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Extreme rainfall deficits — a New South Wales case study

S. Gregory* and T. Cooke†

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The attempt is made to integrate rainfall deficit intensity, duration, frequency and spatial distribution into one end-product. Monthly rainfall data are integrated into all possible events with 3, 4, 6, 12, 24 and 36 months duration, over the 80 years 1900-1979, for 79 stations in New South Wales, and then the individual events are expressed as percentages of the averages of the relevant period. The lowest percentage value of the twelve possible events beginning in each year is taken as an extreme event, and the probability or return period of such events is assessed by fitting the log-Pearson Type III frequency distribution. The latter is used to construct maps of New South Wales showing the spatial pattern of rainfall deficit, for the six different durations mentioned above, for four different return periods — 2 years, 5 years, 10 years, 50 years. Return periods of selected deficits and durations are also mapped and the relationships between deficit intensity, duration and frequency are graphed for 10 stations, forming west-east transects across the northern and southern parts of the State.

Introduction

In the present-day preoccupation with the drought problem, it has to be accepted that this phenomenon is both a climatic and a non-climatic one. Within the latter a wide range of issues related to the society, culture and technology of the communities occupying the areas concerned may modify, or more often intensify, the impact of climatic conditions. In addition, drought impact may be viewed not only in terms of human societies, but also in terms of impact on the ecology of plants and animals in a natural, as well as agricultural, state. Moreover, the climatic influence of reduced precipitation and/or increased evapotranspiration (due to higher temperatures, lower atmospheric humidity or increased wind speed) will affect the biosphere through soil moisture deficits and hydrological conditions rather than directly as climate itself.

Thus the estimation and definition of drought conditions is both a complex and a difficult operation, and any results will have different implications for each different purpose. Underlying all of these considerations, however, is the primary impact of a rainfall (precipitation) deficit, for such a rainfall deficit must be seen as the necessary, though not always a sufficient, cause of drought conditions. In addition, studies of such deficits permit a variety of applied usages, depending on the needs of any particular field or problem, without the non-climatic (and often emotive) connotations of the term 'drought' (see Palmer 1965; Gibbs and Maher 1967; Millan and Yevjevich 1971; Coughlan and Lee 1978; Lee 1980; Bhalme and Mooley 1980).

When considering rainfall deficits, however, as when also considering drought through whatever definition, there are at least four major characteristics that must be included:

(a) deficit intensity, expressed as some amount below the norm or as some proportion of the norm;
(b) deficit duration, in terms of the number of months or years over which such a deficit occurs;
(c) deficit frequency, indicating how often such conditions are likely to occur, this being variously expressed in terms of probability, recurrence interval or return period;
(d) deficit distribution in space for a given intensity, duration or frequency, or for some combination of any two or all three of these.

The purpose of this present paper is to propose a method by which all these four characteristics can be incorporated into one ultimate presentation, and then to illustrate and evaluate this in terms of conditions in New South Wales. One advantage of con-

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* University of Sheffield, UK.
† University of New England, NSW, Australia.

sidering Australian conditions is the long history of drought studies, often based on rainfall deficits, in that the lowest decile of falls are taken to define droughts (Gibbs and Maher 1967). Also, there is continuous monthly monitoring of drought conditions so defined, for advisory and planning purposes (Lee 1978). Within Australia, the State of New South Wales has a very good network of stations with records for the 80-year period 1900-1979, and the 79 stations that are used are shown in Fig. 1. In addition, the extreme seasonality of rainfall that characterises both northern and southern Australia is absent over most of the State, especially for the time duration units used in the analysis, and this eliminates one problem at this stage. Furthermore, an earlier study by Fitzpatrick (1953) has previously considered dry and wet spell probabilities for New South Wales, using daily data and spells of up to 90-100 days in length.

Method

As a rainfall deficit for simply one month, or even two months, would scarcely be recognised as representing drought conditions, the shortest period that was considered was a three-month time unit. For each station, therefore, individual monthly values were integrated into all possible consecutive three-month periods, so that there were twelve such three-month periods beginning in each year, i.e. three months: Jan-Mar; Feb-Apr; . . . Nov-Jan (next year); Dec-Feb (next year). For each of these twelve integrated times series, the average was obtained over the 80 years involved (79 years for the units beginning in November and December), and then each of the values for individual years expressed as a percentage of the average for the respective three-month time unit. Similar integrated calculations were then carried out for successively larger numbers of months, as is also done in the Australian Drought Watch System (Lee 1978), but in the event full analyses are presented for only the following six time units: three months; four months; six months; 12 months; 24 months; 36 months. Thus, in the last case, the periods would be: 36 months: Jan (first year)-Dec (third year) . . . Dec (first year)-Nov (fourth year); through to Jan (78th year)-Dec (80th year); i.e. an average of 77 years for all except the period beginning in January. There would,

