typical values are  $\sim 0.2$ ), then Eqn 8 may be approximated as

$$S_o = 1/2\rho Q \cos\phi (1 - e^{-k H \sec\phi})(1 - 1/2 \beta k_a H) \dots 9$$

Equation 9 is therefore an expression for zero-albedo diffuse-irradiance. With increasing  $k_a H$ , Eqn 9 will tend to underestimate. Replacing  $k_a H$  with  $(1-\rho)k H$  gives

$$S_o = 1/2\rho Q \cos\phi (1 - e^{-k H \sec\phi}) \{1 - 1/2\beta(1-\rho)k H\} \dots 9(a)$$

so that the diffuse irradiance is an analytic function of zenith angle  $\phi$ , of optical depth  $k H$  and the scattering ratio  $\rho$ . Compared to photons scattered once only, the photons undergoing second and subsequent scattering have an increased average path length. So the absorptive attenuation expected from Eqn 5 will be an underestimate of absorption.

Explicitly, the sources of error in the model as expressed by Eqn 9 or 9(a) are as follows: the effect of multiple scattering means that the model overestimates diffuse irradiance when  $\rho < 1$ . With increasing turbidity, most of the initial absorption and scattering occurs in the upper part of the atmosphere and so the assumption of uniformity will break down, leading to an overestimate by the model. Finally, the simplifying approximation introduced at Eqn 9 causes the model to underestimate.

For convenience, allow  $S_o$  to denote the primary diffuse-irradiance. Now let  $S_J$  denote the secondary diffuse-irradiance sourced from ground-reflected irradiance. Assume that the major component is from the reflected direct-irradiance, neglecting any reflected  $S_o$  component. Assume that the albedo is  $A$  and that the ground reflection is Fresnel, where the angle of reflection equals the angle of incidence. Thus there is a surface source of upward irradiance  $Q \cos\phi A e^{-k H \sec\phi}$ .

With argument similar to that above, the interception of ground-reflected radiation  $Int$ , is given by

$$Int = (1 - e^{-k H \sec\phi}) Q \cos\phi A e^{-k H \sec\phi} \dots 10$$

As before, the downward scattered contribution  $dS_J$  at the surface from a thin horizontal uniform scattering slab is:

$$dS_J = 1/2 \rho w Int H^{-1} dy \dots 11$$

$$dS_J = 1/2\rho w (1 - e^{-kH \sec\phi}) Q \cos\phi A e^{-kH \sec\phi} H^{-1} dy \dots 11(a)$$

and  $w$  is the atmospheric depletion of the beam that is scattered from the slab ( $e^{-\beta k_a y}$ ). The final expression

for  $S_J$  after integration and neglecting second order terms is:

$$S_J = 1/2\rho Q \cos\phi \{1 - 1/2\beta(1-\rho)k H\} (1 - e^{-kH \sec\phi}) A e^{-kH \sec\phi} \dots 12$$

Since diffuse irradiance  $S \approx S_o + S_J$  (neglecting subsequent  $S_2, S_3, S_4 \dots$  terms)  $\dots 13$

then

$$S = 1/2\rho Q \cos\phi \{1 - 1/2\beta(1-\rho)k H\} (1 - e^{-kH \sec\phi}) (1 + A e^{-kH \sec\phi}) \dots 14$$

The neglect of the reflected  $S_o, S_J, \dots$ , terms causes an underestimating effect as  $k_s$  or  $A$  increases. This is the fourth source of error introduced by various assumptions and approximations. If isotropic ground reflection is assumed then a slightly different expression for  $S$  results, namely,

$$S = 1/2\rho Q \cos\phi \{1 - 1/2\beta(1-\rho)k H\} \{1 - e^{-kH \sec\phi} + A e^{-kH \sec\phi} (1 + A e^{-\beta k H})\} \dots 14(a)$$

Equations 14 and 14(a) clearly show that  $S$  increases with  $\rho$  or  $A$ , as intuitively expected. The effect of changes in  $\phi$  or  $kH$  is less obvious but is easily plotted for inspection. An alternative perspective is that the analytic model, as a function of  $\phi$ , requires knowledge of the three parameters  $T_z, \rho$  and  $A$ .

A comparison of the variants of the model with symmetrical reflection (Eqn 14) and with isotropic surface reflection (Eqn 14(a)) is provided at Fig. 4 with parameters  $T_z = 0.8, \rho = 0.5$  and  $A = 0.25$ . Inspecting the plots for each variant shows that there is only a small difference between them. Other comparisons with a range of parameters, not shown, support this conclusion. The Fresnel variant for symmetrical reflection is used hereafter mainly because it is more amenable to comparison with the empirical models, and will be referred to as ‘the analytic model’.

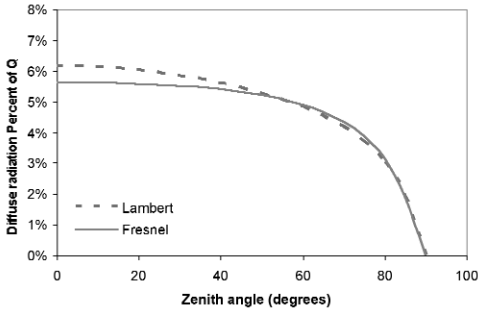
## Validation

Validation was performed in three ways: firstly, against output from a Monte Carlo model, having the same essential physics, but not incorporating the several approximations used above, over a range of values of  $T_z, \rho, \phi$  and  $A$ ; secondly, against observational data of irradiance; thirdly, against estimates from empirical models.

