Analyzes of inertia-gravity waves in upper-air soundings made from Macquarie Island

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Inertia-gravity waves are observed in the lower stratosphere over Macquarie Island in high-resolution upper-air ozonesonde soundings of wind and temperature during 1994 and November 1996 – August 1997. Hodograph analyses of the soundings show that inertia-gravity waves are least common in the stratosphere during the austral winter. Furthermore, the analyses imply that the waves originate in the troposphere.

Most of the study focuses on 27 March 1994. Two gravity wave modes are identified in the ozonesonde sounding using a continuous wavelet transform analysis. These modes have vertical wavelengths of ~2 km and 5-6 km. The smaller vertical wavelength mode is also identified by a modified Stokes’ Parameter method and observed in the hodograph of the horizontal perturbation winds indicating good agreement between the methods.

The source of the two gravity wave modes is investigated using GROGRAT, the ray-tracing model developed by Marks and Eckermann. A cone of rays is released 21 km above Macquarie Island and traced backwards in time. These rays suggest that both gravity wave modes were generated in the jet-front system upstream of Macquarie Island (at the time of the sounding). The vertical wavelength varied along the ray paths with the largest values occurring in the vicinity of the jet core.

Introduction

Upward-radiating internal gravity waves provide an important mechanism for transporting energy and momentum from the troposphere into the middle atmosphere. Subsequent dissipation of the gravity waves at high levels in the atmosphere acts to decelerate the winds at those levels, and in this way, the waves help drive the meridional circulation of the middle atmosphere. It is thought that gravity waves are generated principally in the troposphere by flow over orography (for example see Baines (1995), and Lane et al. (2000)); by convection (for example see Lane et al.

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(2001), and Lane and Reeder (2001a, b)) and by jet-front systems (for example see Griffiths and Reeder (1996), and Reeder and Griffiths (1996)). Orographic gravity wave drag parametrisations have been incorporated into numerical weather prediction and climate models leading to significant improvements in their simulation of winds in the upper and lower stratosphere, although serious deficiencies remain.

Over the Southern Ocean, there are few observations of gravity waves and their variability in the troposphere and the stratosphere. One exception is the study by Guest et al. (2000), who (a) analysed the properties of inertia-gravity waves observed in ozonesonde soundings made over Macquarie Island (54.5°S, 158.9°E) during 1994; (b) examined how those properties varied with season; and (c) investigated the likely source of the identified waves. Guest et al. found that the inertia-gravity wave signal in the lower stratosphere had a seasonal cycle with the minimum during the austral winter. A common synoptic pattern, observed to be present upstream of Macquarie Island for days on which inertia-gravity waves were detected, had a similar seasonal cycle. GROGRAT, the ray-tracing model developed by Marks and Eckermann (1995), was used to investigate the source of waves detected in the 25 October 1994 sounding. The rays suggested that the inertia-gravity waves were generated in the jet-front system southwest of Macquarie Island, which is located in the Southern Ocean away from any orographic sources of low frequency, long wavelength gravity waves.

The present study extends the work of Guest et al. (2000). In particular, results using the analysis techniques described by Guest et al. are compared with those obtained using continuous wavelet transform techniques. For the most part, the analysis focuses on 27 March 1994, as the sounding taken on this day shows a pronounced inertia-gravity wave signal in the stratosphere. GROGRAT is used to identify possible sources on this day and to examine the changes in wave properties along the ray paths. These results support the conclusions reached by Guest et al.

Sounding data

In this study high-resolution upper-air Omega-sonde ozone soundings are used to identify inertia-gravity waves. These soundings were launched approximately weekly from Macquarie Island during the 1994 Airborne Southern Hemisphere Ozone Experiment-Measurements for Assessing the Effects of Stratospheric Aircraft (ASHOE-MAESA) programme. In November 1996, the Cooperative Research Centre for Southern Hemisphere Meteorology (CRC SHM) and the Antarctic Science Advisory Committee funded a project to recommence weekly ozonesonde flights. During 1994, 42 of the 47 flights reached 30 km, and 33 of the 37 flights launched during the period from November 1996 to August 1997 reached this height. Prior to September 1997 the sounding system used Omega navigation receivers to determine the position of the sonde and hence the wind velocity. After this date GPS navigation receivers were used instead. Unfortunately, the GPS signal proved difficult to detect reliably at Macquarie Island. Consequently, we have confined our analysis to soundings made before the changeover.

