Australian Meteorological and Oceanographic Journal 60 (2010) 91-101

A climatology of pressure jumps around the Gulf of Carpentaria

Ilona Nudelman[1], Roger K. Smith[1] and Michael J. Reeder[2]
[1]Meteorological Institute, University of Munich, Germany
[2]School of Mathematical Sciences, Monash University, Melbourne, Australia

(Manuscript received June 2009, revised January 2010)

We present a climatology of pressure jumps, defined as a sharp rise in pressure of at least 0.3 hPa in three minutes, at stations around the Gulf of Carpentaria, based on one-minute data obtained from Bureau of Meteorology automatic weather stations in the region. We examine also data from one inland station, Daly Waters, which lies about 265 km west of the Gulf. Many of the pressure jumps are associated with bore-like disturbances, while others accompany sea-breeze passages, or mark the onset of thunderstorm gust fronts. During the latter half of 2006, data are available at a sufficient number of stations to estimate the speed and direction of the disturbances. Bore-like disturbances tend to have two main directions of travel. The most common propagate predominantly from the northeastern sector and are associated with morning glory disturbances that originate over Cape York Peninsula. However, a number of disturbances originate south of the Gulf and subsequently propagate towards the north or northwest. These southerly morning glory disturbances are associated with the passage of cold fronts across central Australia, or are spawned within the inland trough following strong ridging across the continent. Of the 21 bore-like disturbances that were recorded at three or more stations during the four-month period August-November 2006, sixteen were from the northeastern sector and five originated south of the Gulf. One of the latter was recorded as far west as Daly Waters, where weak sea-breeze disturbances are common also, even though Daly Waters lies some 265 km inland from the Gulf. Only one northeasterly disturbance was identified at Groote Eylandt in the northwest of the Gulf.

Introduction

The Gulf of Carpentaria region of northeastern Australia is noted for the frequent occurrence of a range of cloud-line disturbances. Some of these cloud lines are convective, while others are low-level rolls linked to travelling wave or bore-like disturbances and associated with the so-called morning glory. A review of these cloud lines together with a comprehensive list of references is given by Reeder and Smith (1998).

Morning glory cloud lines are observed most commonly in the late dry season (September-November) and are primarily of two types: those originating from the east coast sea-breeze over Cape York Peninsula and those originating inland from the sector southwest to southeast. Some of these so-called southerly morning glories have been linked to cold fronts crossing central Australia (e.g. Smith et al. 1982, 1986, 1995, 2006; Deslandes et al. 1999; Reeder et al. 2000). Occasionally northeasterly morning glories are observed out of season, but most documented cases of disturbances from the south are confined to the late dry season.

As the name suggests, morning glory cloud lines are confined largely to the morning hours[1]. They are accompanied at the surface by a sharp pressure jump, typically on the order of 1 hPa in a few minutes, followed by a series of waves in the surface pressure and a steady rise in the mean pressure. This behaviour is reminiscent of an undular bore. The passage of the pressure jump is usually followed by an

---

Corresponding author address: Roger K. Smith, Meteorological Institute, University of Munich, Theresienstr. 37, 80333 Munich, Germany.
Email: roger.smith@lmu.de

[1] Around the northwestern coast of Cape York Peninsula the disturbances are observed in the late evening.

abrupt change in wind direction, and a sharp increase in wind speed, but often little change in temperature or dew-point temperature. Normally these disturbances can be distinguished from other types, such as sea-breeze fronts or thunderstorm gust fronts, which tend to be accompanied by a sharp fall in surface temperature also and, in the former case at least, by an increase in the dew-point temperature. Two examples are shown in Fig. 1. The first (panel a) is a typical morning glory signature, in this case at Borroloola, an inland station on the southwestern side of the Gulf. The second (panel b) is an example of a thunderstorm passage at Centre Island, just to the north of Borroloola. (See Fig. 2 for a map of the area and the locations of places mentioned in the text.) In contrast to the morning glory, which has little signature in the temperature or dew-point, the thunderstorm is accom-panied by a sharp fall in temperature and, in this case, the winds gradually return to their pre-disturbance values after about two hours.

Fig.1 Examples of pressure jumps and other data (temperature, dew-point temperature, wind speed and direction) observed by Bureau of Meteorology automatic weather stations: (a) a dry-season morning glory convergence line at Borroloola, (b) a wet-season thunderstorm gust front at Centre Island. Times are local time (Central Standard Time).

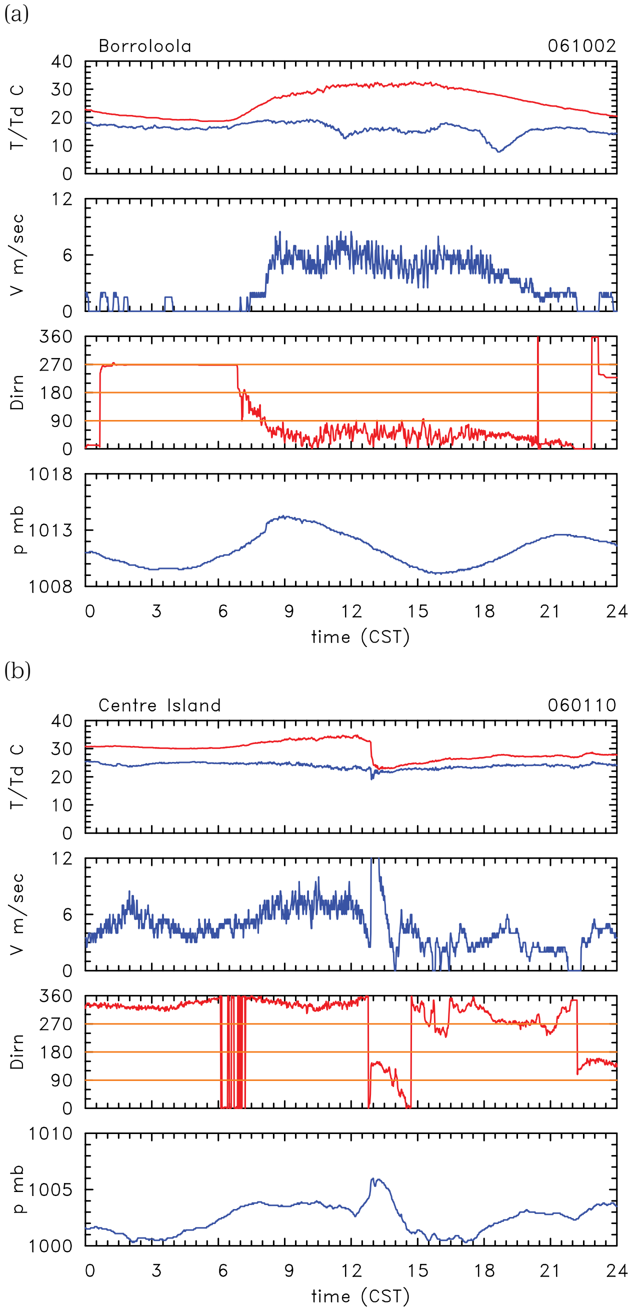
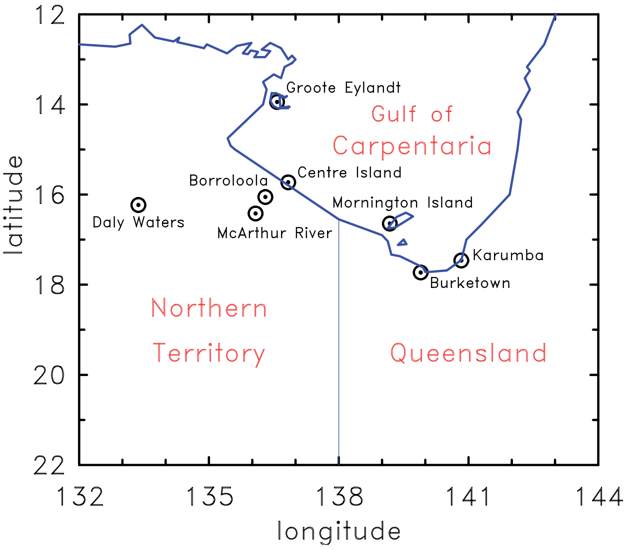


Fig. 2 Map of the Gulf of Carpentaria region with place names mentioned in the text.



