Aust. Met. Mag. 39 (1991) 205–209

Hydraulic jump in a fog bank

Warwick Grace
Regional Office, Bureau of Meteorology, Adelaide, Australia
(Manuscript received October 1990; revised July 1991)

Photographic evidence of a quasi-stationary hydraulic jump occurring in low-level flow and marked by fog is presented. The theory of shallow-water hydraulic flow through a constriction is found to model the essential features reasonably well.

Introduction

Mesoscale phenomena such as strong Antarctic katabatics, violent valley winds, downslope winds, lower level turbulent zones and hydraulic jumps have been explained with some success through analogy with shallow-water hydraulic behaviour. The analogy is most apparent when variations in cloud serve as visual markers. This article presents photographs in which the upper surface of a fog bank exhibits characteristics of a hydraulic jump. Examples of atmospheric hydraulic jumps caused by flow over a topographic obstacle have been documented: the example presented here appears to have been caused by flow through a flat-bottomed constriction. The occurrence and the essential features of the jump are explained with steady-state shallow-water hydraulic theory.

Observations

Sea fog is relatively rare in the Adelaide, South Australia, region. However, during a three-day period (1 to 3 May 1988) with high dew-points at coastal stations and generally light winds, extensive patches of sea fog were reported. These patches were also discernible from the operational visible satellite imagery as obtained by the Bureau of Meteorology.

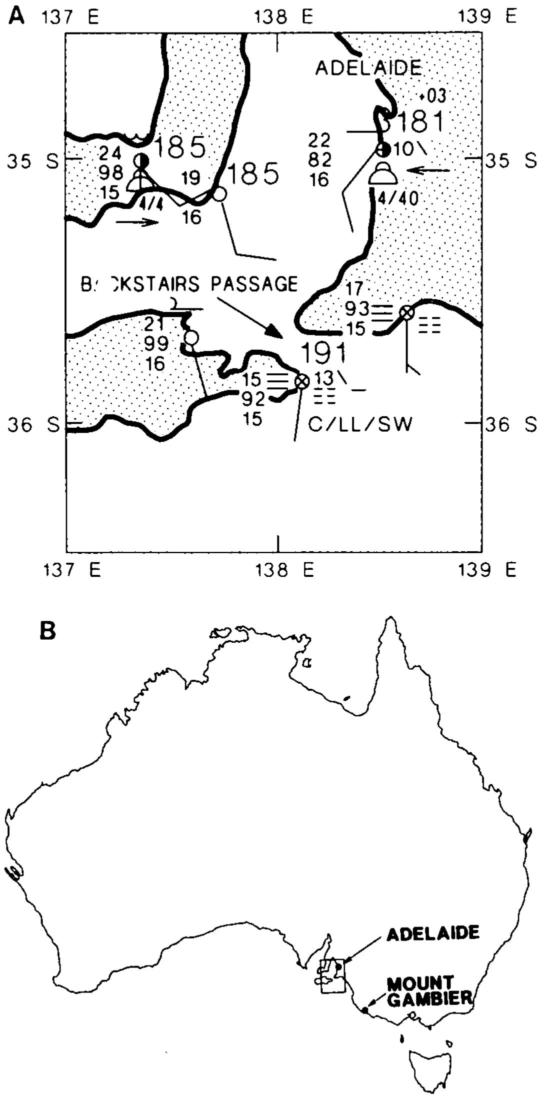
At 1500 local time on 2 May, sea fog was south of, and in, Backstairs Passage as shown on a portion of the relevant synoptic chart (Figs 1 and 2). (All times mentioned are local times.) Sea-surface temperatures either side of the passage are about 15°C at that time of year. Surface air temperatures were generally in the low to mid-twenties, except in the fog-affected areas where it was typically 15°C. The 0830 temperature sounding at Adelaide Airport and the mid-afternoon temperatures indicate that the mid-afternoon environment was approximately adiabatic, at least to 700 hPa. The 600 m wind at 1430 at Adelaide was 1 m/s from 110° and 3 m/s from 070° at Mount Gambier, which is 220 km southeast of Adelaide.