The sonde measures pressure, temperature, relative humidity, dew-point temperature, and horizontal wind direction and speed every 10 s (or roughly every 50 m in the vertical). Although the sonde measures ozone concentration also, those data are not used in analysing gravity waves. Between 3 and 1060 hPa, the pressure sensor on the Omega-sonde has a resolution of ±0.1 hPa and an accuracy of ±0.5 hPa. The temperature has a resolution of ±0.1 K and is accurate to ±0.2 K over the interval 183-333 K. The humidity sensor has a resolution of ±1.0% RH and is accurate to ±2.0% RH. Finally, the random error in each component of the wind is approximately 0.5 m s⁻¹. The date, maximum height (triangles) and tropopause height (circles) for each sounding are shown in Fig. 1. The tropopause height is typically between 8 and 12 km over Macquarie Island. Note that fewer flights occurred in the months from September to January.

Individual soundings are separated into background and perturbation profiles over the interval 12-30 km. The background profiles are defined by fitting least squares fourth-order polynomials to the sounding, while the differences between the soundings and the background profiles define the perturbation profiles. Using a Fourier transform, the perturbation profiles are filtered to remove the very low and very high frequency oscillations. To reduce leakage, a 1.5 km segment at each end of the data segment is tapered using the Hanning window. Only those data that have been detrended in this way are used in the following analyses, and consequently the analysis is confined to the height interval between 13.5 and 28.5 km. (See Guest et al. 2000 for further details.)

Hodograph analysis

Under stable conditions, fluid parcels displaced at some angle to the vertical will oscillate due to the action of the buoyancy force. Propagating oscillations
of this type are called gravity waves, and the low-frequency gravity waves whose motion is also influenced by the Earth's rotation are known as inertia-gravity waves. The key signature of gravity waves is an oscillation in the field variables (such as velocity and temperature). Figure 2 shows vertical profiles of the detrended perturbation zonal wind $u'$, meridional wind $v'$, and temperature $T'$. The oscillations in these profiles are typical of gravity waves. An important property of upward-radiating, plane monochromatic inertia-gravity waves in a uniform background flow is that their horizontal perturbation velocity vector ($u'$, $v'$) rotates anticyclonically with height. Conversely downward propagating waves are characterized by cyclonically rotating perturbation velocity vectors. The curve traced out by the perturbation velocity vector is an ellipse, where the eccentricity, the ratio of semi-major and semi-minor axes, is proportional to the ratio of the intrinsic frequency, $\omega_i$, and the local value of the Coriolis parameter, $f$. The intrinsic frequency, known also as the Doppler-shifted frequency, is the frequency of the wave as measured by an observer moving with the background flow. The orientation of the semi-major axis of the ellipse gives the horizontal direction in which the wave propagates. (See, for example, Gill (1982) for a more detailed discussion of inertia-gravity waves and their properties.)

Figure 3 is a plot of the horizontal perturbation winds in the stratosphere as a function of height (13.5-28.5 km) on 27 March 1994. In this figure the abscissa measures $u'$, while $v'$ is plotted against the ordinate. Such plots are known as hodographs. The crosses mark the height of the balloon above ground level at 1 km intervals. The perturbation wind vector makes about seven complete counterclockwise (i.e., anticyclonic in the southern hemisphere) rotations, implying upward energy propagation, and therefore, a tropospheric source. The height interval taken for the curve to complete a loop gives an estimate of the vertical wavelength. For 27 March 1994, the vertical wavelength estimates from each of the seven loops are: 2.5, 1.9, 3.1, 2.0, 1.5, 1.7, and 2.3 km. These values have an average of 2.1 km.