To our knowledge there have been no climatological studies of the frequency of any type of pressure disturbance and, until recently, the database for such a study was hardly adequate. However, starting in 2005, the Northern Territory and Queensland Regional Offices of the Australian Bureau of Meteorology began installing data loggers on their automatic weather stations (AWSs) in the Gulf region and the one-minute surface data provided by these stations make it possible to undertake a climatological study of morning glory disturbances. The aim of the present paper is to analyse the climatology of pressure jumps at stations around the southern part of the Gulf, using the data available from the AWSs in 2006.

Climatology and diurnal distribution

Figure 3 shows the frequency of pressure jumps recorded at Borroloola, Centre Island and McArthur River during 2006. Here a pressure jump is defined as a sharp rise in pressure of at least 0.3 hPa in a period of three minutes. Pressure jumps were recorded in all months except July, with only one in June and two in May. The months with the largest numbers were January and February.

Fig. 3 Climatology of pressure jumps observed at Borroloola (BL), Centre Island (CI) and McArthur River (MR).

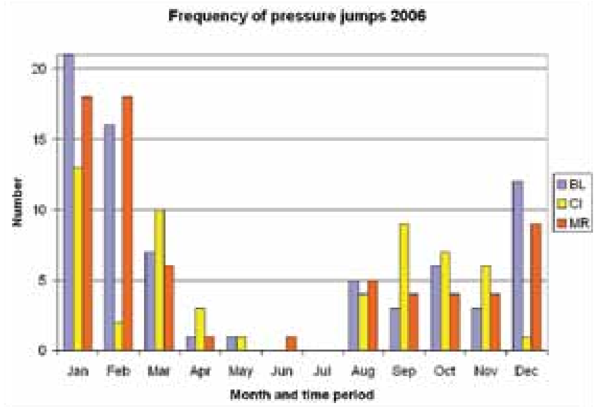


Figure 4 shows the statistics for each station, with frequencies binned into three-hour periods. While a few pressure jumps were detected between midnight and 9 am in January and February, the majority occurred later in the afternoon and evening, and these are likely to have been related to sea-breezes or the gust fronts from thunderstorms, some of which may develop on or near the sea-breeze front. In March there were a few early morning pressure jumps at Centre Island, but none at Borroloola or McArthur River. Between April and July there were only three early-morning pressure jumps, two at Centre Island and one at Borroloola. The one on 23 May was detected at both of these stations. In December there were two early morning occurrences at Centre Island and one at Borroloola. In contrast, all of the pressure jumps between August and November occurred before 9 am and are likely to have been accompanied by morning glory disturbances of one type or another.

A similar situation is found at Mornington Island and Burketown on the southeastern side of the Gulf (Fig. 5 (a), (b)). Data from Mornington Island are available only for the period 1 August to the end of December 2006 and those at Burketown from 15 September to the end of December with two gaps, one of ten days between 14 and 24 October the other of seven days between 12 and 21 November. At Mornington Island, 52 disturbances were identified and 39 were identified at Burketown. When the objective criterion for a pressure jump was reduced to 0.25 hPa in three minutes, the number of disturbances detected at these two stations increased by 27 and 19, respectively, i.e. by about 50 per cent. When the objective criterion was increased to 0.6 hPa in three minutes, the number of disturbances at Mornington Island reduced to 11 and the number at Burketown reduced to 16.

A subset of the data for the period August to December, including the times of passage of bore-like disturbances that were recorded at three or more stations, is detailed in Table 1. For these disturbances it is possible to estimate their speed and direction as discussed later. Table 1 includes a few disturbances that were identified by surface pressure data recorded in Karumba during a small field experiment in late September/early October and a few for which the pressure jump was smaller than the above criteria. All 21 disturbances in the table appear to be morning-glory-type disturbances, having little signature in the temperature and humidity fields. All except one of these was detected by the criterion of a pressure rise of 0.25 hPa or more in three minutes at Burketown and all

Fig. 4 Climatology of pressure jumps observed at (a) Centre Island, (b) Borroloola and (c) McArthur River. Times are local time (Central Standard Time).

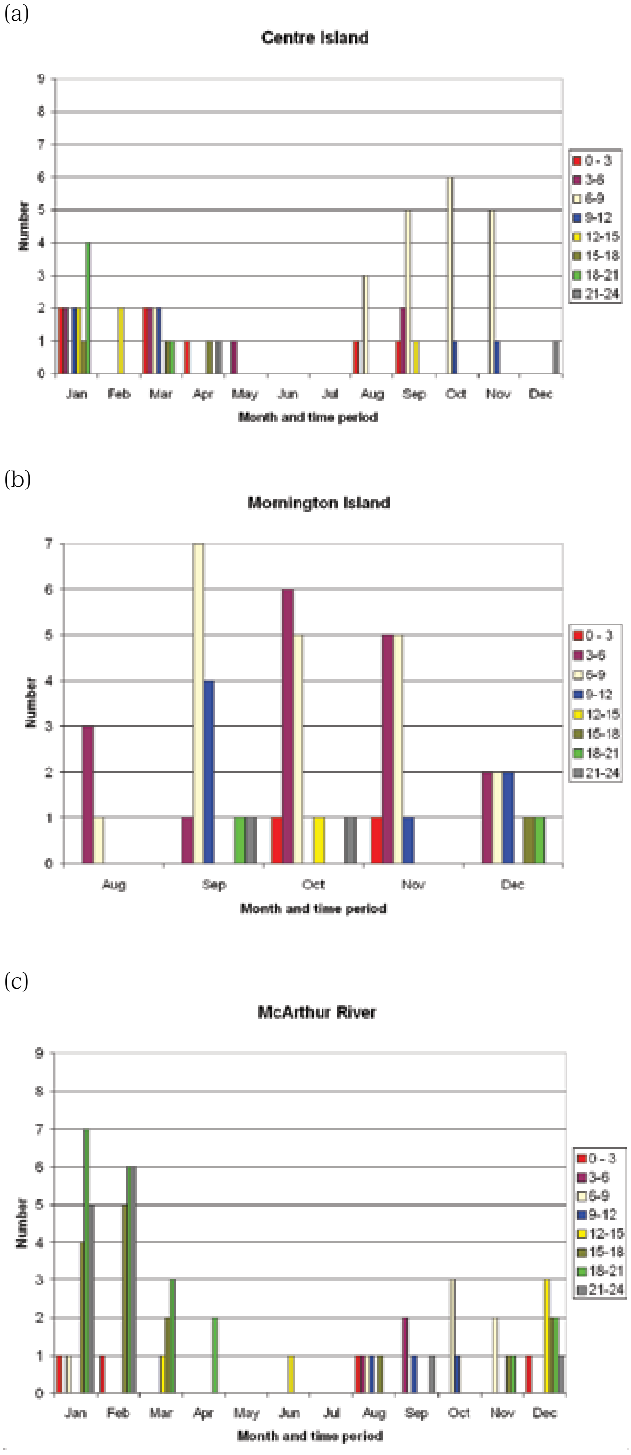
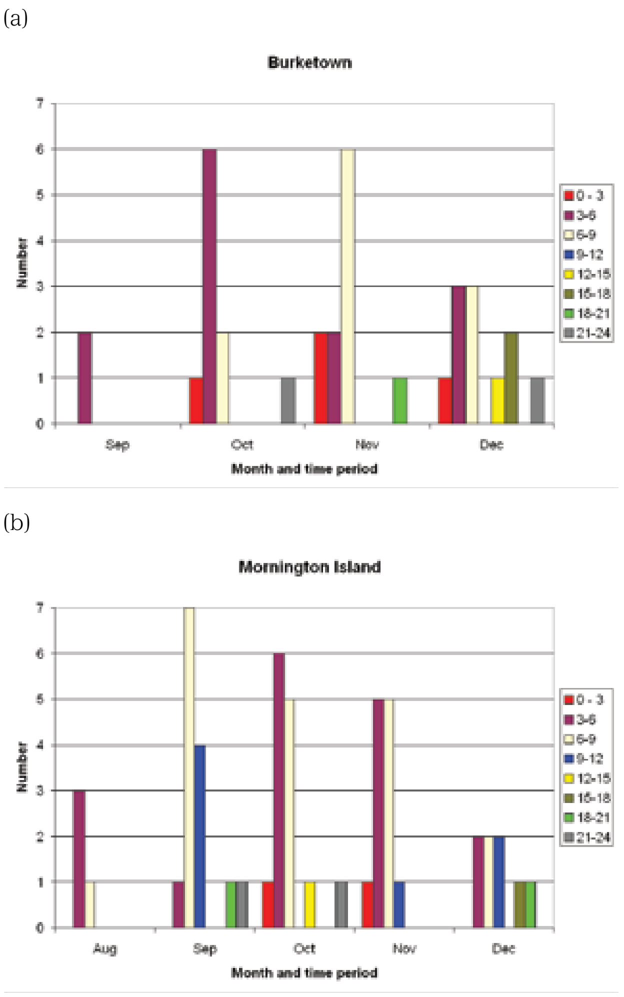