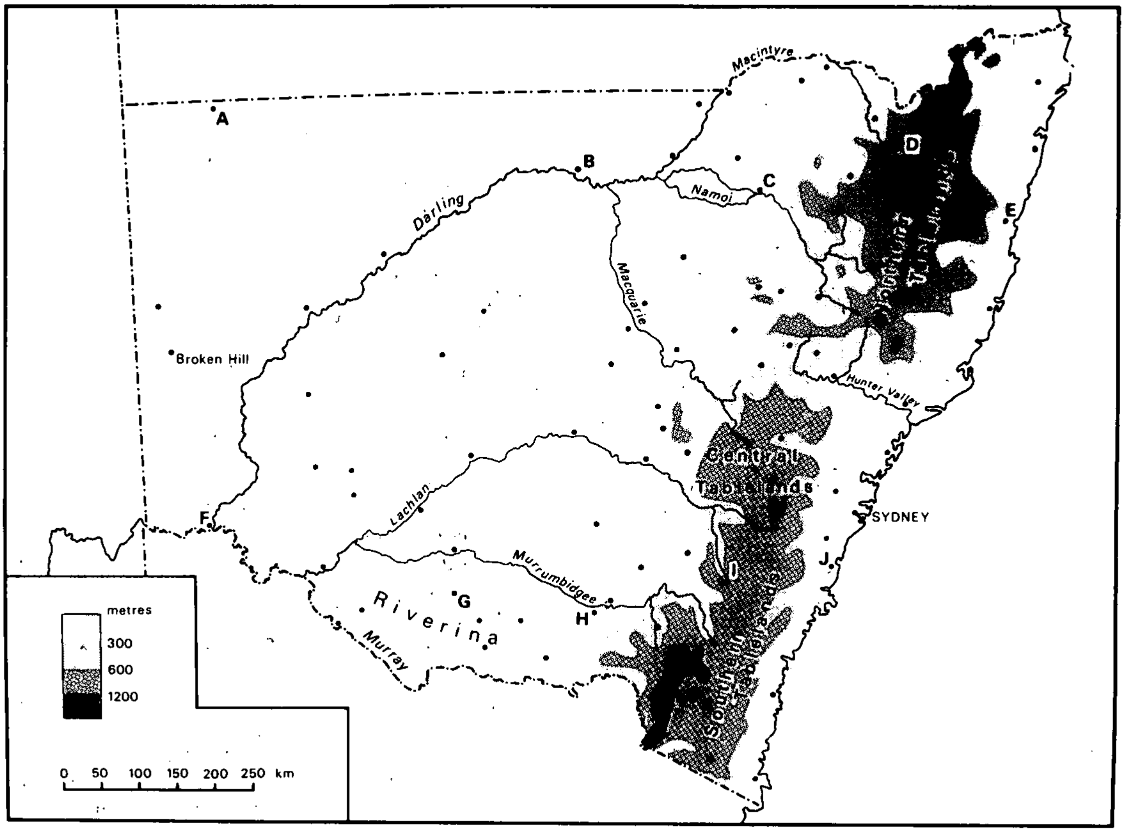


Fig. 1 Base map of New South Wales, showing 79 stations used including the following specifically used in other diagrams: A — Tibbooburra; B — Brewarrina; C — Wee Waa; D — Glen Innes; E — Bellingen; F — Wentworth; G — Conargo; H — Wagga Wagga; I — Gunning; J — Dapto West.

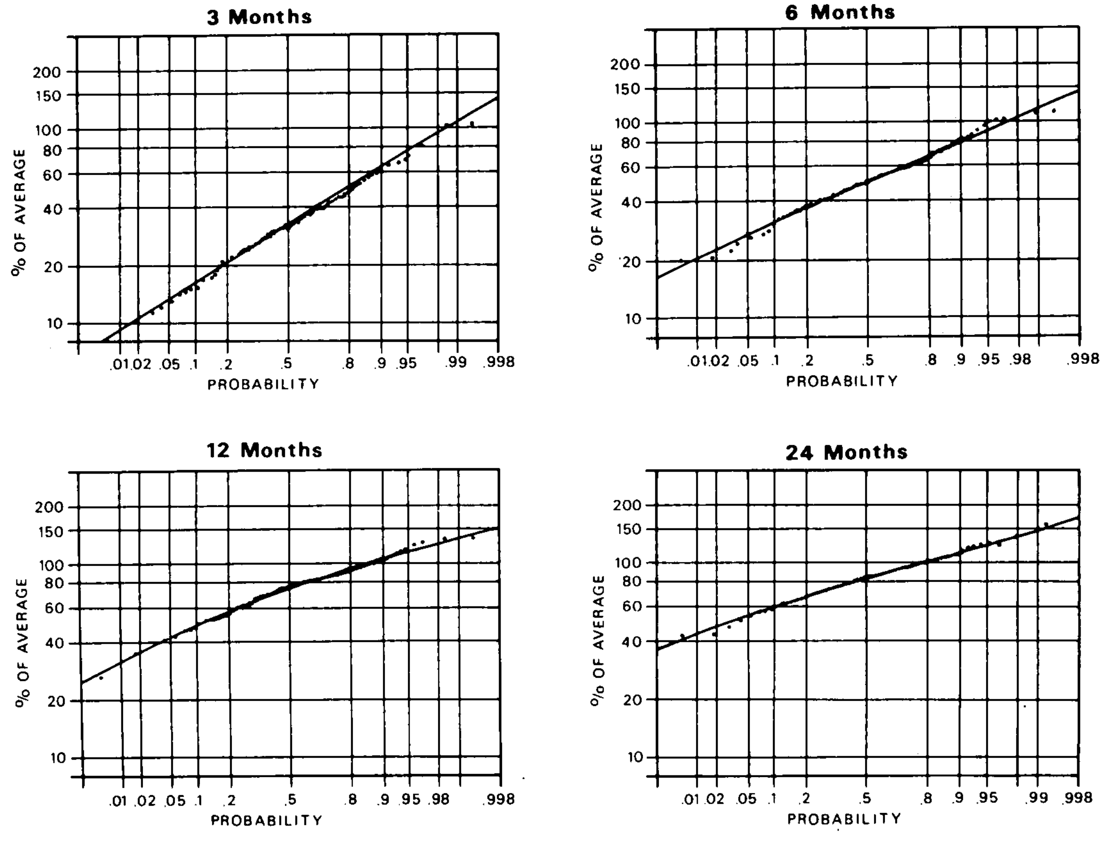
however, still be twelve 36-month periods beginning in each of the 77 years.

With these percentage data it would then be possible to derive probability estimates for given percentages, using the normal frequency distribution, as has been done for actual rainfall values for eastern Africa (e.g. Glover et al. 1954; Manning 1956; Gregory 1969). This would be very comparable to what is done through the Drought Watch System using deciles, however, and although there are merits in adopting such an approach, this is not the purpose of the present study.

Concern has instead been focused on extreme rainfall deficits, adopting the approach that has been so widely used in hydrology, for both flood and high rainfall events, from the work of Gumbel (1941, 1958) to that of Chow (1951, 1954), Hershfield (1962), Haan (1977) and many others. Applications in climatology and meteorology have been summarised by Jenkinson (1975), but low value as distinct from high value extremes have not received much attention (Gumbel 1954; Jenkinson 1954; Huft and Changnon 1960).