Hodographs above and below the tropopause were examined for all the soundings and the results summarised in Fig. 1. This figure shows that upward wave propagation is dominant above the tropopause and that there is a mixture of upward and downward wave propagation within the troposphere. These findings suggest that the waves are generated predominantly in the troposphere. Moreover, fewer soundings show the signature of inertia-gravity waves during the austral winter than at other times of the year. However one needs to treat this conclusion with caution as the hodograph ellipse collapses to a line for higher frequency gravity waves. Consequently, the reduced number of inertia-gravity waves analysed during the austral winter may simply reflect a shift in the gravity waves to frequencies higher than that detectable in hodographs.
Fig. 2  Vertical profiles of the detrended perturbation (a) zonal wind, (b) meridional wind, and (c) temperature. The sounding was made on 27 March 1994 and only the stratospheric data within 13.5-28.5 km are shown.

Fig. 3  A hodograph of the horizontal perturbation winds in the stratosphere on 27 March 1994. The horizontal axis marks the perturbation zonal wind component u' and the vertical axis the perturbation meridional wind component v'. The height (in km) is shown at each cross (×) and the winds have units of m s\(^{-1}\).

Estimating the wave parameters

This section compares the gravity wave parameters determined using a modified form of the Stokes' Parameter method (as described in Guest et al. (2000)) with a Continuous Wavelet Transform (CWT) analysis. It will be shown that the hodograph method, the modified Stokes' Parameter method, and the continuous wavelet transform produce consistent results. However, it is important to bear in mind that the results obtained from the three methods are not independent as they use the same data.

**Modified Stokes' Parameter method**

The method briefly outlined here is described in more detail in Guest et al. (2000), and is essentially a modification of the method developed by Vincent and Fritts (1987) and Eckermann and Vincent (1989). If the sounding is treated as a time series rather than an instantaneous vertical profile, the Fourier transform of the sounding gives the frequency of the wave as seen by an observer moving with the balloon; we call this the apparent frequency \(\Omega\). Then, for a single plane monochromatic wave the vertical wave number \(m\) can be determined from

\[
m = \frac{(\omega_s - \Omega)}{W_b} \quad \ldots 1
\]

where the ascent rate \(W_b\) is calculated directly from the sounding.

If \(u_0\) is the amplitude of the perturbation velocity in the direction of wave propagation, and \(v_0\) is the component normal to the propagation direction, then

\[
\frac{\omega_s}{f} = \frac{u_0}{v_0} \quad \ldots 2
\]

(e.g. Gill 1982). The (RMS) amplitudes of \(\hat{u}_0\) and \(\hat{v}_0\), as well as the direction of wave propagation, are estimated using the Stokes' Parameter method, and Eqn 2 is used to calculate \(\omega_s\). Note that \(u_0/v_0\) is the ratio of the semi-major and semi-minor axes of the hodograph. Thus, the key difference between the Stokes' Parameter method for calculating \(\omega_s\) and the hodograph method described in the previous section is the technique used to estimate \(\hat{u}_0\) and \(\hat{v}_0\).

The apparent frequency is taken to be the frequency of the peak in the power spectrum of the perturbation temperature time series. Next, the horizontal wavenumber \(K_h\) can be determined from the dispersion relation for gravity waves in a rotating atmosphere, which is,

\[
K_h^2 = m^2 \frac{(\omega_s^2 - f^2)}{(N^2 - \omega_s^2)} \quad \ldots 3
\]
where $N$ is the Brunt-Väisälä frequency (e.g. Gill 1982). The density variation with height has been neglected since $\lambda_h = \frac{2\pi}{H}$, where $H$ is the vertical wavelength and $H$ is the density scale height. When $m$ is real, gravity waves can exist only for intrinsic frequencies in the range $|f| < \frac{1}{2\pi} |\omega_0/n|$. The Stokes’ Parameter method and the temperature variations are then used to determine the orientation of the wavenumber vector (see Hamilton 1991).