Fig. 5 Climatology of pressure jumps observed at (a) Burketown and (b) Mornington Island. Times are local time (Eastern Standard Time).



but four were detected at Mornington Island. In contrast, the criterion of a pressure rise of 0.3 hPa in three minutes missed three of the disturbances at Burketown and ten at Mornington Island, suggesting that the criterion of a 0.25 hPa pressure rise is, perhaps, slightly better for capturing morning glory disturbances than 0.3 hPa in three minutes.

Two other stations for which data are available in the second half of the year are Daly Waters and Groote Eylandt. Daly Waters is interesting as it lies far inland, about 265 km from the Gulf coast, and was the site of the Koorin Experiment, a major boundary-layer experiment held in 1974 (Clarke and Brook 1979). As a result, there is something known about the disturbances that occur there. Groote Eylandt, located on the western side of the Gulf, is interesting in view of the remark by Clarke (1983, p. 139) that it is not known whether morning glory disturbances ever occur there. He noted also that 'barograph traces from Alyangula, on the western side of Groote Eylandt, showed no pressure jumps', but did not say over what length of time such charts had been examined. The data from these two stations are discussed briefly below.

Far inland station, Daly Waters

This station is probably too far west to be influenced by northeasterly morning glory disturbances because they would have to cross the land during daylight hours, well after the waveguide provided by the low-level nocturnal inversion has been destroyed by strong, dry convective heating. Nevertheless, the AWS record between 1 July and 30 November 2006 indicates the regular occurrence there of, mostly weak, nocturnal pressure jumps. A large number of these (21 out of 43) were accompanied by an increase in wind speed and a shift in wind direction to the northeast sector. These are almost certainly decaying sea- breezes from the Gulf of Carpentaria. Indeed a few were accompanied also by a fall in temperature and an increase in dew-point temperature, characteristic of a sea-breeze front. Similar sea-breezes were recorded frequently during the Koorin experiment (Garratt 1985; Garratt and Physick 1985). The criterion of a 0.3 hPa rise in three minutes adopted here missed at least an additional 15 disturbances of this type. Of the other 22 disturbances, 13 were probably thunderstorms, which occurred in November and December, eight were hard to classify, and one, the event of 6 October, was clearly associated with a disturbance that was recorded also at stations around the Gulf region. The AWS data for the 6 October event are shown in Fig. 6(a). This event is significant as it has always seemed likely that southerly disturbances observed in the Gulf region should occur also south of the Top End of Australia. In fact, a southerly disturbance was observed at Daly Waters during the Koorin experiment (Clarke 1983), but the event of 6 October is possibly the first real evidence that these disturbances extend all the way to the Gulf.

