The turning winds with height thermal advection rainfall diagnostic: Why does it work in the tropics?

Kevin J. Tory
Centre for Australian Weather and Climate Research, Bureau of Meteorology, Australia

(Manuscript received February 2014; revised July 2014)

For decades forecasters in the Queensland Regional Forecast Office of the Australian Bureau of Meteorology have been using winds that turn with height as a rainfall diagnostic, with good success in the subtropics and tropics. The diagnostic is based on the relationship between thermal advection (with its implied isentropic ascent or descent) and geostrophic winds that turn with height. This paper demonstrates that the above relationship also holds for gradient winds that turn with height, which suggests the relationship should be true for any slowly evolving pressure anomalies. Furthermore, it explains why the diagnostic is successful at low latitudes for most heavy rain bearing mesoscale and larger systems, including tropical depressions, tropical lows, monsoon depressions, tropical cyclones, and hybrid tropical/baroclinic storms such as the often destructive East Coast Low that forms off the eastern coast of Australia.

Equations are derived to demonstrate the relationship between thermal advection and turning gradient winds with height, and the implications of each term are discussed. As with the geostrophic relationship the sign of the thermal advection is determined by the rotation direction, with anticyclonic (cyclonic) turning with height representing warm (cold) air advection and isentropic ascent (descent). In the geostrophic wind relationship the thermal advection magnitude is proportional to the Coriolis acceleration, whereas in the gradient wind relationship the thermal advection magnitude is proportional to the sum of the Coriolis and centrifugal acceleration terms. Alternative forms of the diagnostic are proposed that incorporate the additional centrifugal acceleration.

Introduction

Synoptic and mesoscale thermal advection is often associated with isentropic ascent (warm air advection) and isentropic descent (cold advection), which can be used to diagnose qualitatively precipitation and precipitation free regions. The relationship between thermal advection and turning winds with height has featured in dynamic meteorology text books for decades (e.g. Haltiner and Martin 1957, Wallace and Hobbs 1977, Gordon et al. 1998, Zdunkowski and Bott 2003, Holton 2004, Lynch and Cassano 2006). It is based on the thermal wind equation, which is obtained by taking the vertical derivative of the horizontal momentum equation, while assuming hydrostatic balance. When the vertical gradient is discretised for steady flow a wind shear vector appears that is oriented parallel to the mean temperature gradient (e.g. Holton 2004, Eqn. 3.32). This relationship has proved to be very useful for forecasters to estimate thermal advection from a single sounding of wind data alone.

In Cartesian coordinates the assumption of steady flow implies geostrophic balance, and the thermal gradient is perpendicular to the geostrophic wind shear vector. Forecasters in the Queensland Regional Office of the Australian Bureau of Meteorology have been using thermal advection diagnostics based on this geostrophic relationship for decades to diagnose heavy rain in the tropics and subtropics. There has been some uncertainty
among the Australian forecasting community as to why the geostrophic diagnostics (valid for small Rossby number flows) should work for the many heavy rain bearing synoptic and mesoscale, subtropical and tropical systems identified by the Queensland Regional Office (e.g. Bonell et al. 2005, Callaghan and Bonell 2005, and Bonell and Callaghan 2008). This uncertainty, coupled with a bit of healthy scepticism, led to the diagnostic being dubbed ‘Snake Oil’. The main purpose of this paper is to demonstrate why the turning winds with height can be used to identify thermal advection in these low-latitude rainfall events.

The explanation is based on Forsythe’s (1945) derivation of the thermal wind equation in natural coordinates, which illustrates the relationship between gradient winds that turn with height and thermal advection. It shows that for steady flows of Rossby number order unity, which is likely to include tropical depressions, tropical lows, monsoon depressions, tropical cyclones, and hybrid tropical/baroclinic storms, application of observed winds to the geostrophic relationship can lead to a non-trivial underestimate of the thermal gradient. It follows that the apparent success of the Queensland Regional Office thermal advection diagnostics is in part due to the diagnostics being used qualitatively or at most semi-quantitatively. Despite this limitation a strong relationship between rainfall rate and the magnitude of one of the diagnostics is still evident in a 50 year climatology of rainfall events at two stations, one sub-tropical (Brisbane, latitude 27.4°S) and one tropical (Cairns, latitudes 16.9°S, Callaghan and Tory 2014).

