Temperature variability in a changing climate

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Here we examine probability density functions (PDFs) of temperature variability over multi-decadal periods under idealised global warming scenarios. Such PDFs are expected to be more useful to managers of infrastructure and systems with a finite multi-decadal lifetime, than would PDFs for a specific future year (2030 or 2100 say). We suppose that the PDF of temperature variability at any given time $t_1 > t_0$ retains its initial normally distributed shape, i.e. PDF($T(t = t_0)$), with the same standard deviation, but is shifted to the right by an amount $\mu = \mu(t)$. We refer to each of these PDFs, valid for individual times, as the individual PDFs. The PDF representing variability over the entire period $t_0 \leq t \leq t_1$ and not just at some instant within this period, i.e. the aggregated PDF (APDF), is broader than the instantaneous PDF and so the range of temperatures experienced over a finite period is greater than in an unchanging climate. The APDF is normally distributed if warming occurs at a linear rate, but is skewed towards higher temperatures if warming accelerates over the period of interest. Changes in APDFs of Australian mean minimum temperatures are generally consistent with this simple conceptual model. Idealised warming in which the variance of the individual PDFs increases is also considered.

Popular conceptual models of the impact of global warming on temperature variability are relevant for instantaneous rather than aggregated PDFs. Such models neglect the straightforward but important issues that arise when finite periods and non-linear warming rates are considered.

Introduction

Concern about how climate variability in the future might be affected by global warming is widespread. A simple, powerful and commonly cited (standard) conceptual model for the impact of global warming on temperature variability is embodied by the supposition that current variability has a normal or Gaussian probability density function (PDF) and that global warming shifts this distribution to a higher temperature without altering its shape (e.g. White et al. 2001). This model highlights the possibility that the likelihood of exceeding high temperatures can be increased

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substantially by what at first glance might appear to be a modest overall shift in the mean. Of course this is merely a conceptual model and need not be relevant at all locations. In some locations global warming might affect the shape of the distribution. This can occur in regions where sea-ice or snow cover changes, for example (Yonetani and Gordon 2001).

Strictly speaking the standard conceptual model of two identical normal distributions with different means applies to two different situations. In the first situation there are two different climates in statistical equilibrium. The second situation – of direct relevance to this study and of more practical interest – arises if the climate is in transition. In this situation the two PDFs represent variability at two different times $t = t_0$ and $t = t_1 > t_0$. Here t_0 might correspond to the current year, say 2007, while t_1 might equal say 2030. However, engineers, water managers and others sometimes seek advice from climate scientists on how variability might look over the lifetime of infrastructure they will manage in the future, e.g. the coming 30 years rather than 2030 alone. Despite this, most attention by climate scientists to date has been on specific times in the future: 2030, 2050, 2100, etc. These future PDFs are useful guides but information more explicitly targeted to the full period, covering the period of interest, might be more useful to the decision maker.

Here we examine the statistics of temperature variability over multi-decadal periods for highly idealised global warming scenarios. We will show that the simple standard conceptual model (White et al. 2001) can be readily extended to provide a new model for the variability over multi-decadal periods. The results we obtain may surprise some readers while other readers may find the results obvious. Either way, the extended conceptual model does not appear to have been documented previously, nor does it appear to be widely appreciated. This paper is intended to rectify the situation.

The conceptual model

Suppose that global warming proceeds at a known rate, and that the PDF of temperature variability at any given time $t > t_0$ retains its shape as a normal distribution, but is shifted to a higher temperature $\mu = \mu(t)$, where μ is a monotonically increasing function of t. This means that the PDF at any time $t > t_0$ will have exactly the same normal shape, with the same standard deviation as PDF($t = 0$), though its mean, $\mu = \mu(t)$, will, in general, be different. Suppose that APDF represents the aggregated PDF of variability over the lifetime of the infrastructure whose lifetime extends from $t = t_0$ to $t = t_1$. The aggregated PDF representing temperature variability over this period is equal to a weighted average of a succession of identical but displaced PDFs with means $\mu = \mu(t)$. If we refer to the individual distributions as PDF(t), then APDF is given by:

$$\mathrm{APDF}\{T(t); t_0 \le t \le t_1\} = \frac{\alpha \int_{t=t_0}^{t=t_1} e^{-\left[\{T-\mu(t)\}^2/2\sigma^2\right]} dt}{\int_{t=t_0}^{t=t_1} t\, dt} \qquad \ldots 1$$

where T is the temperature at a given location, or the temperature averaged over a given region, e.g. where the infrastructure or system of interest is located. The symbol α is a normalisation coefficient chosen to ensure that the area under APDF(T) equals 1.0.

Linear and accelerated warming

In this section we calculate the APDF for the linear and accelerated warming scenarios presented in Fig. 1. To do this, the 50-year period was first broken up into 1000 increments with 1000 corresponding individual PDFs. We then averaged these individual PDFs to obtain the APDF. The APDF corresponding to linear warming at a rate of 2σ/50 years is presented in Fig. 2. Here σ is the standard deviation of each individual PDF(t). Three curves are shown (a) PDF($t = t_0$), (b) APDF covering the full period $t = t_0$ to $t = t_1$, and (c) a normal distribution which has the same median as APDF but the same shape as PDF($t = t_0$). As mentioned previously, the APDF represents the PDF of variability over the 50-year period from $t = t_0$ to $t = t_1$. Curve (c) is shown so it can be compared with the APDF. The standard conceptual model applies for a specific time, but does not faithfully represent the PDF of variability that will occur over the lifetime of the infrastructure, i.e. APDF, because the APDF is broader than the PDF($t = t_0$). This must be the case since the APDF represents the average of a succession of normal distributions with different means. Notice, however, that the APDF remains normally distributed in the case of linear warming.

The APDF for the non-linear warming scenario presented in Fig. 1 is shown in Fig. 3. The APDF is again broader than the PDF($t = t_0$), and is now skewed to the right. This is because less time during the 50-year period is spent with high values of μ, and more time is spent with low values for the scenario considered. The PDFs with low values of μ are therefore weighted more heavily in Eqn 1, and so a greater proportion of the area under the APDF is assigned to lower values of T.

Fig. 1 Time series of the mean temperature considered in the simple scenarios, expressed in units of standard deviations per 50 years. In these examples $t_0 = 0$ and $\sigma = 1/\sqrt{2}$. Linear warming (orange) has a changing mean value of $\mu(t) = 2\sigma t/50$. Cubic warming (red) has a changing mean value of $\mu(t) = 2\sigma(t/50)^3$, where t is the time in years and σ is the standard deviation of T variability in PDF($T(t = 0)$).

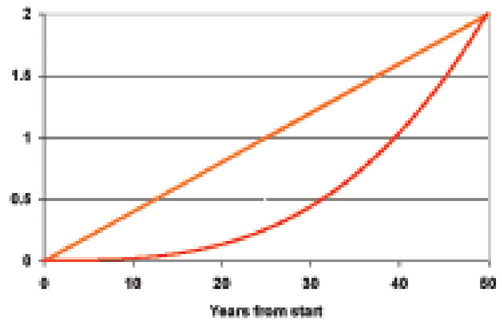
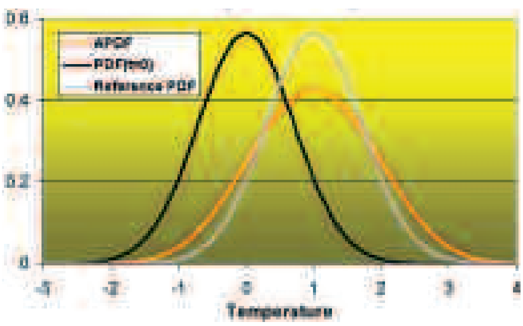


Fig. 2 Probability density functions (PDFs) under linear warming (as depicted in Fig. 1). Individual PDF($t = 0$) representing T variability at time $t = 0$ (black); aggregated PDF, APDF representing variability over the full 50-year period (orange). The standard deviation of PDF(T) has been set to $1/\sqrt{2}$. The reference PDF is a normal distribution with the same shape as PDF($t = 0$) but with the same median value as the APDF. It is shown for comparison (grey).