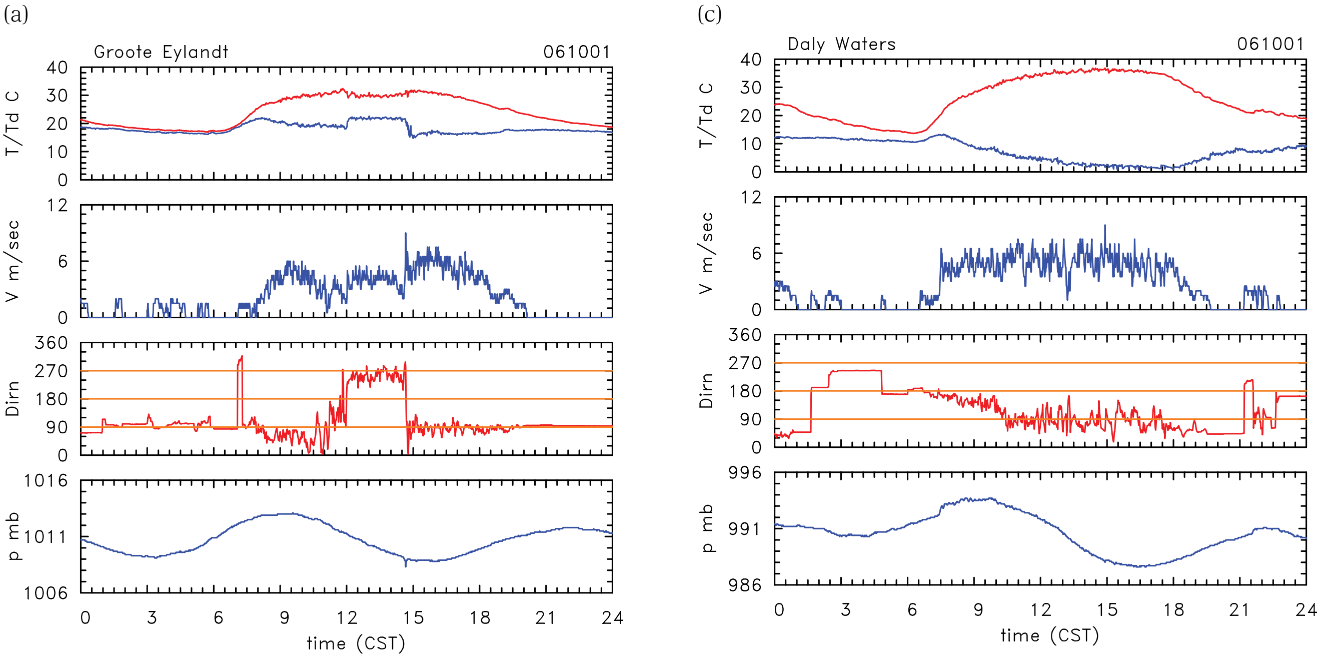
Far northern station, Groote Eylandt

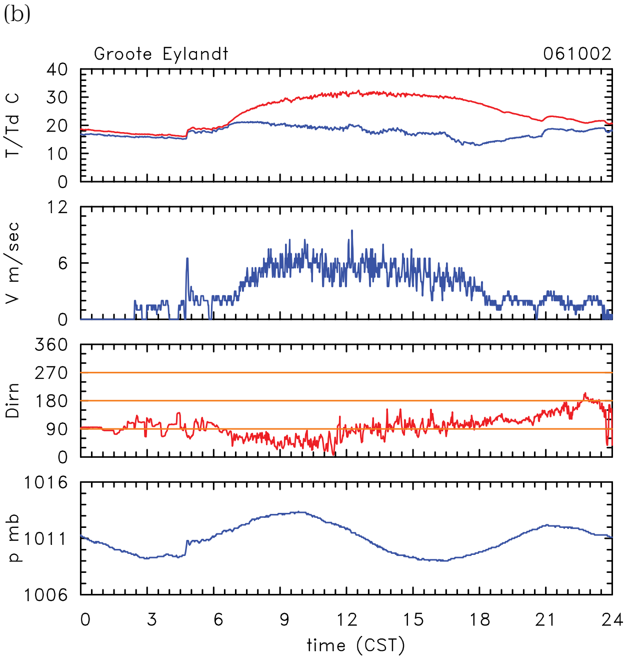
The northernmost station investigated is Groote Eylandt. Thirty-nine pressure jumps were recorded there during 2006. Twenty-five of these occurred between January and March and in November and December. The majority of these wet season events were probably associated with heavy showers or thunderstorms. Most of them occurred in the afternoon or evening. Many of the dry season disturbances at this station did occur in the early morning, but most are difficult to classify in the absence of data from nearby locations. Only one of these appeared to have been connected to a disturbance recorded at stations further south, indicating that morning-glory-like events do not normally extend as far northwest as this station. The exception was the disturbance at 0515 Central Standard Time (CST = UTC + 9.5 hours) on 2 October which was accompanied by a sharp pressure jump of over 1 hPa and a one-minute gust of nearly 7 m s^{-1} (Fig. 6(b)). On occasions there were signatures of a west coast sea-breeze (the AWS on Groote is located a few km inland from the west coast of the island), a good example being that on 1 October (Fig. 6(c)). The westerly sea-breeze became established just after noon, replacing northeasterly winds with westerlies and leading to a sharp increase in dew-point temperature of about 2°C. The sea-breeze retreated at about 1445 CST and was

Table1. Times of passage of bore-like disturbances which were recorded at a minimum of three stations in the southern Gulf region between August and November 2006. The second column gives the times of passage at individual stations: BL Borroloola, BT Burketown, CI Centre Island, DW Daly Waters, GI Groote Eylandt, KA Karumba, MI Mornington Island, MR McArthur River. All times are given in Eastern Standard Time. The third and fourth columns give estimates of the speed and direction of these disturbances, based on all stations or particular groups of stations as indicated. The direction is the usual meteorological format, meaning the direction in degrees from which the disturbance moves measured clockwise from north.

Date	*Stations and times*	*Speed*	*Direction*
11 Aug	MI 0525 CI 0755 BL 0917 MR 1004	18	45
	CI 0735 BL 0917 MR 1004	6	110
18 Aug	MR 0522 BL 0628 CI 0730 MI 0811	18	205
	MR 0522 BL 0628 CI 0730	16	250
28 Aug	MI 0532 MR 0900 CI 0925 BL 0934	17	140
	MR 0900 CI 0925 BL 0930	8	140
07 Sep	MR 0521 BL 0611 CI 0655	24	240
19 Sep	BT 0242 MI 0355 CI 0914 BL 1016	14	105
23 Sep	KA 0415 BT 0711 MI 0802	13	95
24 Sep	KA 0419 BT 0846 MI 0920	9	95
26 Sep	KA 0145 BT 0720 MI 0639 CI 1040	9	60
	KA 0145 BT 0720 MI 0639	7	75
27 Sep	KA 0019 BT 0259 MI 0430 MR 1018	15	120
	KA 0019 BT 0259 MI 0430	13	110
28 Sep	KA 0108 MI 0419 BT 0415	13	85
30 Sep	KA 0150 MI 0445 BT 0518 CI 0830 BL 1020 MR 1130	13	65
	KA 0150 MI 0445 BT 0518	11	75
	CI 0830 BL 1020 MR 1130	5	110
01 Oct	KA 0140 MI 0446 BT 0513 BL 0950 CI 0825 MR 1030	15	70
	KA 0140 MI 0446 BT 0513	11	75
	BL 0950 CI 0825 MR 1030	6	115
02 Oct	KA 0005 MI 0315 BT 0331 CI 0730 BL 0836 MR 0913 GI 0510	12	60
	KA 0005 MI 0315 BT 0331	12	75
	CI 0730 BL 0836 MR 0913	8	110
03 Oct	KA 0010 BT 0255 MI 0300 CI 0714 BL 0837 MR 0913	16	80
	KA 0010 BT 0255 MI 0300	14	85
	CI 0714 BL 0837 MR 0913	5	120
06 Oct	KA 0050 BT 0430 MI 0400	11	75
	MR 0655 BL 0824 DW 0807	18	180
07 Oct	BT 0545 MI 0450 CI 0710 BL 0945 MR 1045	12	50
	CI 0710 BL 0945 MR 1045	3	120
14 Oct	MI 0518 BT 0536 CI 0837	20	75
19 Nov	MI 0420 CI 0700 BL 0836 MR 0930	16	50
	CI 0700 BL 0836 MR 0930	5	110
20 Nov	MI 0358 CI 0808 BL 0925 MR 0955	17	85
	CI 0808 BL 0925 MR 0955	6	120
22 Nov	MI 0630 BT 0652 CI 1011	18	75

Fig. 6 Examples of pressure jumps and other data (temperature, dew-point temperature, wind speed and direction) observed at Bureau of Meteorology automatic weather stations: (a) Daly Waters on 6 October, (b) Groote Eylandt on 2 October, and (c) Groote Eylandt on 1 October 2006. Times are local time (Central Standard Time).





replaced by stronger easterlies, presumably enhanced by the east coast sea-breeze on the island. Note that these sea-breeze onsets are not accompanied by significant pressure jumps, presumably because the island is small and the depth of the cooler air is relatively shallow. Most of the other disturbances in the early morning tended to be accompanied by only weak pressure jumps, with less than a 0.3 hPa rise in three minutes.

Speed and direction of disturbances

For those disturbances that were recorded at three or more stations, it is possible to estimate their speed and direction on the assumptions that the disturbance is unique and that the disturbance front is straight and moves with a uniform speed. Mathematical details of the speed and direction calculations are given in the Appendix. If the passage is recorded at just three stations that do not lie in a straight line, the speed and direction are determined uniquely, while, if it is recorded at four or more stations, it is necessary to apply a minimisation procedure to fit a straight line moving with uniform speed. This procedure yields two possible solutions for the speed and direction. The best fit is the solution that gives the smallest mean-square error in the times of passage at individual stations. The assumption that the disturbance front is straight and moving with a uniform speed will be best satisfied when the disturbance is recorded at three stations that lie relatively close together, but not close to a straight line.