The photographs (Fig. 3) show a mainly smooth expanse of sea fog with the upper surface exhibiting a hydraulic jump. The upper photograph was taken at 1420 looking west from site P (see Fig. 2) which is 270 m above sea level; the composite lower one about an hour later from within 50 m of the same site. (Copses near the coast serve as reference markers.) These photographs are the central ones from panoramic sequences of up to five photographs.

In the earlier photograph the jump is of an undular nature; in the later photograph it is still undular but the first wave, which is in much the same location, is more pronounced and jump-like. The photographer, Mr Geytenbeek, who is an experienced glider pilot and whitewater canoeist, noted that although the undularities gradually waxed and waned, the location of the first wave or jump was fairly steady over the period during which he walked to and from the beach. He estimated that at site P it was about 25°C with a southeasterly wind of 2 to 5 m/s, while down on the beach it was 10 to 15°C with a southeasterly wind of 10 m/s (another photograph shows

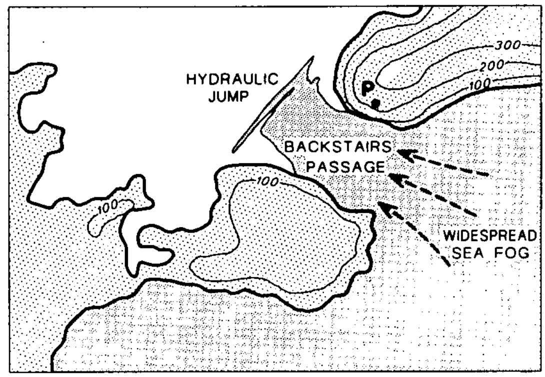
Corresponding author address: Mr W. Grace, Regional Office, Bureau of Meteorology, PO Box 421, Norwood, South Australia 5067, Australia.

Fig. 1 **(a) Synoptic chart for the Adelaide, South Australia, region at 1500 local time 2 May 1988. Stippling indicates land. Adelaide is the north-eastern station. (b) Map showing relation to the Australian continent.**



smoke-drift from a fire near the coast and is consistent with this estimate). He estimated that the waves were 100 to 200 m vertically from crest to trough; he also estimated the altitude of the wave crests to be about 300 to 350 m — inspection of these and several other photographs show these to be plausible estimates. The top of the relatively smooth portion of the fog sheet he estimated as 100 m. In at least qualitative terms many of these observed features may be explained with steady-state shallow-water hydraulic theory.

Fig. 2 **Larger scale projection of Backstairs Passage showing the hydraulic jump position. Contour heights in metres. Heavier stippling indicates the sea-fog bank as determined from observer descriptions and satellite imagery.**



Shallow-water hydraulic theory

Hydraulic flow over two-dimensional obstacles has been investigated by Long (1954) and many others. Summaries of the theory and observations are presented in Lighthill (1978) and Baines (1987). Arakawa (1969) extended the theory to flow through a mountain valley, and Pettre (1982) applied Arakawa's development to a flat-bottomed vertically-walled constriction (the Rhone Valley). A more comprehensive treatment of two-layer flow through a constriction is provided by Armi and Farmer (1986). An outline of the theory pertaining to a shallow homogeneous layer beneath an infinitely deep, less dense layer flowing through a flat-bottomed vertical-walled constriction is presented below.

Consider fluid of constant density flowing in the positive x direction. The fluid is of depth, d(x), and has velocity, u(x), independent of height. This allows the Froude number, F, to be defined as