### Monte Carlo model

A Monte Carlo model is easily constructed, taking less than a page of coding and is conceptually simple in that for the same model conditions, the atmosphere is comprised of 100 layers of thickness,  $h$ , where  $h =$

**Fig. 4** Comparison of Fresnel and Lambert variants of the analytic model with parameters  $T_z = 0.8$ ,  $\rho = 0.5$  and  $A = 0.25$ .



0.01  $H$ . Incoming solar radiation is represented by the  $Q \cos\phi$  term as  $N$  ( $\sim 10^5$ ) photons entering the top layer from above. Within any particular layer, each photon has a probability either of passing through unaffected or of being intercepted. The probabilities for either of these events are easily related to  $kh \sec\phi$ . If a photon is intercepted it must then either be absorbed or scattered, with probabilities related to  $\rho$ . If it is scattered, then under the isotropic scattering condition, it scatters with equal probability at any solid angle. If it is neither scattered nor absorbed, then the photon passes into an adjacent layer where it is subject to the same process as before. A photon reaching the ground is reflected symmetrically with probability equal to the albedo, otherwise it is absorbed. Each photon is tracked until either it is absorbed or it exits from the top of the atmosphere. Counts of photons corresponding to the irradiance components of direct, diffuse, absorbed and that returned to space are easily made.

The Monte Carlo model was run for a comprehensive range of  $T_z$ ,  $\rho$ ,  $\phi$  and  $A$ , taking a few days to run on a PC. To obtain the diffuse irradiance merely requires looking up a 4D table and interpolating as necessary. Other radiation components for any specified combination of  $k$ ,  $\rho$ ,  $\phi$  and  $A$  may be obtained in a similar way.

For a range of  $T_z$ ,  $\rho$ ,  $\phi$  and  $A$  the results from the analytic model as expressed by Eqn 14 were compared to the Monte Carlo estimates of diffuse irradiance (see Figs 5(a) and 6(a)). With low transmittance values, typically  $T_z < 0.33$ , the approximation introduced at Eqn 9 is not used. Figure 5(a) shows comparative plots with zero albedo for a range of transmittance values. It is evident that the simple analytic model compares favourably with the Monte Carlo model at transmittance values greater than  $\sim 0.3$  and with zero albedo. As noted by a reviewer, in the

region of lower  $T_z$  with higher  $\rho$ , the deficiency of the model's handling of multiple scattering becomes apparent and introducing a correction factor will compensate for this deficiency. For the zero albedo case, a correction factor  $F_0$  can be applied as a multiplier to the right-hand side of Eqn 8 where

$$F_0 = 1 + f k H \sec(0.8\phi) \quad \dots 15$$

and

$$f = -(3.85 + 6.1\rho^{5.75}) \quad \dots 16$$

Comparing the analytic model for zero albedo with this correction factor against the Monte Carlo estimates results in a very close agreement as shown at Fig. 5(b).

Figure 6(a) shows similar comparative plots using the analytic model without any correction factor; albedo of 0.25, 0.5 and 0.75; with  $T_z$  limited to 0.6, 0.4 and 0.2. The comparisons are still quite good, although the analytic model tends to strongly underestimate with higher values of  $\rho$  and  $A$ . Thus, the analytic model with its approximations and assumptions fairly represents its simple physics. For completeness, a further correction factor  $F$ , for non-zero albedo cases is provided where

$$F = F_0 F_A \quad \dots 17$$

$$F_A = 1 + 0.75 \rho A k H \sec(0.8\phi) \quad \dots 18$$

As shown by the relevant comparison at Fig. 6(b), the corrected analytic model provides fairly good agreement. As expected,  $F_A$  reduces to 1 when  $A = 0$ . To be clear, the correction factor  $F$  is simply a mathematical construct arrived at by trial and error. It has the advantage of serving as a parametrisation of the Monte Carlo model allowing a full range of possible scenarios to be easily explored. In general it would be difficult to develop a parametrisation with four variables without using the analytic model as the basis of a first-guess estimate.

### Observations

From observational data of the direct irradiance, the value of  $k$  or  $kH$  is easily determined from Beer's Law through Eqn 1(b) by setting  $y = 0$ . The angle  $\phi$  should always be known in practice.  $A$  is known or estimated *a priori*, although its dependence upon zenith angle is ignored. From the observed diffuse values  $\rho$  can be determined from Eqn 14. Albedo  $A$  is estimated as 0.1 at the Adelaide site and as 0.25 at Alice Springs, estimated respectively from values for Los Angeles by Taha (1994) and Coakley (2003). This is done for the sample of cloudless days at Adelaide Airport and Alice Springs Airport. The comparisons of observed and modelled irradiances are shown at Figs 7, 8, 9 and 10.