The times of passage for this subset of disturbances for the period from August to December are summarised in Table 1 and the calculated speeds and directions are shown diagramatically in Fig. 7. A few of the disturbances at some stations listed in Table 1 do not satisfy the criterion that the pressure jump is at least 0.3 hPa in three minutes. In these cases, the disturbance passage was clear from an abrupt change in wind direction and often a sharp increase in wind speed. During the above period, 21 disturbances were recorded at three or more stations. In one case, that of 6 October, two different disturbances were recorded, one from the northeast sector and one from the south. Of the 21

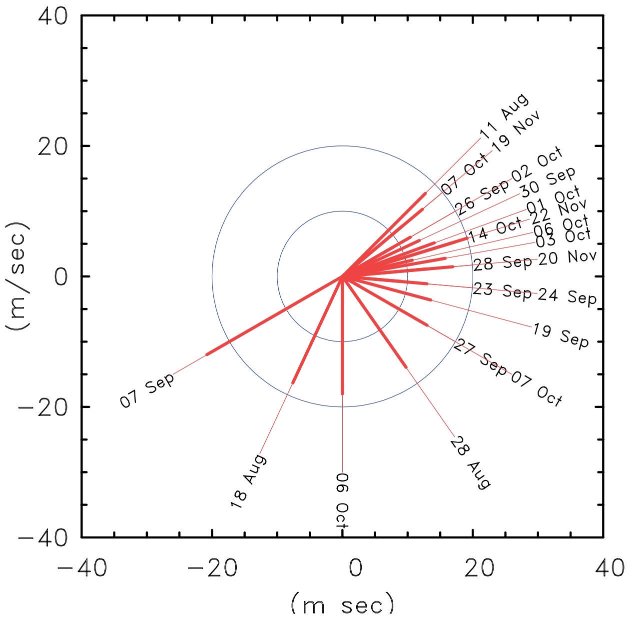


Fig. 7 Summary of disturbance speed and direction calculations. The red lines indicate the speed and direction from which the disturbance is moving. The two circles represent speeds of 10 and 20 m s^{-1}.

disturbances, more than three-quarters (16) were from the sector from northeast to east, the average speed being 15 m s^{-1}. Three disturbances, those of 18 August, 7 September and 6 October, had a south to southwesterly direction and an average speed of 20 m s^{-1}, and two others, those of 28 August and 27 September, were from the southeast or east-southeast with an average speed of 16 m s^{-1}. These speed differences agree with those of previous studies, which showed a tendency for the speed of disturbances with a southerly component to be larger than those from the northeast to east by a few m s^{-1} (see e.g. Smith et al. 1982).

The inherent errors in the speed and direction calculations can be roughly estimated by carrying out an ensemble of calculations in which random errors are added to the times of passage at each station. For the calculations here we assumed a maximum timing error of ±10 minutes and took twenty ensembles. The calculation for the event of 24 September is typical. It was observed at the three southeastern Gulf stations Karumba, Burketown and Mornington Island, and gave a mean speed of 8.7 m s^{-1} from a direction of 89 degrees, with standard deviations of 0.05 m s^{-1} and 0.5 degrees, respectively. The maximum deviations from the ensemble mean speed and direction for this case were 0.24 m s^{-1} and 3.9 degrees, respectively. A similar calculation was made for the event of 11 August, which was observed at the three western Gulf stations, Centre Island, Borroloola and McArthur River. These stations, which lie closer to a line than those in the southeastern Gulf, gave a mean speed of 6.8 m s^{-1} from a direction of 111 degrees, with standard deviations of 0.32 m s^{-1} and 1.0 degrees, respectively. The maximum deviations from the ensemble mean speed and direction for this case were 2.4 m s^{-1} and 8.5 degrees, respectively, values that are appreciably larger than for the southeastern Gulf stations.

The additional lines in the entries for some days in Table 1 show speed and direction calculations for three nearby stations, typically the group Borroloola, Centre Island and McArthur River on the southwestern side of the Gulf, or the group Burketown, Mornington Island and Karumba on the southeastern side. It is interesting to note that disturbance speeds calculated for the southwestern group alone for events that were classified as easterly-northeasterly tended to be significantly smaller (average 6 m s^{-1} compared with 15 m s^{-1}) and the direction always lay between 110 and 120 degrees. While such disturbances tended to be further from their source and were probably decelerating, we have no explanation for the consistent change in direction in this region. This is an interesting problem for future study using a mesoscale numerical model.

Southerly disturbances

The five disturbances with a southerly component occurred on 18 August, 28 August, 7 September, 27 September and 6 October. The disturbances on 7 September and 6 October were detected only at stations on the southwestern side of the Gulf (Borroloola, Centre Island and McArthur River). It is of interest to examine the synoptic situations on these occasions, as indicated by the Bureau of Meteorology mean sea-level pressure (MSLP) analyses at 10.00 pm Eastern Standard Time (EST = UTC + 10 hours) prior to the event and at 10.00 am EST on the day of the event (Fig. 8). A common feature of four of the cases is the ridging overnight across central Australia with a pronounced trough to the north of the ridge that is displaced further northwards. This situation is common to many of the southerly events so far documented (Smith et al. 1982, 1986, 1995, 2006; Thomsen and Smith 2006) and increases support for the mechanisms of formation analysed in detail by Thomsen and Smith (2006) and Thomsen et al. (2009). The exception was the event of 27 September, where, although there was an inland trough analysed to the south of the Gulf, the ridging was confined to the south of the continent. Such events as this have been documented in the literature also (Smith et al. 1982, 1986).

Conclusions

We have presented a climatology of nocturnal or late morning pressure jumps at stations around the Gulf of Carpentaria in northeastern Australia and at one inland station south of the Top End. The climatology is based on one-minute average data from Bureau of Meteorology automatic weather stations. Of the 21 bore-like disturbances that were recorded at three or more stations during the four-month period August-November 2006, 16 were from the northeast sector and five had a significant southerly component. One of the latter was recorded as far west as Daly Waters. Only one northeasterly disturbance was identified at Groote Eylandt in the northwest

Fig. 8 Bureau of Meteorology mean sea-level pressure analyses for the Australian region at 10 pm prior to the five southerly disturbances and at 10 am on the day of occurrence. (a) 10 pm 17 August, (b) 10 am 18 August, (c) 10 pm 27 August, (d) 10 am 28 August, (e) 10 pm 17 September, (f) 10 am 18 September, (g) 10 pm 26 September, (h) 10 am 27 September, (i) 10 pm 5 October, (j) 10 am 6 October. The dashed lines over the continent indicate trough lines. All times Eastern Standard Time (= UTC + 10 hours).