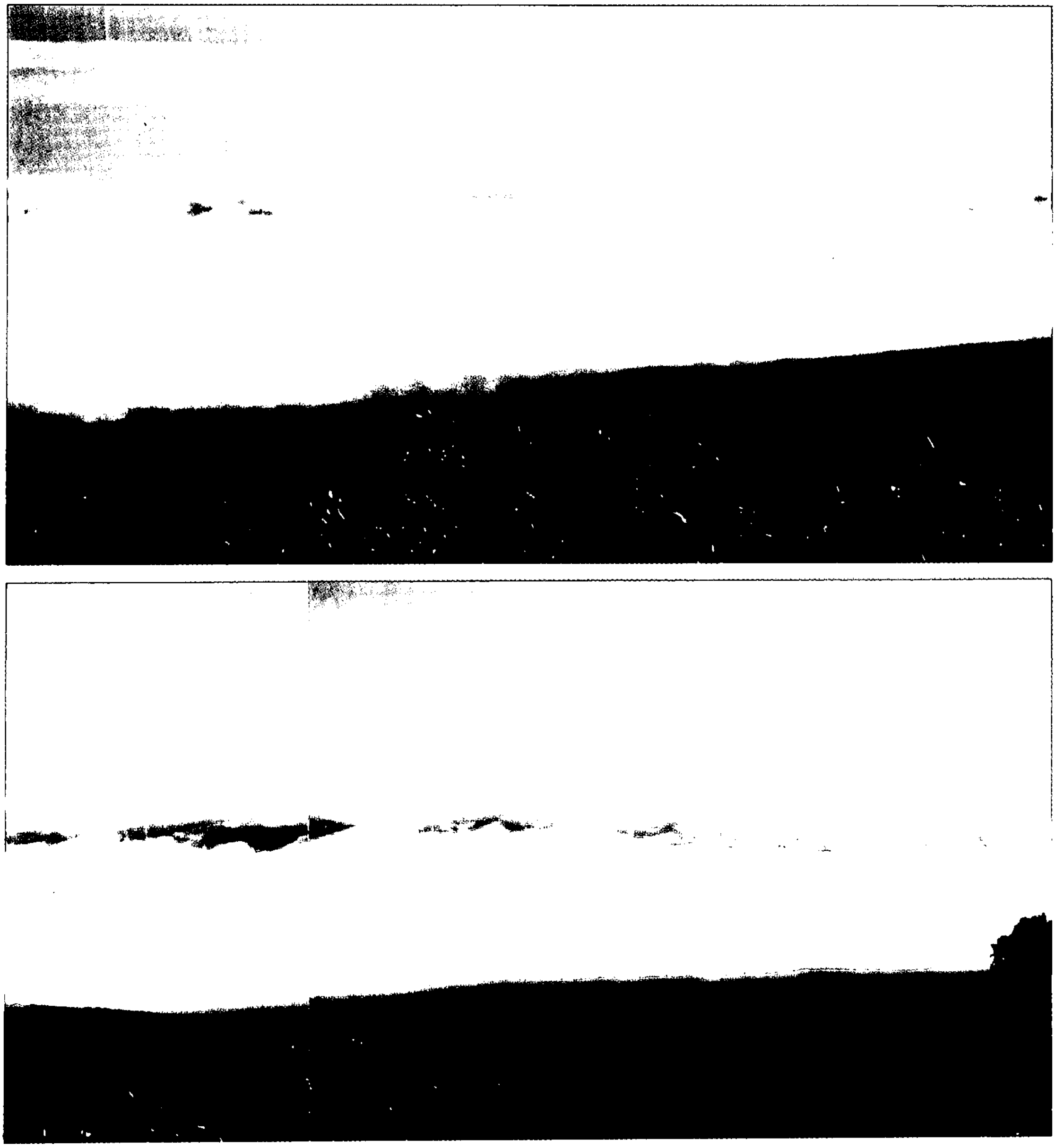
$$F = u(gd)^{-1/2} \qquad \ldots 1$$

where g is gravitational acceleration. However, note that there are several alternative definitions in the literature.

If $F = 1$ the flow is critical; if F is greater (less) than one then the flow is termed supercritical (subcritical). Sometimes supercritical flow is referred to as shooting or rapid flow, and subcritical flow as tranquil. Shallow-water gravity waves have a phase velocity equal to $u \pm (gd)^{1/2}$ and therefore cannot travel upstream in shooting flow.

Now consider flow through a flat-bottomed constriction. There are three flow configurations possible: flow which is everywhere subcritical, flow which is everywhere supercritical, and transitional flow (from subcritical to supercritical).