The extreme events to be analysed in this enquiry, however, are not the primary (annual minimum) series that are normally analysed and in relation to which theory has mainly been devised, but rather various partial duration series. The lowest of the twelve percentage values to be recorded as beginning in each individual year, for the time unit being studied, form the extreme event data. Thus for the three, four, six, and 12-month time units, a full record would provide a series of 79 extreme events; for the 24-month time units this number would be 78 while for the 36-month time unit it would be 77. In all cases, however, any incompleteness in the record would lead to rather fewer extreme events; in the stations used this never falls as low as 70, because they were at least in part selected with this in mind. Another possible limitation should also be indicated, especially in relation to the longer 24 and 36-month time units. Extreme event theory assumes the independence of the extreme events selected and this assumption becomes less valid in the present study as the time unit lengthens. Screening procedures do exist (Chow 1964), but have not been applied in this exploratory study. However, little impact of this, if any, is found in the fitting of the frequency distributions (e.g. Fig. 2) or in the spatial coherence displayed in Figs 3 to 6, but this theoretical limitation should be noted.

Fig. 2 Cumulative frequency distributions of extreme rainfall '% of average' values and the related log-Pearson Type III distributions for four different time units, for Dapto West, south of Sydney (Station J on Fig. 1).



There are some four or five different types of approaches to the analysis of extreme events, summarised in Linsley et al. (1982), but the most widely used of these is now the generation of non-normal frequency distributions to 'fit' the data. The observed values are seen as a sample, the detailed distribution of which is almost certain to differ to some extent from that of the longer-term generalisation that is expressed in the derived equation. It is from the latter that overall probabilities can then be estimated, not from the sample direct. Also, assumptions have to be made about the return period, or probability, of the observed data, so as to derive a 'Plotting Position' in a graphical sense. Several of the available formulae are presented in Table 1, showing the different return periods obtained for a 100-year record. The Weibull approach tends to be adopted in the USA, but Linsley et al. (1982) suggest that 'the true return period of the higher floods is probably longer than the value computed' by the Weibull method. In the UK it is the Gringorten method that has been recommended (Institute of Hydrology 1975), whilst Jenkinson (1954) proposed the modified Hazen method for meteorological data. The latter method also coincides with the usual convention for computing probabilities for the ogive or cumulated frequency curve, and gives values very close to the Gringorten method for all cases except the most extreme one. It is this modified Hazen method that has been adopted in this study.

The numerous possible frequency distribution methods are fully discussed and evaluated in the Flood Studies Report (Institute of Hydrology 1975), and it must be appreciated that 'despite much effort, tests suggests that there is no best distribution for floods' (Benson 1968) — and, by inference, for other extreme event phenomena. All the distributions fit the general form expressed by Chow (1951) as: $Y_m = \bar{Y} + K_m \sigma_y$ where Y is the variable under study, m is the given return period or probability, and K is a coefficient or frequency factor derived from the mathematical characteristics of the frequency distribution.

Previously it was standard practice to plot extreme event data onto graph paper, especially devised with the abscissa (probability) 'warped' in relation to the frequency distribution being used, so as to ensure a straight-line plot for interpolation and extrapolation. Now it is more effective to analyse by the method of moments, especially if the relevant K values have been tabulated, similar to the normal distribution function in most statistical tables. Amongst the various methods available (Institute of Hydrology 1975), the log-Pearson Type III distribution involves first transforming the data into logarithms to the base 10, and then obtaining the average, standard deviation and skewness of this transformed data set. In Linsey et al. (1982) the appropriate K coefficients have been tabulated for a number of return periods, and for skewness values from +3 to −3. It should be noted that if the skewness value =0, then this method is the same as applying the log-normal frequency distribution. The allowance for skewness in this method is of critical importance, for it permits virtually any data set to be fitted very closely indeed. It is nevertheless true that ideally a very large sample is needed to obtain a reliable index of skewness, and with only 75 to 80 items in the present data set this must remain a weakness.

As can be seen from Table 2, the degree of skewness of the transformed data decreased steadily from the 3-month time unit to the 24-month time unit. In the shortest time-period virtually all the stations have negative skewness, 86 per cent of them with a value below −1.0; but by the longest period 40 per cent of them show positive skewness and 90 per cent have values between −0.5 and +0.5. The spatial patterns of these values display considerable regional coherence. Thus at the 3-month period the skewness values below −2.0 occur across the areas west of a line from Station B to Station H to just beyond the Darling River (Fig. 1), with the smallest values along the coastal belt south of the Hunter Valley. At the 12-month period largest negative values extend over the interior from the Mackintyre Valley to the lower Lachlan Valley, whilst positive skewness values occur both in the northwest (Broken Hill northwards) and in patches in the east. At the 24-month period, the largest positive values are in the northwest and the southeast, and the largest negative values southwest for some 500 km from Station D, but excluding the Namoi Valley (Fig. 1).

Table 1. Return periods calculated by several methods, when the number of items (N) = 100, and they are ranked (R) from 1 = most extreme to 100 = least extreme.

Method	*Equation*	*Rank*						
		1	*2*	*3*	*4*	*5*	*6*	*100*
California	N/R	100	50	33.3	25	20	16.7	1
Weibull	N+1/R	101	50.5	33.7	25.3	20.2	16.8	1.01
Gringorten	N+.12/R−.44	178.8	64.2	39.1	28.1	21.9	18.0	1.005
Hazen	2N/R	200	100	66.7	50	40	33.3	2
Modified Hazen	2N/2R−1	200	66.7	40	28.6	22.2	18.2	1.005

Table 2. The degree of skewness of the logarithmically transformed data for the 79 stations used, for six different time periods.