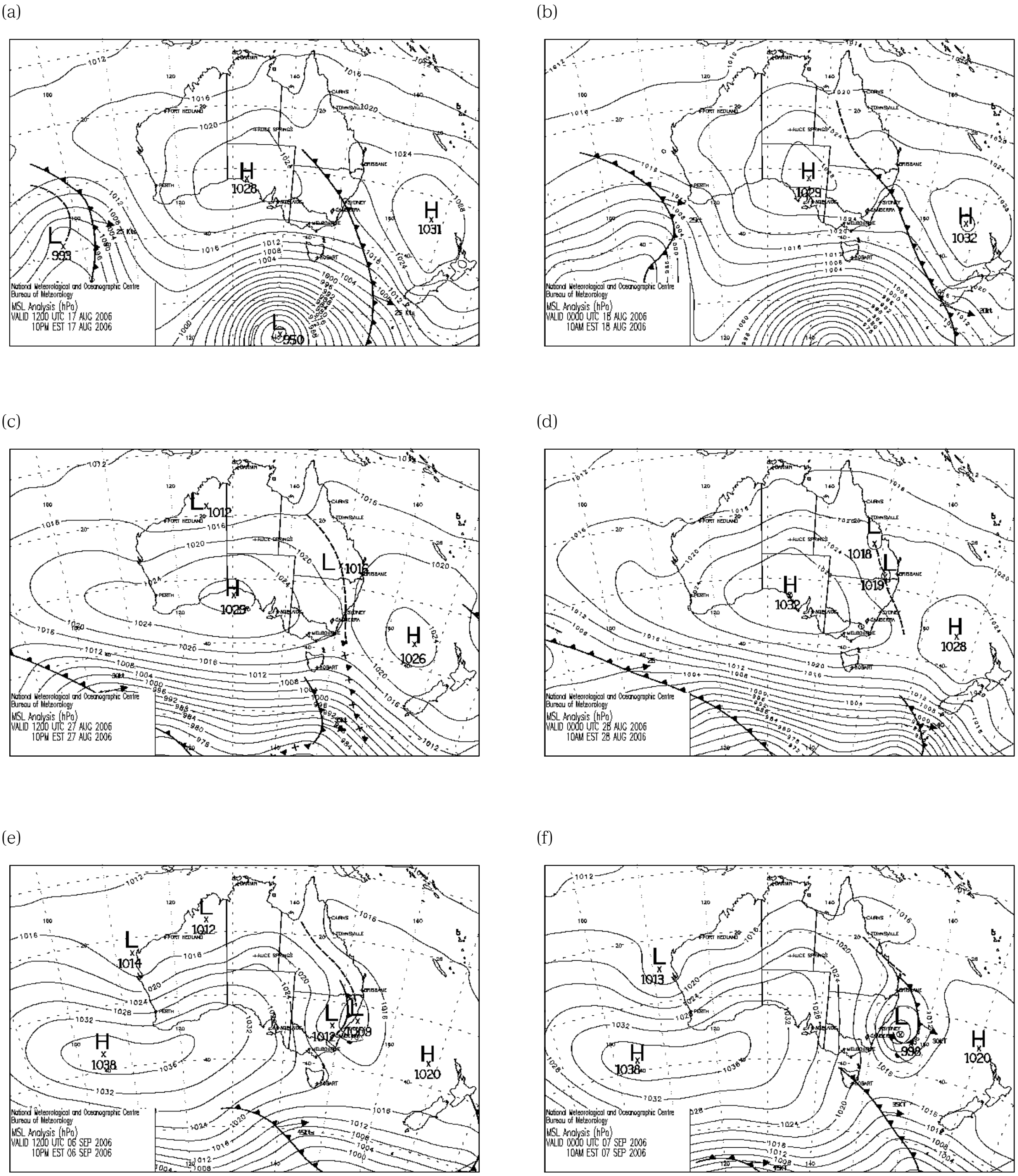
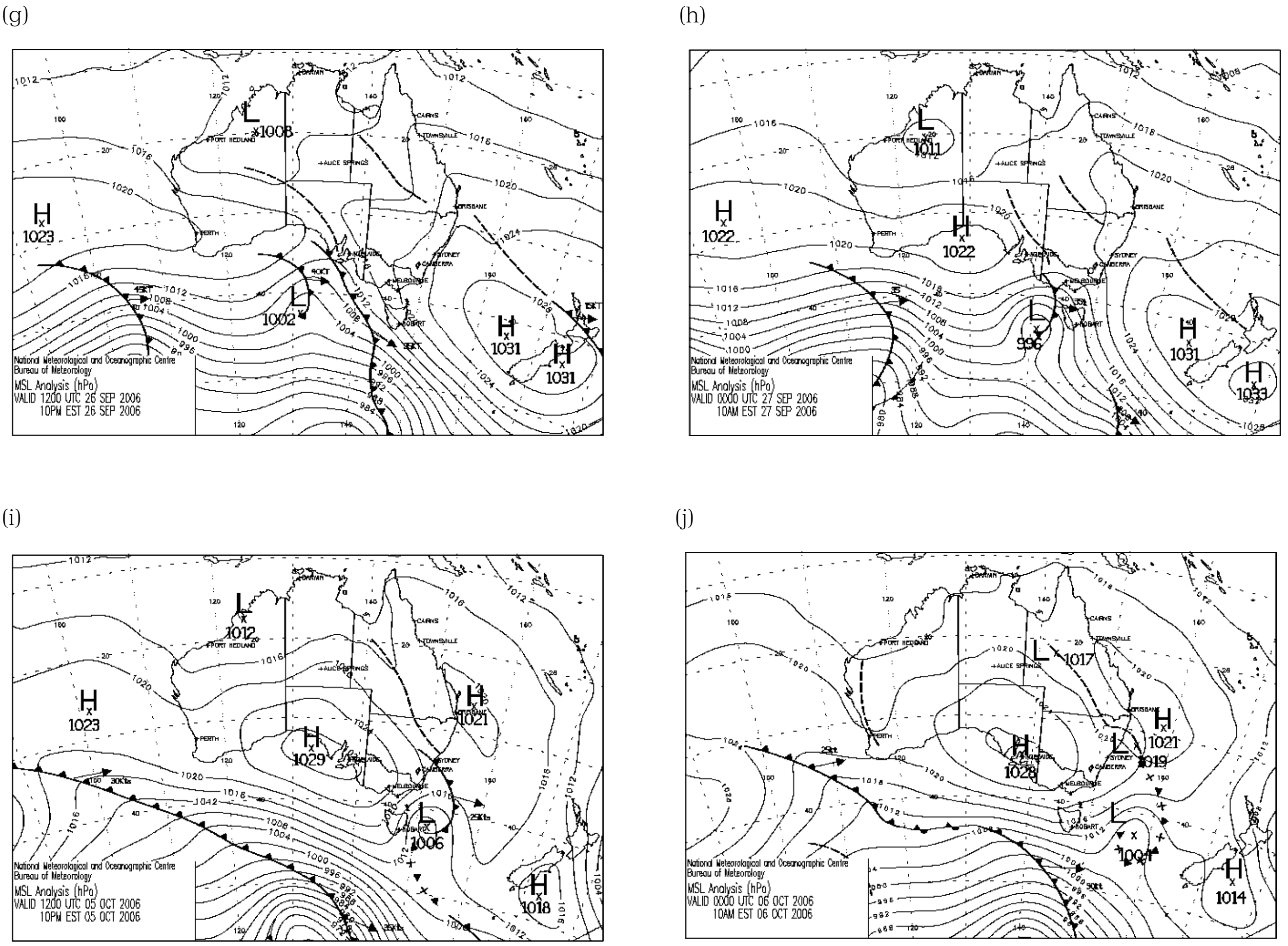


Fig. 8 Continued.



of the Gulf. Our study highlights the value of one-minute data in identifying the passage of all types of low-level disturbances at surface stations. Such passages would be missed with the conventional hourly or even three-hourly data routinely available to forecasters. We find that a pressure jump criterion of about 0.25 hPa in three minutes is suitable for detecting the majority of morning-glory-type disturbances.