Fig. 3 Photographs of quasi-stationary hydraulic jump in a bank of sea fog, taken from site P looking west. Flow in the fog is fresh to strong and from left to right; above the fog the flow is light. The upper one was taken at 1420 local time and the second, a composite, about an hour later from near the same site (within 50 m).



There is a fourth configuration (transitional flow — from supercritical to subcritical) but this is not allowable in steady-state flow because it implies a violation of energy conservation (Long 1954).

As in Fig. 4, consider steady-state transitional flow of water with depth, d(x), through a channel of width, b(x), having a minimum width of b_c at x = c. The flow is from left to right in the positive x direction. The flow velocity is u = u(x) and is constant with height. Differentiation with respect to x is denoted by primed values. Subscripts o, c, 1 and 2 denote upstream, minimum width, and before and after jump values respectively.

The continuity and momentum equations may be expressed as

$$(ubd)' = 0 \qquad \ldots 2$$

$$uu' + gd' = 0 \; . \qquad \ldots 3$$

Eliminating u′ and using

$$F^2 = u^2(gd)^{-1} \quad \ldots 4$$

gives

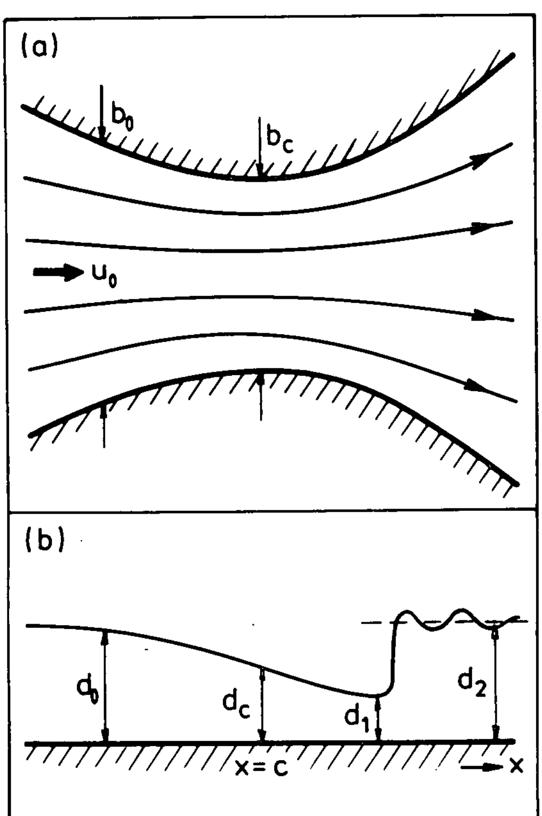
$$d' = -db'b^{-1}F^2(F^2-1)^{-1} \, . \quad \ldots 5$$

Note that Eqn3 is not valid at the jump as energy is dissipated there. Equation 5 shows that for $b' < 0$ and $F < 1$ then $d' < 0$ and hence $u' > 0$. That is, subcritical 'windward' flow is accelerated; similarly supercritical leeward flow is accelerated. To have acceleration on both sides of the obstacle it is apparent that there must be a transition from $F < 1$ to $F > 1$. It follows that $F = 1$ at some point.

However, if d′ is to be continuous and finite, then Eqn5 shows that F may only equal unity at $b' = 0$, that is, at the narrowest cross-section. Application of L'Hôpital's rule will show that d′ is indeed finite if $F = 1$ and $b' = 0$, such that (at the narrowest section)

$$d'^2 = d^2b''(3b)^{-1}, \quad \ldots 5(a)$$

Fig. 4 **Schematic plan and side views of transitional flow through a constricted passage which is flat-bottomed and vertically walled. The flow accelerates both going into and out of the constriction, and then decelerates at the jump.**



where the negative root is the physically appropriate value. The condition for transitional flow is therefore that $F_c = 1$. Exactly how the flow attains such a steady-state condition is discussed by Baines (1987).