Skewness	*3 months*	*4 months*	*6 months*	*12 months*	*18 months*	*24 months*
+0.51 to +1.0	0	0	0	0	3	4
0 to +0.5	2	2	4	18	28	28
0.01 to −0.5	9	13	34	52	42	43
−0.51 to −1.0	23	32	26	8	6	4
−1.01 to −1.5	13	18	12	1	0	0
−1.51 to −2.0	10	3	2	0	0	0
<−2.0	22	11	1	0	0	0

Apart from providing a very close fit to the data, this log-Pearson Type III method also ensures that estimates of the variable cannot fall below zero, as the data are in logarithms. Thus, as Chow (1964) asserts, 'Since there is always a limit to the drought with a minimum of zero, Type III extremity distribution is assumed to be suitable (p.8-35). Indeed, estimates of 0 per cent are only obtained, even for the most extreme event possible, by a decision on the degree of rounding in terms of the number of decimal points to be used. In some of the observed cases, values of 0 per cent were legitimately recorded, especially in the 3-month time unit. To permit the transformation into logarithms it was decided to return these as 0.1 per cent, so that they became −1.00 in logarithms. If a lower value were used, e.g. 0.001 per cent, the resultant logarithmic value of −3.00 created excessive skewness, leading to misleading estimates.

Examples of the degree of fit between the log-Pearson Type III curve and the observed data, plotted according to the modified Hazen assumptions, are illustrated for Dapto West (Station J) in Fig. 2. The correlation between these observed and estimated data (expressed in logarithms) is very high indeed, lying at or above + 0.997 in each of these examples, and with the standard error of the estimate of the '% of average' values being less than 0.02 in logarithms. This will mean that in actual percentage values the error will be larger at the upper end of the scale than at the lower end. It is the latter which is the present area of interest, and error terms here are slight. Similar relationships hold true over the whole area, even in the western half at the three-month time unit, when, as indicated earlier in Table 2, there are many stations with large negative skewness characteristics in the transformed data. Thus at Wentworth (Station F in Fig. 1), where skewness for the three-month time unit was greater than −3.0, correlation between observed and estimated logarithmic data was still +0.980. The standard error in logarithms was nearly 0.2, however, leading to quite marked discrepancies at the upper end of the scale but not at the relevant lower end. This method, applied to the data for any station, will thus provide information in the following form: 'For one year in N, one n-month period starting in that year can be expected to receive no more than Y% of its long-term rainfall average value.'

New South Wales case study

In Figs 3 to 6 are presented the spatial patterns of extreme rainfall-deficit intensity, for specified durations with a given frequency or return period, derived as outlined above. In this way, the objective of integrating these four critical characteristics (as discussed earlier) has been achieved. In all cases a continuous and coherent spatial pattern is observed; sharp discontinuities are absent, and the large number of stations falling between each successive pair of isopleths stresses the essential validity of the spatial end product. Moreover, the basic pattern is constant in its essentials, though not in all details, for all the six duration time units, and all the four return periods, that have been presented in these figures. This is also true of all other durations and frequencies that were considered. These characteristics, together with the spatial coherence of the skewness data outlined earlier, would suggest that, although there are strong arguments for the use of area average rather than individual station data when regional generalisations are being attempted (Vines 1985), these hand-smoothed isopleths based on point values provide a valid picture of progressive spatial change.

In most cases, the smallest extreme percentage deficits, i.e. the largest '% of the average' values, occur over the Northern Tablelands of New England, closely followed by the Central Tablelands to the west of Sydney and the Southern Tablelands to the south. Percentage values decrease, and thus extreme deficits increase, eastwards to the coast and along the Hunter Valley. Similar changes also occur westwards over the slopes and plains of the interior, with extreme deficit intensities increasing steadily to the west and northwest, to a maximum (lowest percentage values) beyond the middle Darling Valley and north of Broken Hill. In the southern part of the interior of the State, across the Riverina between the Murray and the Murrumbidgee, rather less extreme conditions extend westwards, although there is still a steady change from less to more extreme conditions from east to west. Moreover, this is

Fig. 3 Maps of New South Wales showing the '% of average' values for six different durations that will occur with a two-year return period.