Since this study was initiated, considerably more data have become available and will provide a rich source of material to extend this climatology. Our study should provide a useful starting point for analysing these data and indicates that more work needs to be done in refining the criteria for a significant pressure jump for particular types of disturbances.

Acknowledgments

We are especially grateful to the late Dale Ingafield of the Bureau of Meteorology's Northern Territory Regional Office, who acquired the data loggers and arranged for their installation and the data collection therefrom. We thank also Geoffrey Garden, the then Director of the Regional Office, for his enthusiastic support of the project, as well as the Technical Officers in both the Northern Territory and Queensland Regional Offices, who installed the data loggers and retrieved the data. Financial support for some of the data loggers was provided by the Australian Research Council.

Appendix: Disturbance speed and direction calculation

Consider a disturbance that has a straight front and is moving with uniform speed, c, in the direction of the unit vector ($\cos\theta$, $\sin\theta$) in the (x, y)-plane. This disturbance can be represented as the line

$$x\cos\theta + y\sin\theta = d + ct$$

where θ, c and d are constants and t is the time (see Fig. A1). The problem is to determine values of c,d and θ that give the best fit to times of passages at a given number of stations.

Disturbance passing through three points

Suppose that the disturbance passes through the three points (x_i, y_i) at times t_i (i = 1,2,3). Then the three equations

$$x_i\cos\theta + y_i\sin\theta = d + ct_i \quad (i = 1,2,3) \qquad ...1$$

are satisfied and can be solved for the three unknowns θ, c and d, provided that the stations do not lie in a line. To make these three equations more symmetrical we add them to get

$$\bar{x}\cos\theta + \bar{y}\sin\theta = d + c\bar{t} \qquad ...2$$

where $\bar{x} = (x_1 + x_2 + x_3)/3$ and similarly for $\bar{y}$ and $\bar{t}$. Likewise we multiply Eqn 1 by x_i and sum over i to obtain

$$\overline{x^2}\cos\theta + \overline{xy}\sin\theta = d\bar{x} + c\overline{xt} \qquad ...3$$

where $\overline{xy} = (x_1y_1 + x_2y_2 + x_3y_3)/3$ and similarly for $\overline{x^2}$, $\overline{y^2}$, $\overline{xt}$ and $\overline{yt}$. Finally, multiplying Eqn 1 by y_i and summing over i we obtain

$$\overline{xy}\cos\theta + \overline{y^2}\sin\theta = d\bar{y} + c\overline{yt} \qquad ...4$$

Eliminating d from Eqn 3 and Eqn 4 using Eqn 2 gives

$$(\overline{x^2} - \bar{x}^2)\cos\theta + (\overline{xy} - \bar{x}\,\bar{y})\sin\theta = c(\overline{xt} - \bar{x}\,\bar{t}) \qquad ...5$$

and

$$(\overline{xy} - \bar{x}\,\bar{y})\cos\theta + (\overline{y^2} - \bar{y}^2)\sin\theta = c(\overline{yt} - \bar{y}\,\bar{t}) \qquad ...6$$

Assuming that the three stations do not lie in a straight line, Eqns 5 and 6 may now be solved for $\cos\theta$ and $\sin\theta$ in terms of c, whereupon

$$\cos\theta = cX_1 \text{ and } \sin\theta = cY_1 \qquad ...7$$

where

$$X_1 = \frac{(\overline{xt} - \bar{x}\,\bar{t})(\overline{y^2} - \bar{y}^2) - (\overline{yt} - \bar{y}\,\bar{t})(\overline{xy} - \bar{x}\,\bar{y})}{(\overline{x^2} - \bar{x}^2)(\overline{y^2} - \bar{y}^2) - (\overline{xy} - \bar{x}\,\bar{y})^2}$$

and

$$Y_1 = \frac{(\overline{yt} - \bar{y}\,\bar{t})(\overline{x^2} - \bar{x}^2) - (\overline{xt} - \bar{x}\,\bar{t})(\overline{xy} - \bar{x}\,\bar{y})}{(\overline{x^2} - \bar{x}^2)(\overline{y^2} - \bar{y}^2) - (\overline{xy} - \bar{x}\,\bar{y})^2}$$

Finally, c is determined from Eqn 7 using the equality $\cos^2\theta + \sin^2\theta = 1$, giving

$$c^2 = \frac{1}{X_1^2 + Y_1^2} \qquad ...8$$

Note that it is necessary to take only the positive sign for c as taking the negative sign will simply reverse the direction of the unit vector ($\cos\theta$, $\sin\theta$) determined by Eqn 7. Note also that if the three stations lie in a straight line, $y_i = \alpha x + \beta$, where α and β are constants. In this case it can be shown that the denominators in the expressions for X_1 and Y_1 are zero.

Disturbance passing through four or more points

Suppose that the disturbance passes through the n points (x_i, y_i) at times t_i, (i = 1,2,3...n). We project the locations (x_i, y_i) onto the line in the ($\cos\theta$, $\sin\theta$) direction with coordinate $X = x\cos\theta + y\sin\theta = d + ct$ (Fig. A1). The unknowns are the constants $|c|$, d and θ. We seek the best fit to these quantities by minimising the distance $\sum(X - X_i)^2$, i.e.

$$E^2 = \sum(d + ct_i - x_i\cos\theta - y_i\sin\theta)^2 \qquad ...9$$

If E^2 is an extremum, then $\partial E^2/\partial\theta$, $\partial E^2/\partial c$ and $\partial E^2/\partial d$ are zero, i.e.

$$\frac{\partial E^2}{\partial\theta} = 2\sum(d + ct_i - x_i\cos\theta - y_i\sin\theta)(x_i\sin\theta - y_i\cos\theta) = 0 \qquad ...10$$

$$\frac{\partial E^2}{\partial c} = 2\sum(d + ct_i - x_i\cos\theta - y_i\sin\theta)\,t_i = 0 \qquad ...11$$

$$\frac{\partial E^2}{\partial d} = 2\sum(d + ct_i - x_i\cos\theta - y_i\sin\theta) = 0 \qquad ...12$$

Equation 12 gives

$$d = \bar{x}\cos\theta + \bar{y}\sin\theta - c\bar{t} \qquad ...13$$

and Eqn 11 with Eqn 13 gives

$$c\,(\overline{t^2} - \bar{t}^2) = (\overline{xt} - \bar{x}\,\bar{t})\cos\theta + (\overline{yt} - \bar{y}\,\bar{t})\sin\theta \qquad ...14$$

Finally, Eqn 10 reduces to

$$\begin{aligned}&\overline{xy}\,(\cos^2\theta - \sin^2\theta) + (\overline{y^2} - \overline{x^2})\cos\theta\sin\theta\\ &\quad + d(\bar{x}\sin\theta - \bar{y}\cos\theta)\\ &\quad + c(\overline{xt}\sin\theta - \overline{yt}\cos\theta) = 0 \qquad ...15\end{aligned}$$

which, using Eqn 13, gives

$$\begin{aligned}&\overline{xy}\,(\cos^2\theta - \sin^2\theta) + (\overline{y^2} - \overline{x^2})\cos\theta\sin\theta\\ &\quad + (\bar{x}\cos\theta + \bar{y}\sin\theta - c\bar{t})(\bar{x}\sin\theta - \bar{y}\cos\theta)\\ &\quad + c(\overline{xt}\sin\theta - \overline{yt}\cos\theta) = 0 \qquad ...16\end{aligned}$$

or

$$\begin{aligned}&(\overline{xy} - \bar{x}\,\bar{y})(\cos^2\theta - \sin^2\theta)\\ &\quad + [(\overline{y^2} - \bar{y}^2) - (\overline{x^2} - \bar{x}^2)]\cos\theta\sin\theta\\ &\quad + c[(\overline{xt} - \bar{x}\,\bar{t})\sin\theta - (\overline{yt} - \bar{y}\,\bar{t})\cos\theta] = 0 \qquad ...17\end{aligned}$$