Fig. 5(a) Comparison of analytic model (lines) with Monte Carlo model (diamonds) for a range of  $T_z$ ,  $D$  and  $n$  with albedo  $A = 0$ . Transmittance  $T_z = 0.1, 0.2, \dots, 0.9$  for each panel reading left to right from top-left panel. The scattering ratio  $\rho$ , varies within each panel from top to bottom as 1.0, 0.75, 0.50, 0.25. Zenith angle  $\phi$  is shown in degrees along the abscissa; the diffuse irradiance received at the surface as a percentage of the solar constant  $Q$  is shown on the coordinate axis. Note that the coordinate scale varies in order to better show relative differences. The analytic model compares favourably provided  $T_z > 0.3$ .

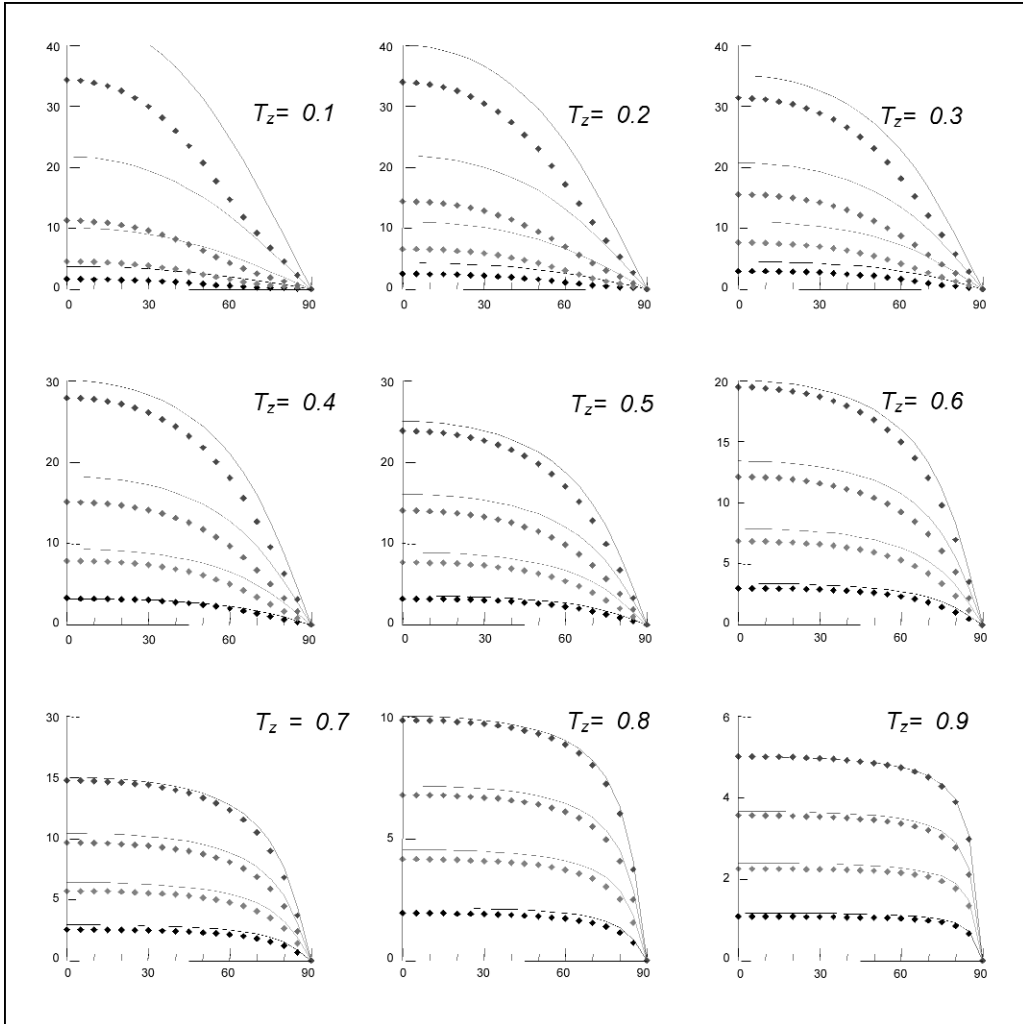
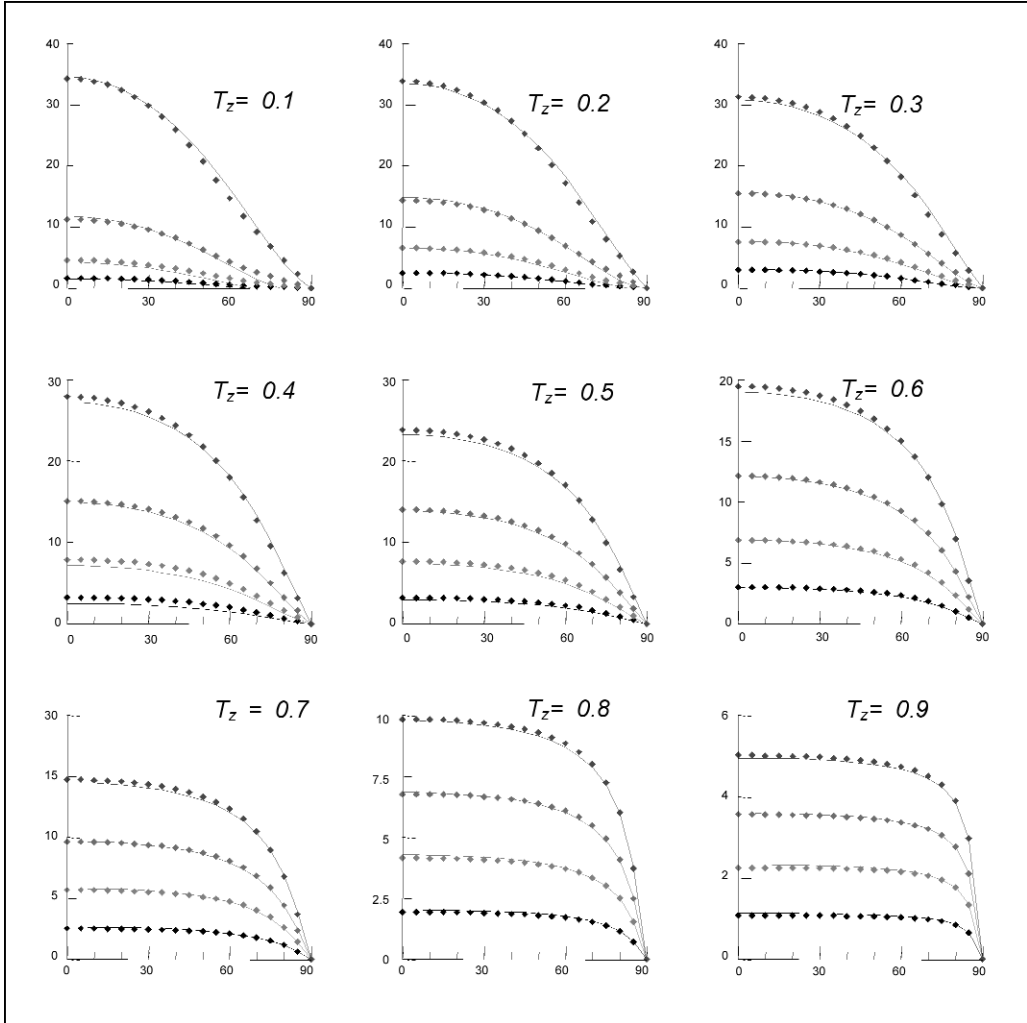