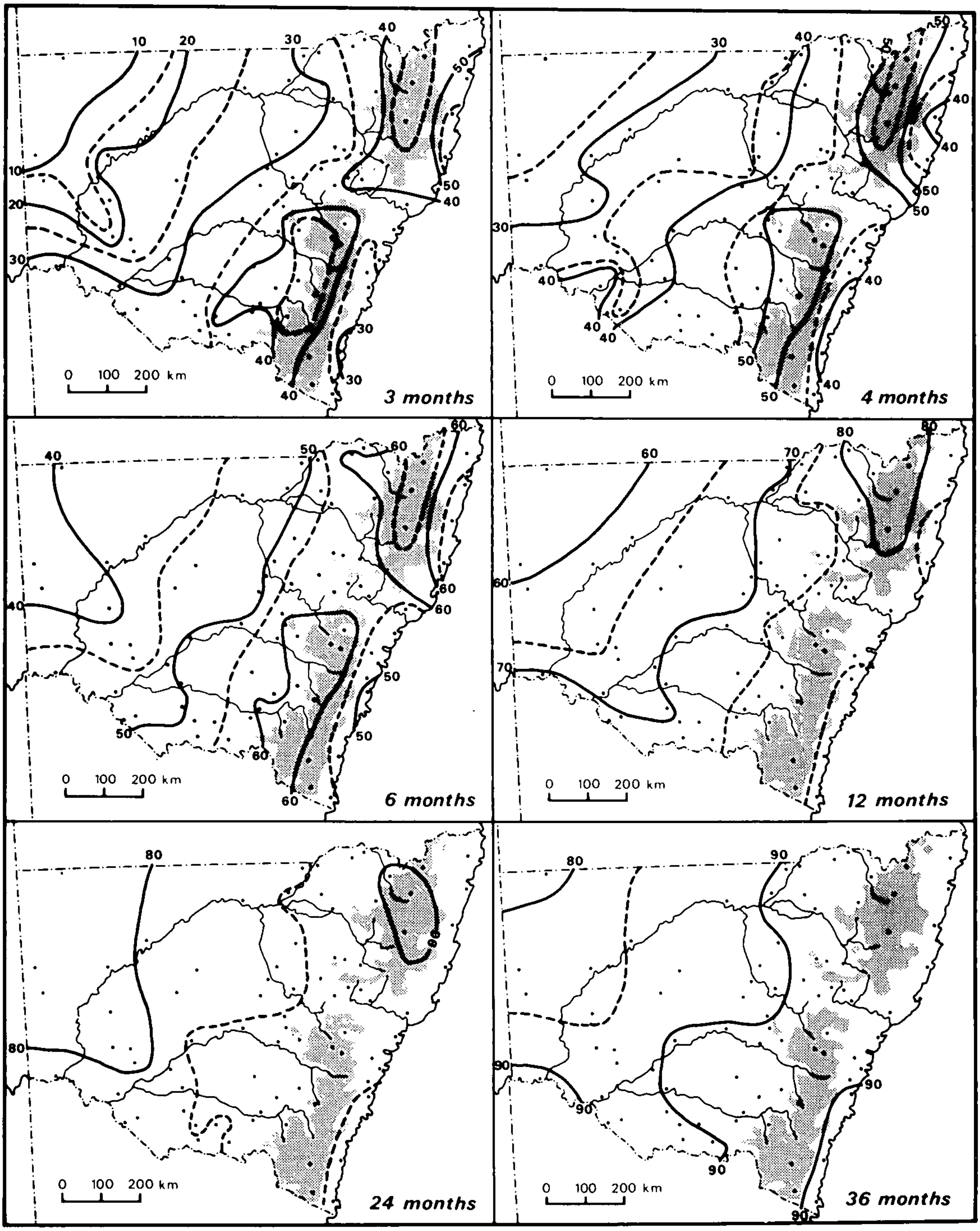


Fig. 4 Maps of New South Wales showing the '% of average' values for six different durations that will occur with a five-year return period.

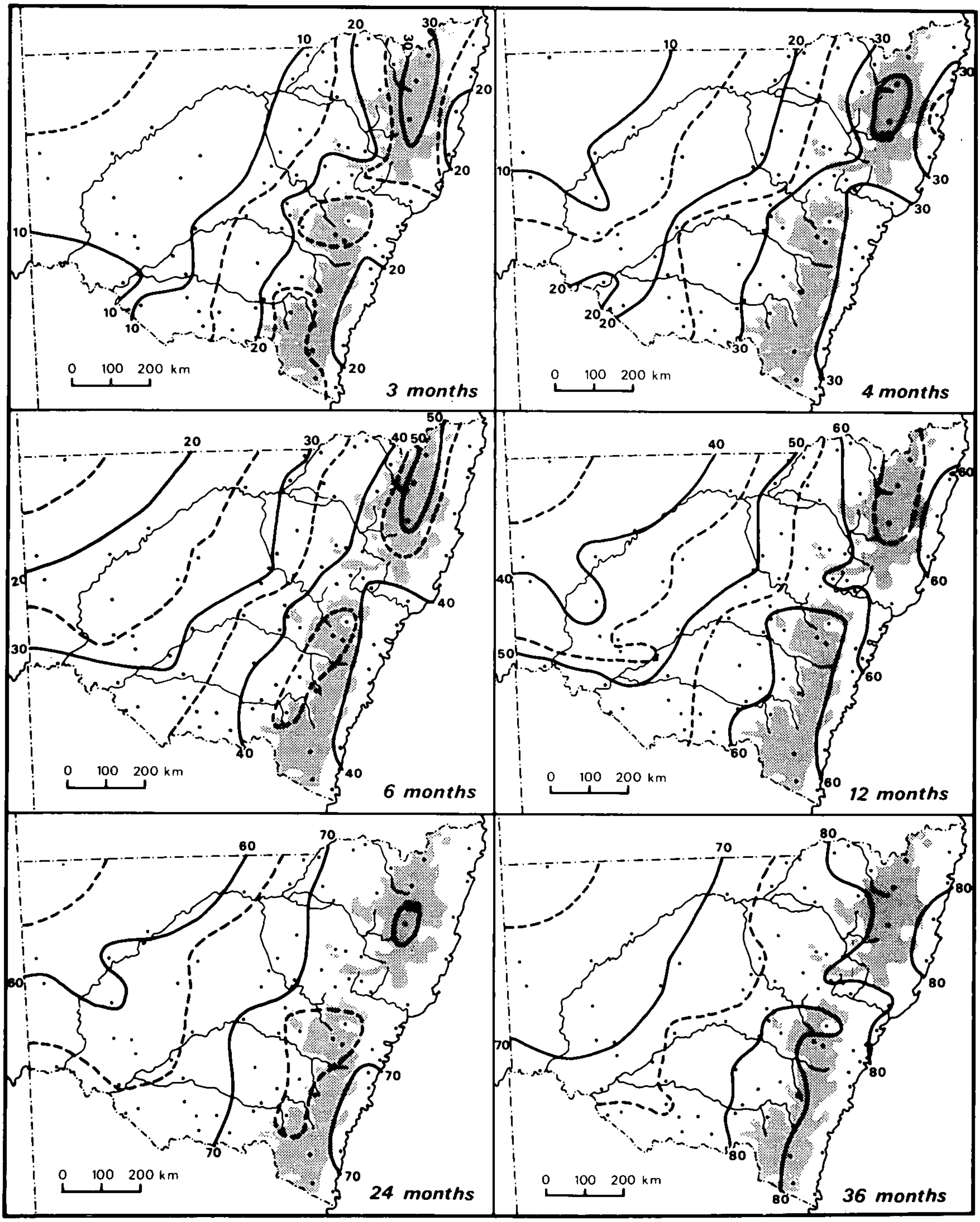


Fig. 5 Maps of New South Wales showing the '% of average' values for six different durations that will occur with a ten-year return period.