The broadening and skewing effect leads to a marked increase in the likelihood of exceeding high values of T compared with expectations based on PDF($t = 0$) or a normal distribution with either the same median or the same mean as the APDF. While this effect occurs in the standard conceptual model, it is more marked if variability over the lifetime of the

Fig. 3 Probability density functions (PDFs) for non-linear (cubic) accelerated warming (as depicted in Fig. 1). Individual PDF($t = 0$) (black); aggregated PDF, APDF (red). The standard deviation of PDF(T) has been set to $1/\sqrt{2}$. The reference PDF is a normal distribution with the same shape as PDF($t = 0$), but with the same median value as the APDF. It is shown for comparison (grey).

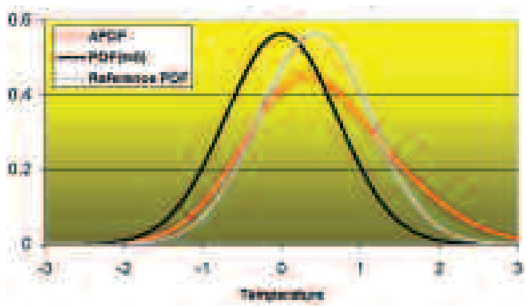
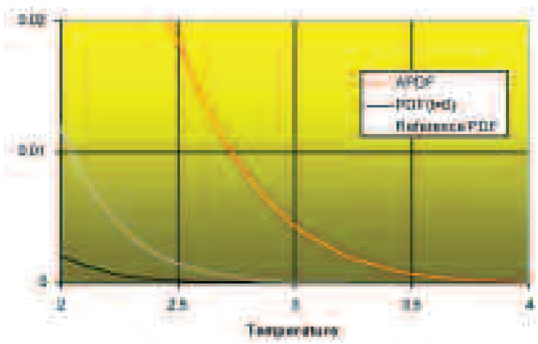


Fig. 4 Three different estimates of the chance of exceeding extreme (scaled) temperatures using: (a) the individual PDF($t = 0$) (black); (b) the aggregated PDF, APDF (red); and (c) a reference PDF that is identical to PDF($t = 0$) but has a median value that is the same as the median value of the APDF (grey). The chances of exceeding 2.5σ, where σ is the standard deviation of PDF($t = 0$) are approximately: (a) 0.00018, (b) 0.0015, and (c) 0.018, respectively. Note that the last value is 100 times larger than the value for (a), and approximately 12 times larger than the value for (b). The APDF corresponds to the accelerated warming scenario considered in Fig. 3.



infrastructure is considered. This is illustrated in Fig. 4 which shows the chance of exceeding high temperature values estimated using PDF($t = t_0$), APDF($T(t)$; $t_0 \le t \le t_1$) and a normal distribution with the same shape as the PDF($t = t_0$), but with the same median value as the APDF. The case of accelerated warming is again considered here. Notice how the chance of exceedance for any given high value of T is substantially enhanced relative to the shifted normal distribution, i.e. if the standard conceptual model is applied.

Note that if σ increases with time then the broadening of the APDF will be greater and so the probability of exceeding a given threshold will increase further. If a changing σ is accompanied by linear warming then the APDF is not necessarily normally distributed. This is illustrated in Fig. 5. In this example mean warming occurs at the same rate as before, but the standard deviation of the individual PDFs increases linearly with time, with the variance approximately tripling over the 50-year period. In this case the deviations from normality are modest, but this is not necessarily the case if changes in σ are more dramatic. Non-linear changes in the variance will also lead to further deviations from normality in the APDF. For example, if the variance changes rapidly to a different value at year 25, then the APDF (not shown) will then have two peaks.