Applying the method to the sounding taken on 27 March 1994 gives $\omega_0 = (1.3 \pm 0.1) f$, $\lambda_h = 1.9 \pm 0.2$ km, and $\lambda_h = 378 \pm 116$ km, where $\lambda_h = 2\pi K_h$ is the horizontal wavelength. Like the hodograph analysis, the results are based on measurements taken between the heights 13.5 and 28.5 km. The derived vertical wavelength compares very well with the estimate from the hodograph (Fig. 3).

**Continuous Wavelet Transform**

The Continuous Wavelet Transform (CWT) is a useful technique for identifying localised variations in the power of a signal. In this section, the CWT is applied to the sounding taken on 27 March 1994 interpolated to 100 m resolution.

The CWT is defined as the convolution of the signal $s$ and a basis function $g$. In this section, the signal will be either the perturbation wind or the perturbation temperature profile. The basis function, known as the Morlet wavelet, is defined by

$$g(z) = e^{i\omega_0 z} e^{-z^2/2}$$  

where $\omega_0 = \pi \sqrt{2/\ln 2} = 5.336$. The Morlet wavelet represents an amplitude-modulated plane wave in which the modulating function is Gaussian. Mathematically, the CWT can be written as

$$\text{CWT}(\lambda_{\gamma}, z) = \frac{a}{\sqrt{\lambda_{\gamma}}} \int s(\xi) g^\ast \left( \frac{\xi - z}{\lambda_{\gamma}/a} \right) d\xi$$  

where (*) indicates the complex conjugate, $\xi$ is the dummy variable of integration, $a$ is a constant equal to twice the vertical resolution (= 2 x 100 m). Note that the CWT is a function of two parameters: the vertical wavelength and the height.

Like the modified Stokes’ Parameter method, the analysis is performed in a coordinate system that is rotated horizontally through an angle $\theta$. The new coordinate system is defined such that one of the coordinate axes is aligned with the direction of wave propagation. Then, the amplitudes $u_0$ and $v_0$ are estimated from the CWTs of the rotated perturbation wind profiles, $u_0$ and $v_0$, and $\omega_0$ is calculated from Eqn 2. The horizontal wavelength is calculated from Eqn 3.

For gravity waves, there are specific relationships between the field variables. For example: (1) $u_0$ and $v_0$ are in quadrature, (2) $T'$ and $u_0$ are in quadrature, and (3) $T'$ and $v_0$ are in phase (in the southern hemisphere). If the CWTs satisfy these conditions to within 5°, the signal is assumed to be that of an inertia-gravity wave (for details see Chane-Ming et al. 2000a, 2000b, 2002).

The CWT and the Morlet wavelet have been used previously by Chane-Ming et al. (2000a, 2000b) to analyse inertia-gravity waves in Rayleigh lidar profiles and ozone soundings taken above La Reunion Island (20.8°S, 55.3°E).

Figure 4 shows the total energy (the sum of the kinetic and potential) for identified gravity wave modes as a function of the vertical wavelength determined from the 27 March 1994 sounding. The analysis identifies three gravity wave modes with vertical wavelengths: ~2 km (mode A), 5–6 km (mode B), and 8–9 km. As the analysis height interval is 15 km, wave perturbations with vertical wavelengths greater than 7.5 km are not properly resolved. For this reason, the following discussion will be confined to the origin of modes A and B. Interestingly, all three methods detected mode A, the mode with the smaller vertical wavelength. In the following section we will investigate possible source(s) of modes A and B.

**Wave source**

We investigate now the source(s) of the inertia-gravity waves identified in the preceding sections by examining the synoptic conditions close to the time of the flight, and using a ray-tracing model. Macquarie
Island is a ridge, roughly 32 km long and 4 km wide, and 300 m high, situated in the Southern Ocean far from other land masses. Orographically produced gravity waves have been detected downstream from Macquarie Island (for example, see Mitchell et al. (1990)), but they were stationary and possessed horizontal wavelengths much smaller than those analysed on 27 March 1994.