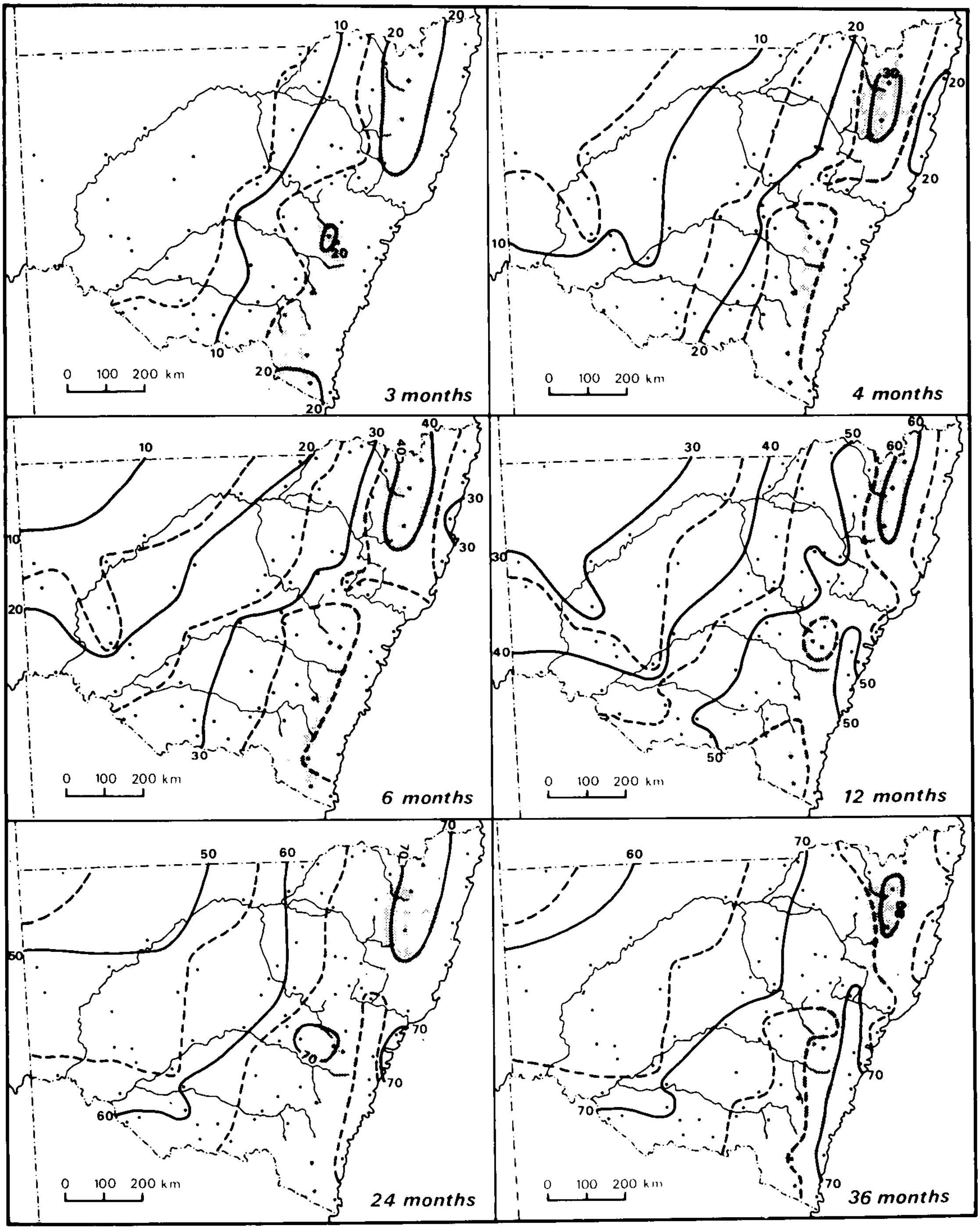
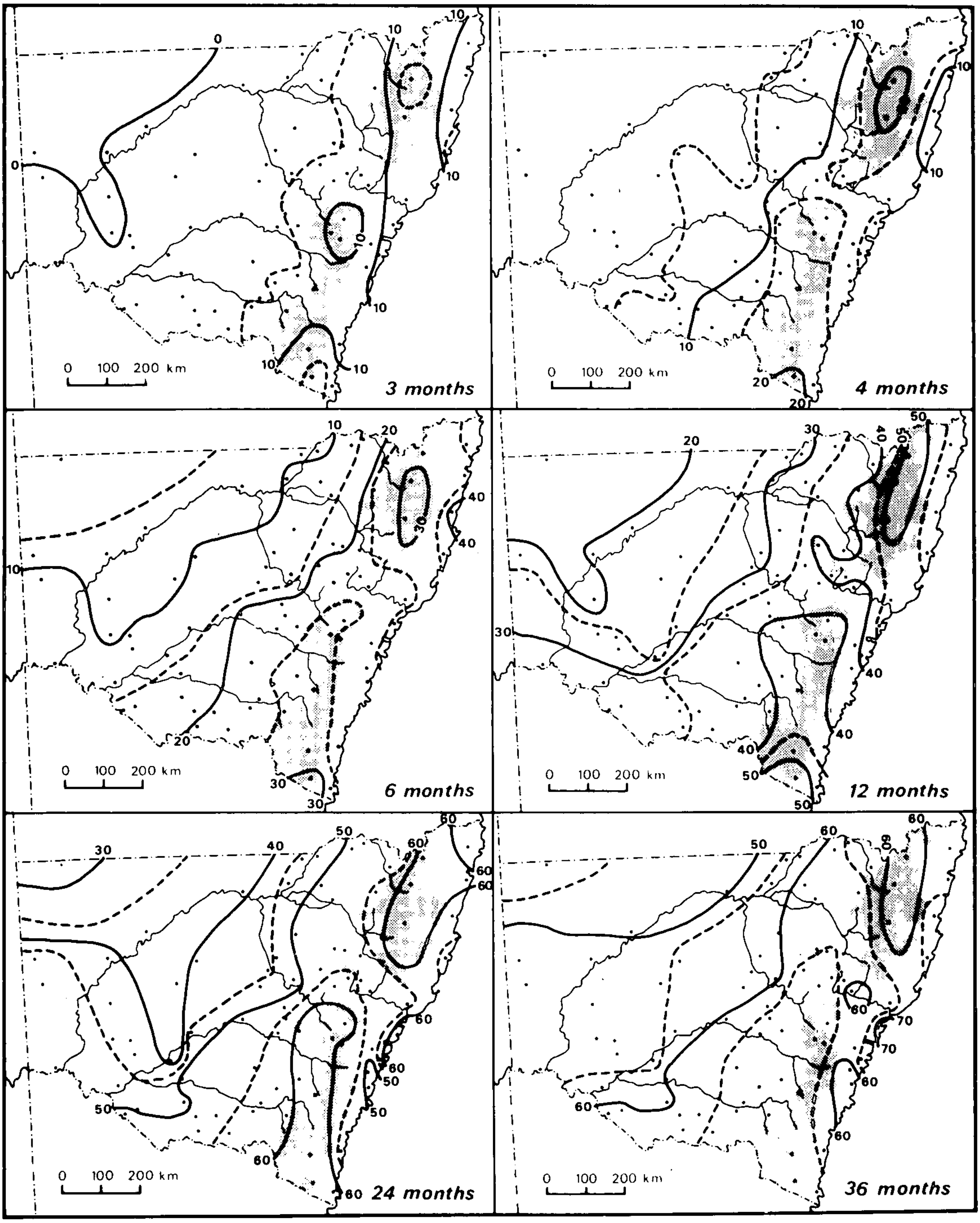