Fig. 5 **Probability density functions (PDFs) for linear warming in the case where the standard deviation of temperature variability in the individual PDFs increases linearly, with the variance increasing by a factor of 2.92 over 50 years. Individual PDF(t = 0) (black); aggregated PDF, APDF (red). The standard deviation of PDF(T) is $1/\sqrt{2}$. A normal distribution with the same mean and variance as the APDF is shown for comparison (grey).**

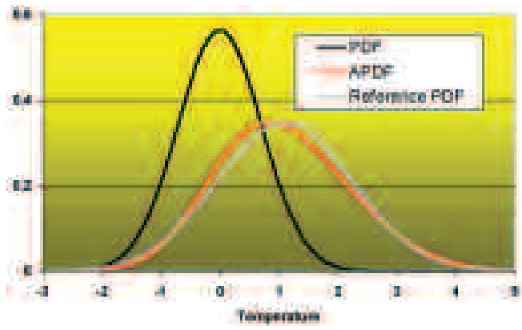
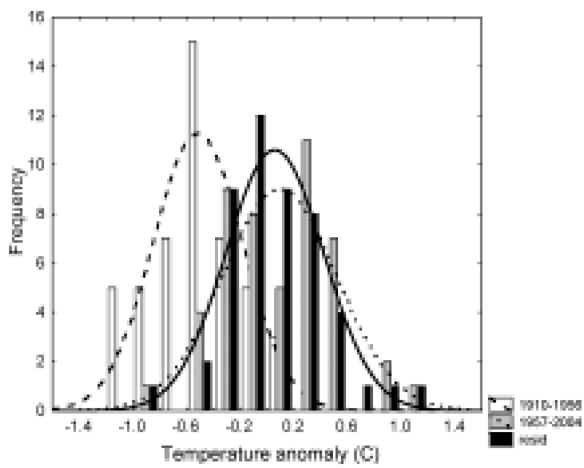


Fig. 6 **Aggregated PDFs of observed all-Australia mean minimum temperature for the periods 1910-1956 (open columns, dashed line) and 1957-2004 (shaded columns, dotted line). The third aggregated PDF (black columns, solid line) represents detrended variability during the period 1957-2004. The normal distributions represent best fits to each subset of the data.**



Observational evidence in Australia

In this section we will show that interdecadal changes in PDFs of Australian temperatures can be explained using the conceptual model developed above. We will analyse PDFs of high-quality annual mean minimum temperatures, averaged across Australia. The data are available from http://www.bom.gov.au/silo/products/cli_chg/. These data show an approximately linear warming trend of about 0.5ºC since the middle of the 20th century. To examine the effects of this warming trend on the APDF, the data were split into two periods, 1910-1956 and 1957-2004. The first period shows little if any warming trend, whereas the second exhibits strong warming. The APDFs of the two subsets of the data are shown in Fig. 6, which indicates that normal distributions provide adequate though imperfect representations of the data for both periods. The APDF of the second subset is broader and flatter than the APDF of the first subset. How much of this broadening is due to the trend? To estimate this, the linear regression between year and temperature was calculated using the 1957-2004 data. Then for each year 1957-2004 the temperature estimated from this linear trend was subtracted from the actual temperature leaving a 'residual' temperature after removal of the warming trend. The APDF of this residual subset is also shown in Fig. 6. The APDF of the residual subset is narrower than that of the original 1957-2004 subset. The standard deviations of the 1910-1956, 1957-2004, and residual subsets are 0.33, 0.43, and 0.36 respectively. That is, the residual dataset variability is closer to the variability exhibited in the 1910-1956 dataset than is the case with the original

1957-2004 subset. Thus, the trend has contributed to a broadening of the APDF in the second half of the 20th century. If, in the mid-20th century, an engineer had estimated the APDF of temperature expected with a warming of about 0.6ºC simply by shifting the mean by 0.6ºC and using the shape and variability of the 1910-1956 APDF, then this would have misrepresented the actual APDF of the second half of the 20th century. If temperatures continue to rise through the 21st century, as they are expected to do (IPCC 2007), then simply assuming a shift in the location of either PDF($t = t_0$) or past APDFs will also result in an underestimation of the variability over the coming century. Note again that this is the case even if there is no change in the shape of the instantaneous PDF that applies at any given time t.