Integrating Eqns 2 and 3, then

$$ubd = Q \text{ (a constant)} \quad \ldots 6$$

and

$$0.5\, u^2/g + d = K \text{ (a constant)}. \quad \ldots 7$$

Eliminating u from Eqns 6 and 7 gives

$$Q^2/(2gb^2) + d^3 = Kd^2. \quad \ldots 8$$

Differentiating with respect to x gives

$$-Q^2b'/(gb^3) + 3d^2d' = 2Kdd' \quad \ldots 9$$

so for $b_c' = 0$ then either $d_c' = 0$ (corresponding to the non-transitional flow) or $d_c = (2/3)K$ (corresponding to the transitional flow).

In the transitional flow case the flow must eventually return to subcritical in the wide region downstream. It is known that this may only be accomplished via a hydraulic jump; that is, by a sudden increase in height and consequent decrease in velocity. Using continuity and momentum flux considerations it may be shown that

$$d_2 = 0.5d_1\,((1 + 8F_1{}^2)^{1/2} - 1). \quad \ldots 10$$

For a homogeneous layer with potential temperature, Θ_L, and depth, d, overlaid by a deeper layer with temperature of Θ_U, it is known (Baines 1987) that the above equations remain valid provided g is replaced by 'modified' gravity, g*, where

$$g^* = g(\Theta_U - \Theta_L)\,/\Theta_U. \quad \ldots 11$$

Discussion and conclusions

The constriction model is not strictly applicable to Backstairs Passage since the topographic height on the southern side is less than d_c, so the results are anticipated to be primarily qualitative. Applying the model and using the available observations and estimates it is apparent that Θ_L = 15°C while Θ_U is in the low to mid-twenties, say 22°C; so $g^* = 0.23$ m/s². Next we set $d_1 = 100$ m, $u_1 = 10$ m/s. Equation 7 then shows that $K = 320$ m, hence $d_c = 210$ m and $u_c = 7$ m/s. The value of d_o lies between d_c and K, but depends upon the value of u_o. These calculated values are plausible but unfortunately there is no independent check available. The mean post-jump height, d_2, is expected to be comparable to, but less than, d_o. The value of d_2 is determined from Eqn 10 as 250 m, which is in accord with both the expected range of d_o (between 210 and 320 m) and with the observer's estimates. Equation 6 implies that $u_2 = 4$ m/s, a value also consistent with the available observations.

It is concluded that steady-state shallow-water hydraulic flow through a flat-bottomed vertically-walled constriction models the essential features of a hydraulic jump observed in fog cloud streaming northwestward through Backstairs Passage on 2 May 1988. This particular event was documented only because it was marked by relatively rare sea fog and then photographed and carefully noted by an astute observer in the right place at the right time: many otherwise similar events must pass unobserved.

Acknowledgments

R. Geytenbeek of Marion High School, South Australia is thanked for photographs and notes and for bringing the event to the author's attention. Colleagues at the Bureau of Meteorology in Adelaide and Melbourne aided in drafting the text and diagrams. Dr R. Byron-Scott and Dr Y. Shao of Flinders University provided many useful comments. Advice from Dr P. Baines of CSIRO considerably improved the mathematical exposition and Dr P. Manins of CSIRO helped clarify the presentation.

References

Arakawa, S. 1969. Climatological and dynamical studies on the local strong winds, mainly in Hokkaido, Japan. *Geophys. Mag.*, *34*, 359–425.

Armi, L. and Farmer, D.M. 1986. Maximal two-layer exchange through a contraction with barotropic net flow. *J. Fluid Mech.*, *164*, 27–51.

Baines, P. 1987. Upstream blocking and airflow over mountains. *Ann. Rev. Fluid Mech.*, *19*, 75–97.

Lighthill, J. 1978. *Waves in Fluids*. Cambridge University Press, Ch 2&3, 89–245 and Epilogue, 437–69.

Long, R.R. 1954. Some aspects of the flow of stratified fluids II. Experiments with a two fluid system. *Tellus 6*, 97–115.

Pettre, P. 1982. On the problem of violent valley winds. *J. Atmos. Sci.*, *39*, 542–54.