In the next section the Forsythe thermal wind equation is introduced and the thermal advection diagnostic is re-derived assuming gradient wind balance. In section three the relationship between thermal advection and turning winds with height is further discussed, the limitations of various forms of the thermal advection diagnostic are raised, and an alternative form of the diagnostic is discussed. The paper is summarised in the last section.

Theory

(a) Thermal advection and vertical motion

The simplest way to demonstrate the relationship between horizontal thermal advection and diabatic vertical motion is through the rearrangement of the adiabatic temperature tendency equation (e.g. Eqn. 3.42 of Holton 2004),

$$\frac{\partial T}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \mathbf{u} \cdot \nabla T \right)$$

...(1)

Here, $\omega$ is the vertical velocity (Pa s$^{-1}$), $S = -\nabla \theta$ (difference of static stability and potential temperature. Equation 1 shows that any diagnostic based on the horizontal thermal advection ($\mathbf{u} \cdot \nabla T$) alone (including the thermal advection diagnostics introduced in this paper) is not a complete vertical motion diagnostic\(^1\). Such diagnostics will be most reliable when $|\mathbf{u} \cdot \nabla T|$ is small compared to $|\mathbf{u} \cdot \nabla T|$. In practice, this is true when the thermal structure of the atmospheric feature in question is translating and evolving relatively slowly compared to the earth relative flows that contribute to the thermal advection term. In these scenarios the isentropes are approximately stationary relative to the flow $\mathbf{u}$, which means air parcels must ascend or descend sloping isentropes (i.e. isentropic ascent or descent is present where $|\mathbf{u} \cdot \nabla T| \gg |\partial T/\partial t|$\(^2\)

(b) Thermal wind

As noted in the introduction insight into why the geostrophic thermal advection diagnostic appears to work for heavy rain bearing systems of Rossby number order unity, can be gained from an early paper by Forsythe (1945). Forsythe noted that when the observed wind shear is used to estimate the geostrophic windshear vector and thus the orientation of the thermal gradient, non-trivial errors are often introduced with deviations of up to 30° occasionally observed. Forsythe proposed corrections to the method to improve estimates of the geostrophic wind shear, based on the thermal wind equation expressed in natural coordinates (see also Neimann and Shapiro 1989).

The main difference between the Cartesian and natural coordinate forms of the thermal wind equations is the introduction of a centrifugal parameter ($VR$),

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \times \mathbf{K} + \frac{\partial}{\partial z} \left[ \frac{\partial \mathbf{u}}{\partial z} \right]$$

...(2)

Here,

$$F = f + \frac{V}{R} = f (1 + R_n)$$

...(3)

is the sum of the coriolis ($f$) and centrifugal ($VR$) parameters respectively\(^3\), $V$ is the wind speed, $R$ is the flow radius of curvature, $R_n = V/R$ is the Rossby number, $s$ is the unit vector in the direction of the wind, $R$ is the gas constant for dry air, $k$ is the vertical vector, $\mathbf{v}$ is the horizontal gradient operator, $T$ is the air temperature, $\frac{\partial T}{\partial t}$ is the time rate of change moving with the air parcels, and $\mathbf{n}$ is the unit vector perpendicular to $\mathbf{s}$ (positive to the left of the wind).

\(^1\)Equation 1 does not include any vertical motion associated with diabatic processes either: Diabatic contributions are deliberately excluded in order to focus on meso- to synoptic-scale vertical forcing, which may subsequently lead to broad-scale convective initiation.