Some skewing towards the right is also evident in the APDF of the raw 1957-2004 subset. This is again consistent with the extended conceptual model with accelerated warming. The degree of skewing evident in Fig. 6 is not great, so we cannot rule out the possibility that it is merely fortuitous. However, accelerated warming in the late 20th century is also evident in climate model simulations (IPCC 2007).

Conclusions

In this study we have extended an existing simple, but very powerful, conceptual model (White et al. 2001) to provide an estimate of the probability density function that applies to temperature variability over extended multi-decadal periods in a warming climate. We referred to this as the aggregated PDF or the APDF. The APDF may be of more interest to managers of infrastructure that will exist over the extended period than will individual PDFs appropriate for future individual years – say 2030 or 2010.

In order to provide a useful conceptual model which we can use to estimate the APDF we assumed that the expected mean temperature at any given time in the future (i.e. the 'expectation value' of temperature at time t) is given by $\mu = \mu(t)$, where μ is a monotonic increasing function of t, and that the individual PDF at any given time $t > t_0$ has the same shape and standard deviation as PDF($t = t_0$) i.e. the PDF that applies at time $t = t_0$.

We showed that the APDF is broader than the PDF($t = t_0$) if warming occurs (i.e. $\mu > 0$), and that the APDF is normally distributed if warming occurs at a linear rate. However, the APDF becomes skewed towards higher temperatures if warming accelerates over the period of interest. The range of variability that infrastructure must withstand is necessarily increased. Warming therefore renders temperature PDFs estimated from long historical data inaccurate. Changes in multi-decadal PDFs of Australian mean minimum temperatures are consistent with this simple model – the APDF for the period 1957-2004 is broader than the APDF for the period 1910-1956. The broadening is due to the linear trend that occurs during the latter period. The PDF covering the latter period also exhibits skewing towards higher temperatures, consistent with accelerated warming.

Warming, especially accelerated warming, can lead to large underestimates in the probability of exceeding high temperatures over future multi-decadal periods, if historical records are used to estimate probabilities. This amounts to using a past APDF to estimate a future APDF. These issues also influence the quantification of how exceptional very recent high temperature records actually are (e.g. Trigo et al. 2005). Suppose, for example, that a new record T^* is set at time $t = t_2$. If we estimate the likelihood of obtaining or exceeding T^* using the historical record developed over the period t_0 to t_1, where $t_0 < t_1 < t_2$, then this amounts to using APDF for the period $[t_0, t_1]$ to estimate APDF(t_2). This will again lead to an underestimate of the probability that T reaches or exceeds T^*, especially if the warming is accelerating. In this instance, this may in fact be the point of the exercise, i.e. to show that recent 'extraordinary' events may seem less extraordinary if global warming is taken into account. Nevertheless the framework outlined here, namely explicit consideration of the two different kinds of probability density functions – the aggregated PDF (APDF) and the individual PDF (PDF(t)) – again helps to clarify the nature of the analysis being undertaken. When PDFs are presented confusion will arise unless the precise nature of the PDF being discussed is specified.

While some readers may regard these results as surprising, others may regard them as obvious extensions of the existing standard conceptual model (as espoused by White et al. (2001) for example). Whatever the case, the potentially important extension provided here does not appear to have been noted previously, and does not appear to be widely appreciated.

Finally, note that the APDF, the probability density function of temperature variability aggregated over a given multi-decadal period, only provides part of the story. Its use would, for example, yield over-estimates of the magnitude of the year-to-year variability possible, for example, since the extremely low values of T are more likely early while the extremely high values of T are more likely later in the period of interest. The full story is given by a sequence of individual PDFs, one for each time t. This particular issue might be an important consideration in some contexts.

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