Table 2 shows the parameters  $T_z$  as determined from direct irradiance observations,  $\rho$  as determined from diffuse irradiance observations and  $A$  estimated *a priori*. The diffuse model estimates are quite good but with the caveat that the validation necessarily has the shortcoming that the unknown atmospheric parameters  $T_z$  or  $\rho$  are derived from the observational data. Assuming, as above, that  $A$  is known or estimated independently, then in theory only two observations at a

given point in time – one for direct and one for diffuse irradiance – are required to determine  $T_z$  and  $\rho$ . In practice, all available observations for a given day were used to determine the values of  $T_z$  and  $\rho$ . The important point is that the shapes of the model curves closely match those of the observed data throughout the day. This is unlikely to happen if either the model is inadequate or the value of  $T_z$  or  $\rho$  vary markedly over the course of the day.

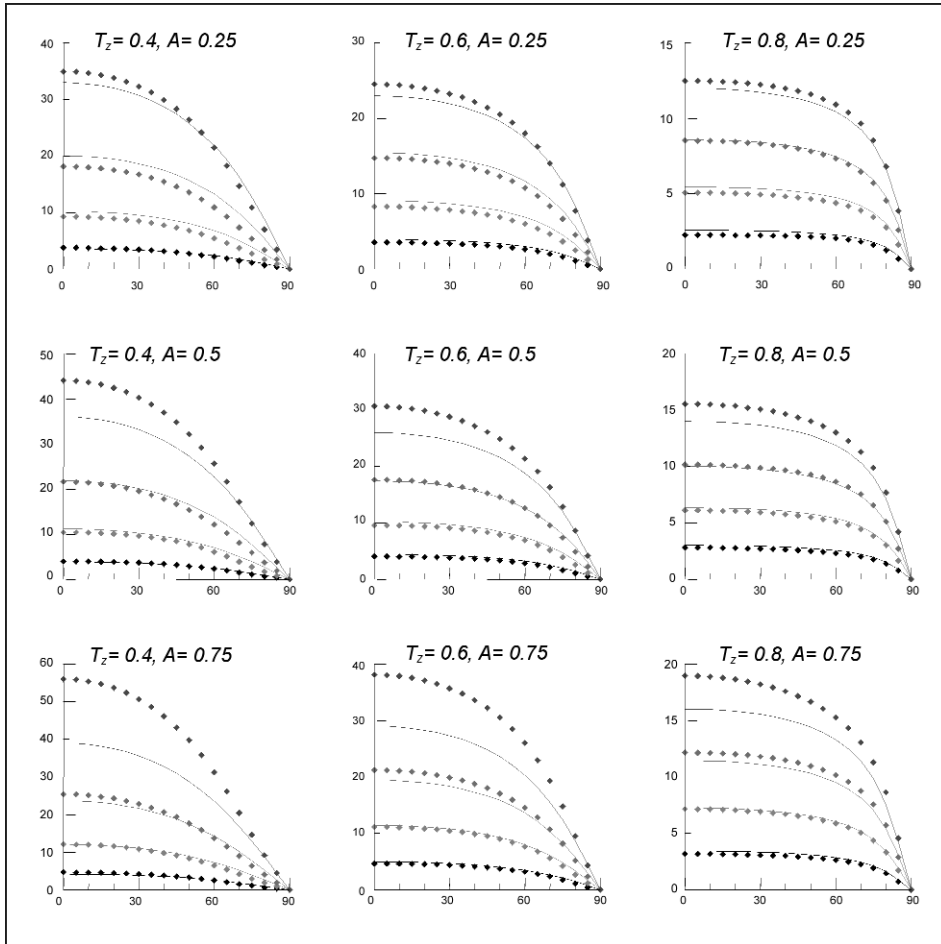
Fig. 5(b) Comparison of corrected analytic model with Monte Carlo model as in Fig. 5(a).



