Surface Waves

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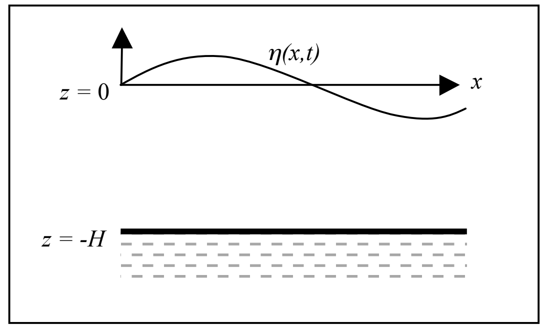
1. Governing equations

Linear wave theory – small amplitude assumption



Solve: $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$ subject to

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \quad \text{at} \quad z = 0$$

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -H$$

$$\frac{\partial \phi}{\partial t} = -g\eta \quad \text{at} \quad z = 0$$

Solution: $\phi(x,z,t) = \frac{a\omega}{k} \frac{\cosh k(z+H)}{\sinh kH} \sin(kx - \omega t)$

2. Dispersion relationship

$$\omega^2 = gk \tanh kH$$

Deep water:	$kH >> 1$;	$\omega^2 = gk$;	$c_p = 2c_g$	Waves are dispersive
Shallow water:	$kH << 1$;	$\omega^2 = gHk^2$;	$c_p = c_g$	Waves are non-dispersive

3. Basic definitions

Wave spectrum: $F(f,\theta)$ used to characterise sea-surface elevation

Significant Wave Height: $H_s = 4\sqrt{\overline{F(f,\theta)}}$

= average value of the $\frac{1}{3}$ highest waves in a sample

4. Operational wave modelling

WAM, WAVEWATCH III^{TM}, SWAN

$$\frac{\partial F}{\partial t} + \nabla.(\mathbf{c}_g F) = S_{in} + S_{nl} + S_{ds} + S_{bot}$$

Typical *rms* error for short-range forecasts from global scale wave models is ~0.5 m.

5. References

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