\(^2\)Paradoxically, the relationship between vertical motion and the horizontal advection term in Eqn. 1 is most accurate when the actual horizontal thermal advection (relative to some reference frame) is zero, since horizontal thermal advection implies translation of cold or warm air, which must be accompanied by horizontal isentrope motion. For consistency with historical papers and text books we continue to refer to the process as thermal advection, instead of a more physically appropriate name such as isentropic ascent and descent.

\(^3\)According to convention $VR$ for anticlockwise flows, thus $f$ and $VR$ are of the same sign for cyclonic flows and of opposite sign for anticyclonic flows in both hemispheres (i.e. the Coriolis and centrifugal parameters combine to enhance the magnitude of $F$ for cyclonic flows, and they oppose one another for anticyclonic flows).
For geostrophic and gradient flow the second term on the right hand side of Eqn. 2 is zero, and for geostrophic flow (small $R_f$) $F$ reduces to $f$. Thus, when discretised the term on the left hand side of Eqn. 2 for geostrophic flow becomes the wind shear vector (since $f$ is constant with height), which is aligned perpendicular to the mean thermal gradient (as mentioned in the introduction). For gradient flow, however, the thermal gradient is instead proportional to the vertical gradient of the sum of the Coriolis and centrifugal accelerations, which is not necessarily aligned with the wind shear vector. Hence the need for Forsythe’s (1945) correction terms to identify the thermal gradient orientation. The orientation of the acceleration terms relative to the thermal gradient becomes evident after expanding Eqn. 2 into component form (M. Reeder 2014, personal communication),

$$\frac{\partial F}{\partial \ln \rho} = \frac{\partial F}{\partial \Phi} = \frac{\partial \Phi}{\partial \ln \rho} \frac{\partial \Phi}{\partial \rho}$$

For geostrophic and gradient flow the second term on the right hand side of Eqn. 2 is zero, and for geostrophic flow (small $R_f$) $F$ reduces to $f$. Thus, when discretised the term on the left hand side of Eqn. 2 for geostrophic flow becomes the wind shear vector (since $f$ is constant with height), which is aligned perpendicular to the mean thermal gradient (as mentioned in the introduction). For gradient flow, however, the thermal gradient is instead proportional to the vertical gradient of the sum of the Coriolis and centrifugal accelerations, which is not necessarily aligned with the wind shear vector. Hence the need for Forsythe’s (1945) correction terms to identify the thermal gradient orientation. The orientation of the acceleration terms relative to the thermal gradient becomes evident after expanding Eqn. 2 into component form (M. Reeder 2014, personal communication),

$$\frac{\partial F}{\partial \ln \rho} = \frac{\partial F}{\partial \Phi} = \frac{\partial \Phi}{\partial \ln \rho} \frac{\partial \Phi}{\partial \rho}$$

The mean thermal advection ($MTA$) between two pressure surfaces can be expressed as,

$$MTA = -u_m \cdot \nabla \langle T \rangle$$

Where $u_m$ is some representation of the mean layer horizontal wind vector, and $\langle T \rangle$ represents the vertical average temperature (Holton 2004; MTA is defined here to be positive for warm air advection). Using the hydrostatic relationship the mean layer temperature gradient between two pressure levels can be expressed as a function of the difference in geopotential gradients on those levels such that,

$$MTA = \frac{C}{R_d \ln \left( \frac{p_0}{p_1} \right)}$$

and the subscripts 1 and 0 represent upper and lower pressure surfaces respectively. The gradient term in Eqn. 8 can be expressed using Eqn. 6 as,

$$\nabla \langle \Phi \rangle = \left[ (f_0 v_0 - f_1 v_1), -(f_0 u_0 - f_1 u_1) \right]$$

Note the left hand side of Eqn. 10 is proportional to the thickness gradient and is thus proportional to the mean temperature gradient between the two pressure levels. The right hand side of Eqn. 10 is the difference between the sum of the Coriolis and centrifugal acceleration vectors on the two pressure levels. The familiar relationship between the geostrophic wind shear vector and the thickness gradient is evident in Eqn. 10, when the $F$ terms are replaced with the constant $f$, and the right hand side of the equation reduces to the product of the wind shear vector and the Coriolis parameter.