**Table 2.** Values of the parameters  $k$  and  $T_z$  determined from direct observations;  $\rho$ , determined from diffuse observations and knowledge of  $k$ ; and  $A$ , estimated a priori, for two cloudless days at Alice Springs Airport and Adelaide Airport.

Site	Latitude	Date	$k \cdot 10^{-6} m^{-1}$	KH	$T_z$	$\rho$	$A$
Alice Springs Airport	23.8 °S	1 Jul 04	27	0.22	0.81	0.35	0.25
Alice Springs Airport	23.8 °S	11 Dec 04	30	0.24	0.79	0.40	0.25
Adelaide Airport	35.0 °S	17 Oct 04	35	0.28	0.76	0.50	0.10
Adelaide Airport	35.0 °S	17 Dec 04	34	0.27	0.76	0.55	0.10

Fig. 6(a) Similar to Fig. 5(a) but with albedo  $A = 0.25, 0.5$  and  $0.75$  in rows from the top to bottom and  $T_z = 0.4, 0.6$  and  $0.8$  in columns from left to right. With lower transmittance, relatively more diffuse irradiance is received. With higher albedo, the irradiance increases and the increase is greater with larger  $\rho$ . The analytic model still compares well but tends to strongly underestimate for higher albedo as  $\rho$  increases toward 1.0.



**Empirical models**

Two empirical expressions for the diffuse irradiance from CN and PD are, respectively

$$S = 0.3Q \cos\phi (1 - e^{-kH \sec\phi}) \quad \dots 19$$

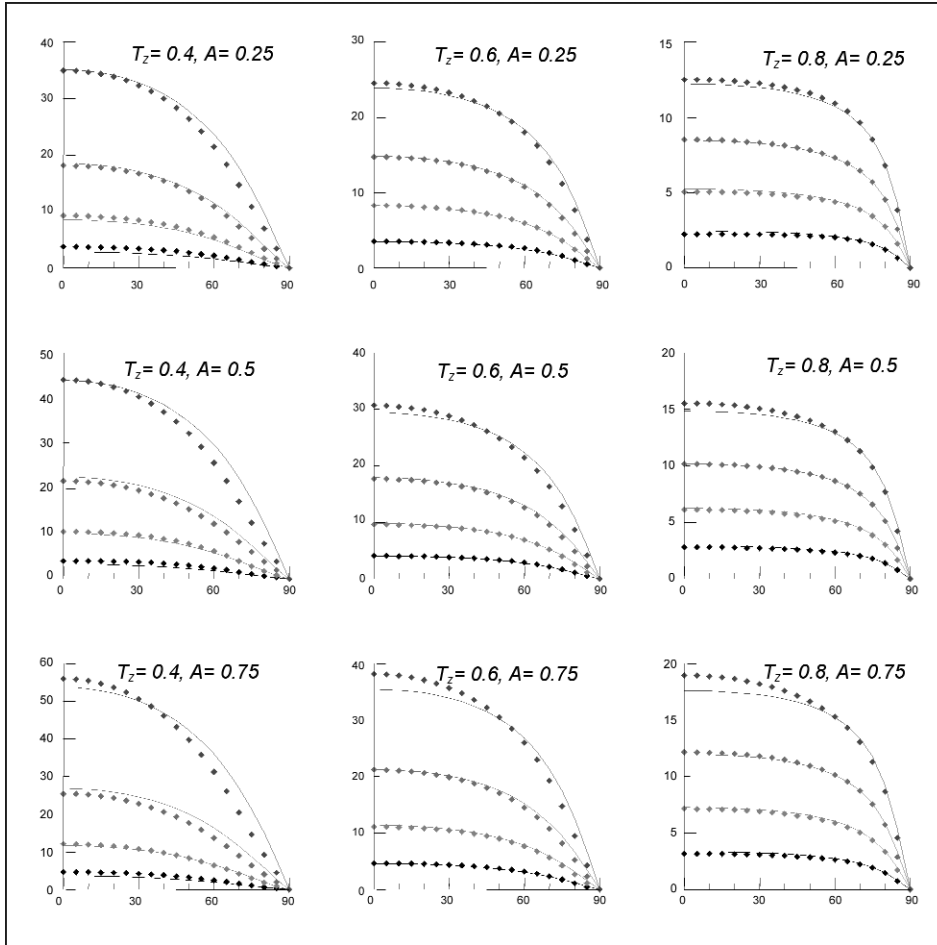
$$S = R Q e^{-kH \sec\phi} \quad \dots 20$$

In these models,  $T_z = e^{-kH}$  and is estimated or empirically determined using Eqn 1(e) from readings of direct irradiance.