Fig. 6 Maps of New South Wales showing the '% of average' values for six different durations that will occur with a 50-year return period.



partially counterbalanced by a tongue of more extreme conditions often extending southeastwards from the lower Darling to the lower Murrumbidgee.

The detailed spatial differences apparent between maps is often no more than a function of the exact isopleth interval, but there is a real difference in the gradient of change both between different time units for the same return period, and between return periods themselves, as measured by the difference between actual percentage values of the stations with the highest and lowest values. Thus the greatest intensity of spatial contrast (the steepest gradient) for the two-year return period (Fig. 3) is at the three and four-month level. As the return period lengthens, so the time unit of maximum spatial contrast also lengthens, from the six-month unit for a five-year return period (Fig. 4) to the six and 12-month units for a 10-year return period (Fig. 5), and to the 12-month unit when a return period of 50 years is being considered (Fig. 6). Also, for the shorter return periods, gradients are least for the longest duration time units (i.e. 24 and 36 months), but this no longer holds true as longer return periods are considered. These trends also continue further if a 100-year return period is mapped.

An alternative way of considering these relationships is presented in Fig. 7 where, for a given duration time unit, the return period is mapped for a specific '% of average' or deficit value. Four examples are given, the first of which shows the location of various return period isopleths for 90 per cent deficit (10 per cent of average) rainfall over a three-month period. The two-year return period isopleth lies in the far northwest and the 100 year isopleth encircles the Northern Tablelands and crosses the far southeast of the State. Comparable changes are also shown in the other three examples.

From these analyses, it is also possible to graph the inter-relationships between deficit intensity, duration and frequency for individual locations (Fig. 8). These graphs, which comprise two east-west cross-sections of the State, provide an alternative