Using an overbar to define an average value on pressure levels $0$ and $1$ (e.g., $\bar{u}=(a_0+a_1)/2$) $u_m$ can be replaced by $\bar{u}$ in Eqn. 8 and combined with Eqn. 10 to give,

$$MTA = \frac{C}{2} \left( \bar{u}_1 + u_0, v_1 + v_0 \right) \left[ (f_0 v_0 - f_1 v_1), -(f_0 u_0 - f_1 u_1) \right]$$

which reduces to,

$$MTA = C. \sqrt{R_f} \left[ u_1 v_0 - u_0 v_1 \right].$$

This equation quantifies the $MTA$ between two pressure levels for a given distribution of gradient winds on those pressure levels. Note, this representation of $u_m$ is introduced here to illustrate the turning winds with height in the next section. Wind on an intermediate pressure level is used in the actual diagnostics (e.g. Eqns. 13, 14 and 15) as the approximation $u_m \approx u$ can be inaccurate for large wind rotation angles.

---

*Thickness is defined as the difference in geopotential height between two pressure levels ($Z_1-Z_0$). Thus $\nabla (\Phi_1 - \Phi_0) = g \nabla (Z_1 - Z_0) = g \nabla (\text{thickness})$.**
Discussion

Qualitative relationship
The $MTA$ (Eqn. 11) can be expressed as,

$$MTA = C[f(1 + \overrightarrow{R_o})] [k_0 (u_1 \times u_0)], \quad \ldots (12)$$

where the mean rotation term $[\overrightarrow{R}]$ is expressed as a function of the mean Rossby number $[\overrightarrow{R_o}]$, derived from Eqn. 3. This form of the equation best illustrates the difference between the gradient and geostrophic ($\overrightarrow{R_o} = 0$) mean thermal advection. Equation 12 shows that both are comprised of a positive constant $C'$ (Eqn. 9), a rotation term in curly brackets, and a variable wind term in square brackets. In a regular low pressure system (i.e. with cyclonic flow) the Rossby number, as defined in Eqn. 3, is positive, which shows that the neglect of the centrifugal parameter leads to an underestimate of the thermal advection magnitude, as mentioned in the introduction (Eqn. 5). Furthermore, for a given hemisphere, the sign of the variable wind term determines the nature of the thermal advection: either cold or warm. Finally, since the variable wind term is independent of the Rossby number, the relationship between winds turning with height and thermal advection is the same for both the geostrophic and gradient wind $MTA$. The variable wind term represents the magnitude of the cross product of the upper and lower winds. It is positive (negative) for flows that turn clockwise (anticlockwise) with height respectively. Thus when considering the sign of the rotation term in Eqn. 12 for a regular low, warm (cold) air advection corresponds to positive (negative) $MTA$ for flows that turn anti-cyclonically (cyclonically) with height in both hemispheres.

Towards a quantitative relationship
The turning winds with height climatology for Cairns and Brisbane documented in Callaghan and Tory (2014) suggests a correlation between rainfall rate and $MTA$ does exist. However, it must be emphasised here that: (i) the turning winds with height represented by the square bracketed term is only one of three contributions to the $MTA$ (Eqns 11 and 12), (ii) the $MTA$ is not a complete vertical motion diagnostic (Eqn. 1), (iii) the $MTA$ magnitude is reference frame dependent, and (iv) precipitation that results from any associated isentropic ascent is very much dependent on the amount of moisture in the environment and the depth of the ascent. Nevertheless, forecasters in the Brisbane office, using expert knowledge and intuition to account for these limitations, have had good success with the Snake Oil (derived below), which quantifies the contribution from the variable wind term in Eqn. 12 (e.g. Bonell et al. 2005, Callaghan and Bonell 2005, and Bonell and Callaghan 2008). A physical understanding of this term can be obtained by considering the cross product of the upper and lower winds, which shows that the $MTA$ magnitude increases with increasing wind speeds, but it is also modulated by the angle between the two winds. It is important to note that the $MTA$ magnitude in this formulation (i.e. with $u_0 = \overrightarrow{R}$) is zero at an angle of 0° and increases to a maximum value at an angle of 90°, and decreases again to zero at an angle of 180°, whereas if $u_0$ is independent of $u_1$ and $u_0$, (see examples in the next section) non-zero values of $MTA$ are possible for wind angles of any angle other than 0°.