CN's model, represented by Eqn 19, is an adaptation of empirical relations presented by Liu and

Jordan (1960). In this form the diffuse irradiance is proportional to the amount of reduction in the direct irradiance, that is, proportional to the amount that has been intercepted from the direct beam. That this is a plausible model is seen from a consideration of the special case of no absorption, no albedo and high transmittance. In such circumstances then it would be expected that half of the scattered radiation would reach the ground. Thus Eqn 19 would be applicable but with the coefficient being 0.5. For the other extreme case of only absorption, a coefficient of zero would apply. An intermediate value between the two extremes is 0.25 compared to CN's value of 0.3.

Fig. 6(b) Similar to Fig. 6(a) but with corrected analytic model.

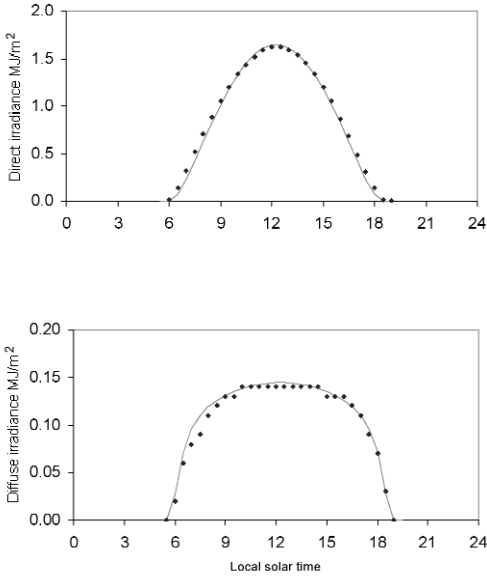


For PD's model, represented by Eqn 20, the heuristic is that the ratio of the diffuse irradiance to the direct irradiance is constant for most of the day. The ratio may vary from day to day with different air masses and from season to season with different ground albedo, but throughout any given cloudless day, the ratio is conservative. Peterson and Dirmhirn (1981) reported ratios ranging from five to nine per cent with the higher values being associated with snow cover. A form of this heuristic is used for quality control purposes by the Australian Bureau of Meteorology (B. Forgan, personal communication). At Fig. 11 both models are plotted against  $\phi$  and then

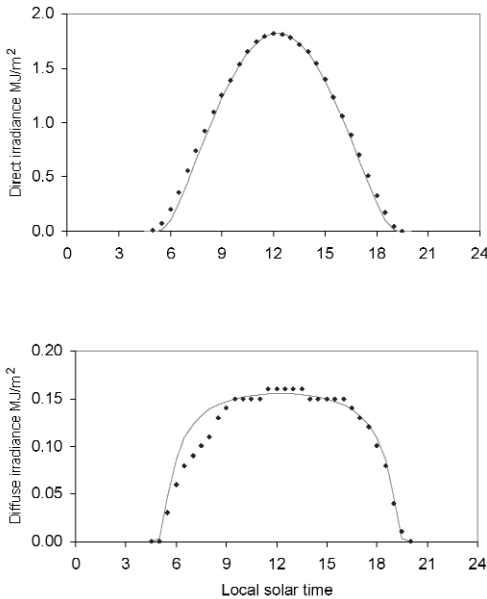
compared to the analytic model, with  $T_z = e^{-kH} = 0.76$ ,  $\rho = 0.53$  and  $A = 0.1$  being average values for the Adelaide Airport data.

Evidently each of the models is in approximate agreement. For the CN model, the empirical factor is 0.3 and it is possible to anticipate this value from consideration of the analytic model with likely input values. For convenience the Fresnel variant of the model at Eqn 14, which was seen from Fig. 4 to be very similar to the Lambertian variant, is used to compare with the CN model at Eqn 19. Clearly the empirical factor of 0.3 corresponds to the term  $1/2\rho\{1-1/2\beta(1-\rho)kH\}(1+A e^{-kH \sec\phi})$

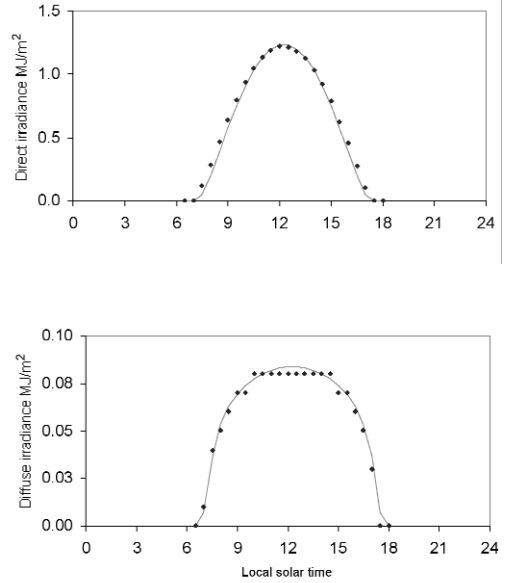
**Fig. 7** Half-hourly exposures of observed and modelled direct (upper panel) and diffuse (lower panel) irradiance in MJ/m<sup>2</sup> (on a horizontal surface) at Adelaide Airport on 17 Oct 2004. Observed denoted by diamonds, model estimates shown by solid line.