We may now eliminate c from Eqns 14 and 17 to obtain an equation for θ. First we define

$$c_1 = \frac{\overline{xt} - \bar{x}\,\bar{t}}{\overline{t^2} - \bar{t}^2} \text{ and } c_2 = \frac{\overline{yt} - \bar{y}\,\bar{t}}{\overline{t^2} - \bar{t}^2} \qquad ...18$$

so that

$$c = c_1\cos\theta + c_2\sin\theta \qquad ...19$$

Then Eqn 17 becomes

$$\begin{aligned}&(\overline{xy} - \bar{x}\,\bar{y})(\cos^2\theta - \sin^2\theta) + [(\overline{y^2} - \bar{y}^2) - (\overline{x^2} - \bar{x}^2)]\cos\theta\sin\theta\\ &+ (c_1\cos\theta + c_2\sin\theta)[(\overline{xt} - \bar{x}\,\bar{t})\sin\theta - (\overline{yt} - \bar{y}\,\bar{t})\cos\theta)] = 0\end{aligned}$$

which may be written in the form

$$A \cos^2\theta + B \sin^2\theta + 2C \cos\theta \sin\theta = 0 \qquad \text{...20}$$

where

$$A = (\overline{xy} - \bar{x}\,\bar{y}) - c_1(\overline{yt} - \bar{y}\,\bar{t})$$
$$B = -\,(\overline{xy} - \bar{x}\,\bar{y}) - c_2(\overline{xt} - \bar{x}\,\bar{t})$$

and

$$2C = (\overline{y^2} - \bar{y}^2) - (\overline{x^2} - \bar{x}^2) + (\overline{xt} - \bar{x}\,\bar{t})c_1 - (\overline{yt} - \bar{y}\,\bar{t})c_2.$$

Note that

$$(\overline{t^2} - \bar{t}^2)\,A = (\overline{xy} - \bar{x}\,\bar{y})\,(\overline{t^2} - \bar{t}^2) - (\overline{yt} - \bar{y}\,\bar{t})\,(\overline{xt} - \bar{x}\,\bar{t})$$
$$(\overline{t^2} - \bar{t}^2)\,B = -\,(\overline{xy} - \bar{x}\,\bar{y})\,(\overline{t^2} - \bar{t}^2) + (\overline{xt} - \bar{x}\,\bar{t})\,(\overline{yt} - \bar{y}\,\bar{t})$$

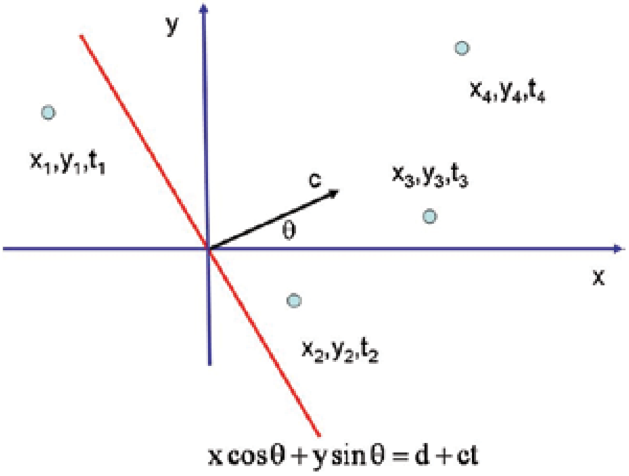
whereupon $A + B = 0$. Then Eqn 20 gives

$$\tan 2\theta = -\frac{A}{C} \qquad \text{...21}$$

where C is given by

$$(\overline{t^2} - \bar{t}^2)2C = [(\overline{y^2} - \bar{y}^2) - (\overline{x^2} - \bar{x}^2)](\overline{t^2} - \bar{t}^2) + (\overline{xt} - \bar{x}\,\bar{t})^2 - (\overline{yt} - \bar{y}\,\bar{t})^2$$
$$= [(\overline{y^2} - \bar{y}^2)(\overline{t^2} - \bar{t}^2) - (\overline{yt} - \bar{y}\,\bar{t})^2] - [(\overline{x^2} - \bar{x}^2)(\overline{t^2} - \bar{t}^2) - (\overline{xt} - \bar{x}\,\bar{t})^2].$$

Fig. A1 A line disturbance moving with constant speed *c* at angle θ to the x-axis.



References

Clarke, R.H. and Brook, R.R. 1979. *The Koorin Expedition - Atmospheric Boundary Layer Data over Tropical Savannah Land*, Dept. of Sci., Canberra, Australia, 495 pp.

Clarke, R.H. 1983. Fair weather nocturnal inland wind surges and atmospheric bores: Part I Nocturnal wind surges. *Aust. Met. Mag., 31*, 133-45.

Deslandes, R., Reeder, M.J. and Mills, G.A. 1999. Synoptic analyses of a subtropical cold front observed during the 1991 Central Australian Fronts Experiment. *Aust. Met. Mag., 48*, 87-110.

Garratt, J.R. 1985. The inland boundary layer at low latitudes: I The nocturnal jet. *Bound. Layer. Met., 32*, 307-27.

Garratt, J.R. and Physick, W. 1985. The inland boundary layer at low latitudes: II Sea-breeze influences. *Bound. Layer. Met., 33*, 209-31.

Reeder, M.J. and Smith, R.K. 1998. *Mesoscale Meteorology. Meteorology of the Southern Hemisphere*. Eds. D. Vincent and D.J. Karoly. American Meteorological Society, Boston, Mass., 201-41.

Reeder M.J., Smith, R.K., Deslandes, R., Tapper, N.J. and Mills, G.A. 2000. Subtropical fronts observed during the 1996 Central Australian Fronts Experiment. *Aust. Met. Mag., 49*, 181-200.

Smith, R.K., Crook, N.A. and Roff, G. 1982. Morning Glory: an extraordinary atmospheric undular bore. *Q. Jl R. Met. Soc., 198*, 937-56.

Smith, R.K., Coughlan, M.J. and Evans-Lopez, J. 1986. Southerly nocturnal wind surges and bores in northeastern Australia. *Mon. Wea. Rev., 114*, 1501-18.

Smith, R.K., Reeder, M.J., Tapper, N.J. and Christie, D.R. 1995. Central Australian cold fronts. *Mon. Weath. Rev., 123*, 19-38.

Smith, R.K., Reeder, M.J., May, P. and Richter, H. 2006. Low-level convergence lines over northeastern Australia. II. Southerly disturbances. *Mon. Weath. Rev., 134*, 3109-24.

Thomsen, G. and Smith, R.K. 2006. Simulations of low-level convergence lines over north-eastern Australia. *Q. Jl R. Met. Soc., 132*, 691-707.

Thomsen, G., Reeder, M.J. and Smith, R.K. 2009. The diurnal evolution of cold fronts in the Australian subtropics. *Q. Jl R. Met. Soc., 135*, 395-411.