**Synoptic analyses**

National Centers for Environmental Prediction (NCEP) reanalyses have been used to produce cross sections of the synoptic flow. Figure 5(a) shows the 300 hPa geopotential height and wind speed fields at about the time of the ozonesonde sounding. On 27 March 1994, the 300 hPa surface is just below the tropopause, which is located at 9.8 km. Figure 5(b) shows the potential vorticity and the velocity vectors relative to the motion of the trough on the 330K surface at the same time. The tropopause on the 330K surface is defined to be the region bounded by the -2.0 \times 10^{-6} and -2.5 \times 10^{-6} K m^2 kg^{-1} s^{-1} potential vorticity contours. To the north of the tropopause the 330K surface resides in the tropopause while to the south it resides in the stratosphere. Together these plots show that there is a tongue of stratospheric air with a cyclonic circulation associated with the upper-level trough. Macquarie Island is located downstream of the upper-level trough in the vicinity of a jet.

Figure 6 shows the vertical cross-section of wind speed and the horizontal potential temperature gradient at 155°E (just upstream of Macquarie Island) at the same time as Fig. 5. Frontal zones, defined by regions of enhanced potential temperature gradient, are located above and below the intense polar jet core as a consequence of thermal wind balance. The synoptic pattern in Figs 5 and 6 is typical of that on all days in which upward propagating inertia-gravity waves have been detected (Fig. 1).

**Ray tracing**

GROGRAT, a ray-tracing model developed by Marks and Eckermann (1995), is the main tool used here to identify possible wave source(s). The model can accommodate internal gravity waves of any frequency propagating in a rotating, compressible, and slowly varying atmosphere described by numerically grided analyses of horizontal wind, pressure, and temperature. A subset (20°–80°S, 80°–180°E) of the global six-hourly NCEP reanalyses, with resolution 2.5°×2.5° in the horizontal and 1 km (0-30 km) in the vertical, is used to represent the time-varying background flow.

For the 27 March 1994 sounding, the zonal wavenumber, the meridional wavenumber and the ground-based frequency are estimated using the modified Stokes' Parameter method to be \( k_0 = 1.29 \times 10^{-5} \text{ m}^{-1}, \) \( l_0 = 1.05 \times 10^{-5} \text{ m}^{-1}, \) and \( \omega_0 = 1.01 \times 10^{-5} \text{ rad s}^{-1} \) respectively. Five values centred around \( k_0, l_0, \omega_0 \) are chosen for each of \( k, l, \omega \) to define a discrete spectrum of 125 individual components. In particular, \( k = k_0 \pm 3 \times 10^{-6} n, \) \( l = l_0 \pm 3 \times 10^{-6} n, \) and \( \omega = \omega_0 \pm 5 \times 10^{-6} n, \) where \( n = 0, 1, 2. \) The waves comprising the spectrum have horizontal wavelengths in the interval 250–760 km, and vertical wavelengths between 1.3 and 5.9 km. Note that the spectrum includes waves with the vertical wave-
lengths of modes A and B identified by the CWT. The rays were released 21 km above Macquarie Island at the midpoint of the analysis height range 13.5-28.5 km, and then integrated backwards through time to possible source locations.

Ray tracing requires the background flow, and hence the properties of the wave packet, to vary slowly in time and space. In the ray-tracing model, backward integration of a ray is stopped when \( \delta = 1 / m^2 \) \( | \partial m / \partial z | \geq 1 \) (Marks and Eckermann 1995). In other words, the vertical wavelength cannot become too large or vary too quickly in the vertical. Regions where \( \delta \geq 1 \) are not necessarily the source region for the waves observed in the stratosphere above Macquarie Island. On the other hand, such regions tend to be associated with jets and fronts, which are the likely wave sources. (For example, see Reeder and Griffiths (1996)).