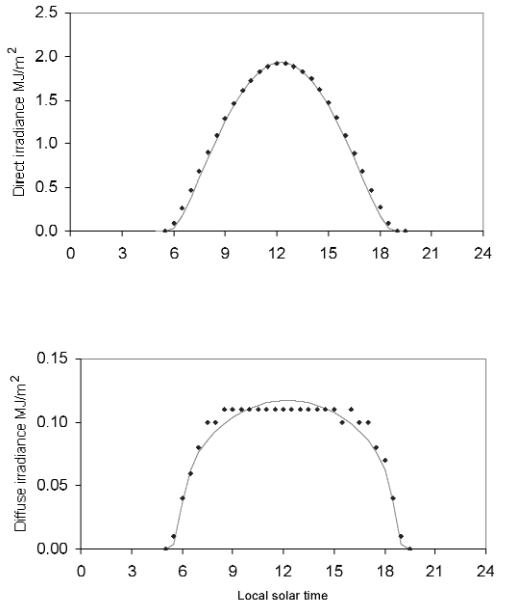
**Fig. 8** As for Fig. 7 but for Adelaide Airport on 17 December 2004.



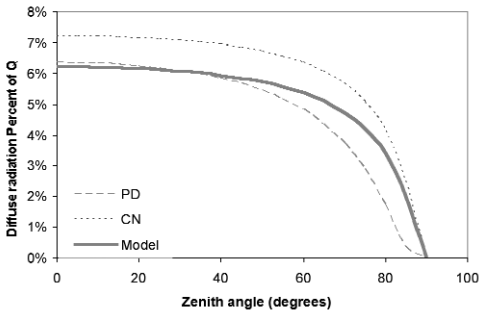
**Fig. 9** As for Fig. 7 but for Alice Springs Airport on 1 July 2004.



**Fig. 10** As for Fig. 7 but for Alice Springs Airport on 11 December 2004.



**Fig. 11** Comparison of the analytic model (Fresnel variant) with two empirical models, CN and PD, using parameters  $T_z = 0.76$ ,  $\rho = 0.5$  and  $A = 0.1$ . The values of  $T_z$  and  $\rho$  were chosen as representative of the values determined from the two sets of observations at Adelaide Airport (see Figs 7 and 8). The value for  $A$  was chosen from tables of representative values in the literature.



Assuming  $T_z = e^{-kH} = 0.76$ , the median value in Table 2; that  $A = 0.25$ , being an average value for soils, grasses and deserts taken from McIlveen (1992) and that  $\rho = 0.5$ , as per Table 1; then the ‘predicted’ proportionality factor is 0.28 at  $\phi = 0^\circ$  and reducing to 0.22 for  $\phi = 90^\circ$ . Although an underestimate of the factor 0.3 in this scenario, for realistic values  $T_z$ ,  $\rho$  and  $A$ , the analytic model is in broad agreement with CN’s model.

Assuming that PD’s model at Eqn 20 approximately corresponds to the analytic model as is evidently reasonable from inspection of Fig. 13, then some manipulation of Eqn 14 shows that this requires that

$$R = 1/2\rho\{1 - 1/2\beta(1-\rho)kH\}\cos\phi\{e^{+kH\sec\phi} - 1 + A(1 - e^{-kH\sec\phi})\} \dots 21$$

Assuming that  $kH \sec\phi \ll 1$  and using first terms of exponential expansion, then

$$R \approx 1/2\rho\{1 - 1/2\beta(1-\rho)kH\}kH(1+A) \dots 22$$

Consistent with PD, Eqn 22 shows that  $R$  is independent of zenith angle, at least while  $kH \sec\phi \ll 1$ . Setting  $T_z$ ,  $\rho$  and  $A$  as above, then  $R \sim 0.06(1+A) = 7.6$  per cent. Modifying  $A$  as appropriate for snow cover results in an estimate of  $R$  of  $\sim 11$  per cent. Thus it is evident that under at least some realistic circumstances the analytic model is consistent with PD’s empirical model.

## Summary and discussion

A model of diffuse irradiance for an homogeneous cloudless atmosphere has been presented. The model’s idealised atmosphere contains scattering and absorbing agents in a constant ratio and the scattering is isotropic. This model was extended to include reflection of the direct beam from the ground and secondary re-scattering.

In developing the model some explicit approximations were made. These were that the interception of the direct beam is uniform with height, reasonable for high transmittance; that the effective increase in a scattered photon’s path length, and therefore chance of absorption, due to multiple scattering is negligible; that  $\beta k_a H < 1$  or  $\beta(1-\rho)kH < 1$  so that an approximation to exponential of  $\beta kH$  is valid, again, reasonable for high transmittance; and that scattered light was not reflected from the ground, reasonable if the initial amount of scattering or albedo is low. Bearing in mind the model’s idealised physics and its main purpose – to provide a simple analytic expression – the simplifying assumptions are maintained at the expense of excluding the more extreme scenarios. The model was validated against a Monte Carlo model based upon the same physical concepts and this showed that the analytic model was able to represent its physics at least over the range of parameter values anticipated for cloud-free skies. The model was used to develop a parametrisation of the Monte Carlo model by applying a correction factor, which then allows the more extreme scenarios to be easily explored.