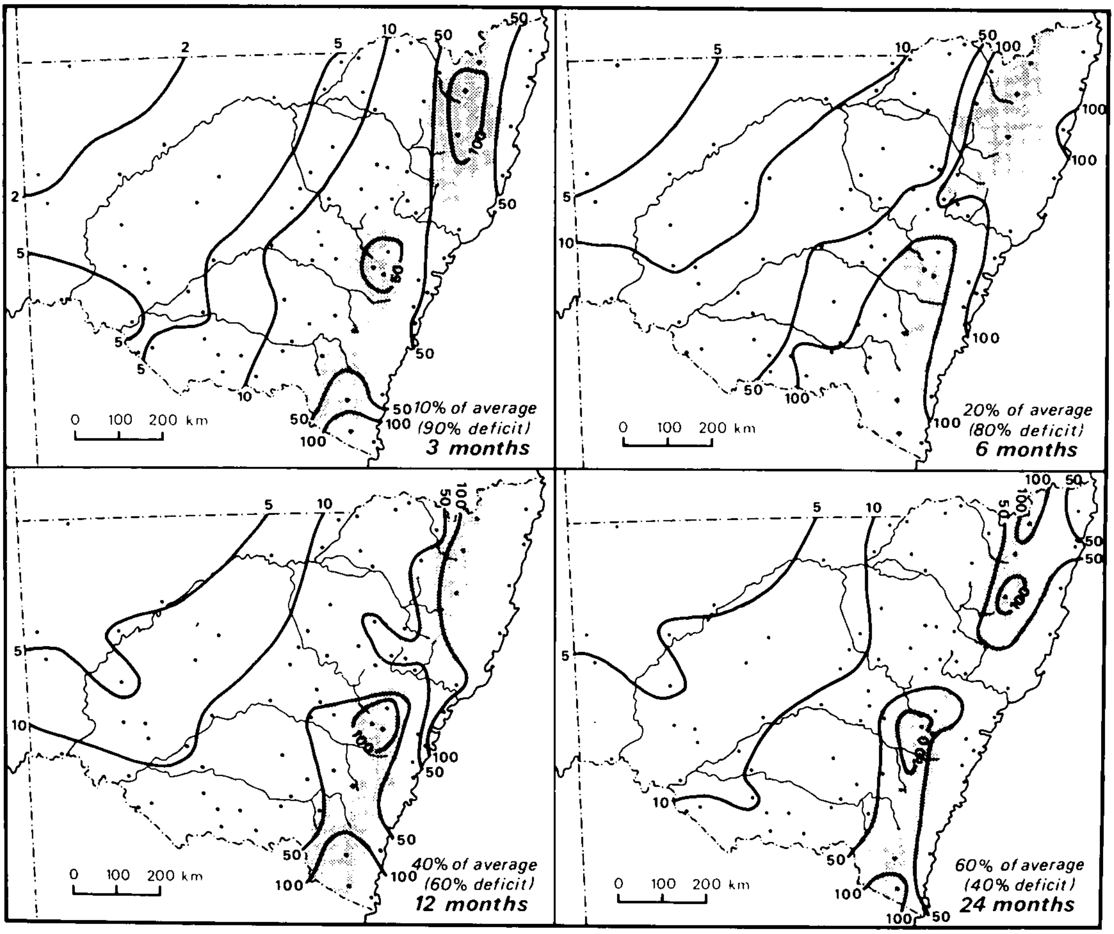
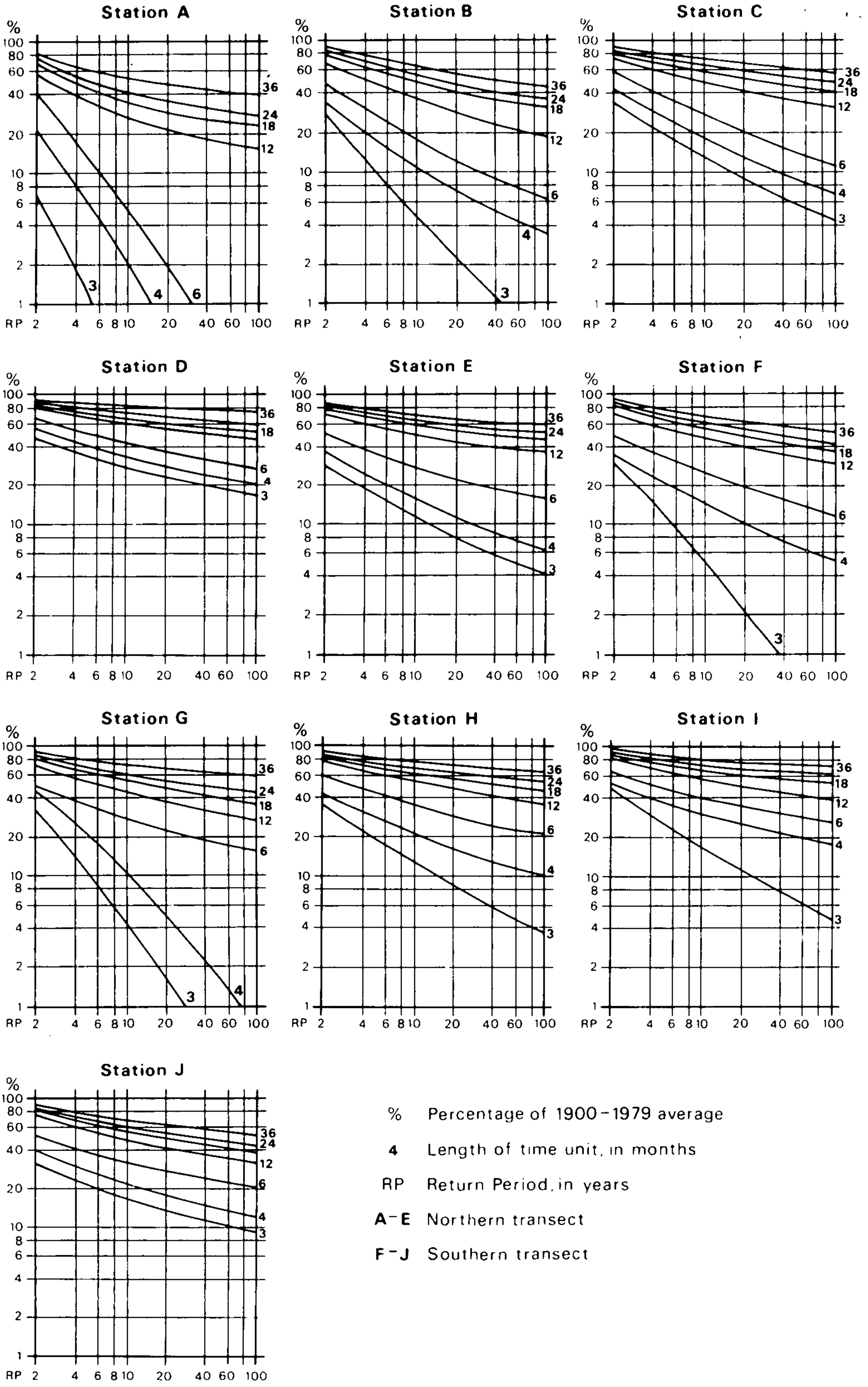


Fig. 7 Selective examples of the location of different return periods for a given '% of average' value for a specified duration.

Fig. 8 Graphs of the inter-relationships between '% of average' values (or deficit), duration of such conditions in months, and the return period in years, for ten selected stations (for location, see Fig. 1).



view of spatial change from one station to another, and also permit interpolation for different (unmapped) durations and frequencies. In this way, they provide a useful adjunct to the maps themselves, depending on the use to which they are put.

Of the maps presented, those for the two and five-year return periods must be seen as extreme conditions occurring sufficiently frequently for them to be envisaged as part of the operational environment of such activities as arable and livestock farming. For example, not to prepare a strategy to cope with a three-month period with at least a 55 per cent deficit on the Northern Tablelands or at least a 65 per cent deficit in the Riverina, when such a three-month situation can be expected to occur on average with a return period of two years, could be seen as most improvident. At the 10-year return period range, a farmer may begin to think more in terms of an insurance strategy rather than an operational one, but a water supply organisation would clearly have to plan for this, e.g. a six-month deficit of at least 65 per cent one year in ten in the Sydney area or at Wagga Wagga (Station H), or of at least 80 per cent at Broken Hill. Once the 50-year to 100-year return periods are being considered, policy and operational decisions become even more difficult, with, for example, deficits over a 24-month period across the western half of the State exceeding (and in some locations considerably exceeding) 50 per cent of the average one year in 50, and with deficits over a 36-month period exceeding 40 per cent one year in 100 along many of the coastal areas of the State.

The maps and graphs thus present information on extreme rainfall deficits in an easily assimilable form, the relevance and usefulness of which can best be evaluated by individual potential users. For some of the latter, an additional critical issue is the specific time of the year that a three, four or six-month deficit occurs, as this will obviously affect its impact on particular crops. Again, in areas of very marked seasonality of rainfall, an extreme deficit in an already very dry period of the year has little practical relevance. Both of these issues require further study before the method can be more widely applied in a useful way. However, when the longer time periods of a year or more are considered these issues become minor or irrelevant ones, and the spatial patterns of long-duration extreme rainfall deficits with different return periods become of critical importance for all water-using activities.

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