Figure 7 shows a latitude-altitude cross section of the wind speed at 155°E, 14 h after the rays were released with all the ray paths projected onto this plane. Most rays can be integrated back to within the latitude range 145-160°E and the time interval 4 to 26 h after release. As the zonal flow only varies slowly in the zonal direction, the wind speed cross-section shown is a reasonable representation of the flow at the end points for most rays. Three distinct termination heights, ~11 km, ~4 km, and the surface, can be seen in the figure. A comparison of Fig. 7 with Fig. 6 shows how slowly this cross-section varies at this time and location, and that the waves can be traced back to the frontal zones associated with the polar jet downstream of an upper trough.

As shown in Fig. 8, some of the rays terminate above the jet core while others can be traced back to heights near the surface. A horizontal plane section of pressure and wind speed at an altitude of 11 km, 8 h after release is shown in Fig. 8. This set of rays all ended close to the jet core on the anticyclonic side. Figure 9 shows a plane-section of pressure and potential temperature at a height of 1 km, 20 h after release. The rays that are traced backwards to this plane appear to terminate within the surface cold front that extends downstream over Macquarie Island. These plots (Figs 7-9), together with an examination of the end point locations for all rays, shows that the rays almost all terminate in the vicinity of the jet or one of the associated frontal regions; i.e. within the same jet-front system.

Figure 10 shows the variation with height of the vertical wavelength of the rays between the release point at an altitude of 21 km over Macquarie Island, and the possible source regions within the jet-front system. Over Macquarie Island the rays had an average vertical wavelength of 3.4 km. The vertical wavelength is typically greatest near the altitude of the jet where it has an average value of 24.6 km. Those rays that reached the surface had an average vertical wavelength of 8.5 km.

Mode A (with vertical wavelengths ~2 km) waves terminate at all three heights mentioned above. However, the mode B waves (with larger initial wavelengths ~5.5 km) have greater vertical wavelengths at all heights, all terminate above 3 km, and have a greater vertical group velocity.
Summary and conclusions

This study used high-resolution upper-air Ome- 
sonde soundings launched over Macquarie Island during 1994 as part of the ASHOE-MAESA programme and during 1996-97. Inertia-gravity waves identified in the lower stratosphere during 1996-97 showed a seasonal cycle similar to that previously identified using the 1994 soundings (Guest et al. 2000). In particular, inertia-gravity waves appeared to be much less common during the austral winter than at other times of the year. Hodograph analysis suggested also that the waves were generated in the troposphere.

The rest of the study focussed on 27 March 1994, as the sounding taken on this day showed strong evidence of inertia-gravity waves in the stratosphere. The hodograph showed upwardly propagating inertia-gravity waves in the stratosphere with an approximate vertical wavelength of 2.1 km.

The gravity wave parameters were determined using a modified form of the Stokes’ Parameter method, and a Continuous Wavelet Transform (CWT) analysis. The Stokes’ Parameter method estimated the vertical wavelength to be 1.9 ± 0.2 km, which agreed well with the estimate from the hodograph. The CWT analysis identified gravity waves with wavelengths of approximately 2 km and 5-6 km. Mode A, the mode with the smaller vertical wavelength, was identified by each of the three methods.

Synoptic analyses showed Macquarie Island located downstream of an upper-level trough in the vicinity of a jet stream on 27 March 1994. There was a tongue of stratospheric air with a cyclonic circulation associated with the upper-level trough and frontal zones located above and below the jet core. GROGRAT, the ray-tracing model developed by Marks and Eckermann, was used to search for possible source regions for the identified inertia-gravity waves. A cone of rays was released 21 km above Macquarie Island and traced backwards in time. These rays had a range of vertical wavelengths that included those of modes A and B, identified by the three analysis methods. The jet-front system upstream of Macquarie Island appears to be the source of the inertia-gravity waves. Rays with the same vertical wavelengths as modes A and B, terminated within the same jet-front system indicating both modes could have originated within the same jet-front system. Guest et al. (2000) used GROGRAT to investigate the source of inertia-
gravity waves identified in the 25 October 1994 sounding. A jet-front system southwest of Macquarie Island, similar to that identified in this study, was suggested as the source of the waves. The vertical wavelength varied along the ray paths with the largest values occurring in the vicinity of the jet core.

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References