The model was validated against two empirical models and against observations, albeit noting that the analytic model still requires knowledge of the atmospheric parameter  $T_z$  and  $\rho$  that need to be determined empirically. In practice, model estimates of irradiance require  $T_z$  and  $\rho$  which are back-determined through the model itself from radiation observations. In theory, they could be determined from experiment or knowledge of the intercepting agents as in Table 1, so in this sense the model may be regarded as an a priori analytic model. As noted by a reviewer, a practical solution to this problem might be to use the model to determine  $T_z$  and  $\rho$  from radiation data and then attempt to obtain relationships between  $T_z$  and  $\rho$  with other variables such as water vapour, and aerosol loading obtained independently.

With the above qualification, the model provides predictions of diffuse and direct irradiance for scenarios with measured or hypothesised values of  $T_z$  and  $\rho$  (or  $k_a$ ,  $k_s$ ) and  $A$ . The model explains why the empirical models are successful and even provides an indication of their empirical coefficients.

If zenith transmittance  $T_z$  is assumed known (say  $\sim 0.75$  being a typical value), then rough estimates of the proportion of scattering agents to intercepting agents (say  $\rho \sim 0.5$ ) and albedo (say  $A \sim 0.2$ ) provide the input parameters for the analytic model of Eqn 14. The model then provides a simple expression to estimate clear sky diffuse irradiance for any desired zenith angle,  $n$ . In this case, Eqn 14 reduces to

$$S = 0.22 Q \cos\phi \{1 - 0.8(0.75^{\sec\phi}) - 0.2(0.75^{2\sec\phi})\} \dots 23$$

In summary, the model provides a simple analytic theory and expression to explain or to estimate broadband diffuse irradiance.

## Acknowledgments

Without the constructive criticism and comments of Dr Bruce Forgan of the Australian Bureau of Meteorology, this paper would not have been written. Professor Peter Schwerdtfeger of Airborne Research Australia is thanked for useful dialogue. Staff of the South Australian Office of the Bureau of Meteorology provided radiation data. The reviewers' criticisms have significantly improved this paper.

## References

Bird, R. and Riordan, C. 1984. *Simple Solar Spectral Model for Direct and Diffuse Irradiance on Horizontal and Tilted Planes at the Earth's Surface for Cloudless Atmospheres*. Solar Energy Research Institute, Colorado.

- Campbell, G.S. and Norman, J.M. 1998. *An Introduction to Environmental Biophysics*. 2nd Ed, Springer Verlag, NY.
- Coakley, J.A. 2003. *Reflectance and Albedo, Surface in Encyclopedia of Atmospheric Sciences* (ed. Holton, J., Pyle, J. and Curry, J.) Elsevier, Academic Press, London.
- de Wit, H. 2000. *On Approximations for the Computation of Solar Position*. Internal publication available from the Bureau of Meteorology, Adelaide, pp.12.
- Goody, R.M. and Yung, Y.L. 1989. *Atmospheric Radiation: Theoretical Basis*. Oxford University Press, New York.
- Iqbal, M. 1983. *An Introduction to Solar Radiation*. Academic Press, New York.
- Kondratyev, K.Y. 1969. *Radiation in the Atmosphere*. Academic Press, New York.
- List, R.J. 1984. *Smithsonian Meteorological Tables*, 6th ed. Smithsonian Institution Press, Washington, D.C., USA.
- Liu, B.Y. and Jordan, R.C. 1960. The interrelationship and characteristic distribution of direct, diffuse and total solar radiation. *Solar Energy*, 4, 1-19.
- McIlveen, R. 1992. *Fundamentals of Weather and Climate*. Chapman and Hall, London.
- Mischenko, M.I., Travis, D.L. and Lacis, A.A. 2002. *Scattering, Absorption, and Emission of Light by Small Particles*. Cambridge University Press, Cambridge.
- Mischenko, M.I., Travis, D.L. and Lacis, A.A. 2003. Radiative Transfer/Scattering. In *Encyclopedia of Atmospheric Sciences* (ed. Holton, J., Pyle, J. and Curry, J.) Elsevier, Academic Press, London.
- Peterson, A.W. and Dirmhirn, I. 1981. The ratio of diffuse to direct solar irradiance (perpendicular to the sun's rays) with clear skies—a conserved quantity throughout the day. *Jnl appl Met.*, 20, 826-8.
- Schwerdtfeger, P. 1995. *Radiation in the Atmosphere*. Flinders University of South Australia, Adelaide.
- Solar Radiation Monitoring Laboratory 2002. University of Oregon <http://solardat.uoregon.edu/EducationalMaterial.html>
- Taha, H. 1994. How Shiny is L.A.? Recent Research in the Building Energy Measurement and Performance Analysis, Lawrence Berkeley laboratory available at: <http://eetd.lbl.gov/ea/buildings/RRResearch/LA.html>