Snake Oil
The thermal advection diagnostic used by the Queensland Regional Office was originally derived from Eqn. 8. The geostrophic relationship (e.g. $F = f$ in Eqn. 6) was used to replace the geopotential terms in Eqn. 8 with wind gradients on the two pressure levels (850 and 500 hPa). They used the wind on an intermediate pressure level (700 hPa) to represent the advecting wind in Eqn. 8 (i.e., $u_0 = u_{0g}$), such that,

$$MTA_g = C[f(u_{1g} (v_{g0} - v_{g1}) + v_{g1} (u_{g1} - u_{g0}))]. \quad \ldots (13)$$

Here the subscripts $g$ and 1/2 refer, to geostrophic wind, and the intermediate pressure level, respectively. Eqn. 13 shows that the thermal advection magnitude is proportional to the Coriolis parameter. However, forecaster experience, using observed winds, showed a systematic difference in rainfall amount between the deep tropics and subtropics for given values of this diagnostic (with greater rainfall in the deep tropics). The difference was attributed to a general equatorward increase in moisture between the subtropics and tropics. As the diagnostic was used only semi-quantitatively, a simple and effective correction for the assumed latitudinal variation in moisture content was to use a constant Coriolis parameter representative of the latitude 30°S,

$$WAA = -\Omega C [u_{1/2} (v_0 - v_1) + v_{1/2} (u_1 - u_0)], \quad \ldots (14)$$

where $\Omega$ is the earth angular rotation rate, and $WAA$ (warm air advection) is the original name used by the Queensland Regional Forecast Office, prior to the colloquial term ‘Snake Oil’. The subscript $g$ is dropped in Eqn. 14 as the diagnostic was applied using the actual winds. In practice the diagnostic is multiplied by 86 400 to give more useable units of K day⁻¹. An example $WAA$ plot is provided in Fig. 1 (upper panel) during a heavy rain event near the town of Bourke (indicated by the star in Fig. 1) in outback New South Wales (145.9°E, 30.1°S) on 13 February 2009. Almost 200 mm of rain fell in a 24 hour period at Bourke airport. For comparison, a gradient wind version of the diagnostic most similar to Eqn. 14 is plotted in Fig. 1 (lower panel),

$$WAA_{gr} = C [u_{1/2} (F_{g0} v_0 - F_{g1} v_1) + v_{1/2} (F_{g1} u_1 - F_{g0} u_0)]. \quad \ldots (15)$$

The technique for calculating the $V/R$ term in $F$ (Eqn. 3) is described in Appendix 1. The pressure levels corresponding to the indices 0, ½, 1 are 850, 700 and 500 hPa. The red shades in Fig. 1 correspond to positive $WAA$ and blue shades

---

5The values in Cairns were often too low to alert forecasters of significant rainfall events (J. Callaghan 2010, personal communication)
represent negative values. The associated anticyclonic and
cyclonic turning winds with height can easily be inferred
from the 850 and 500 hPa wind vectors. As expected
from the discussion above the main difference between the
two panels in Fig. 1 is the $WAA$ magnitude, with the greatest
differences where there is strong cyclonic curvature, e.g.

near Bourke where $R_o$ calculations yielded a peak value
of 0.8 (not shown). A map of the 24 hour rainfall
distribution (Fig. 2) shows good agreement with the positive $WAA$
distribution in eastern Queensland and northern New
South Wales, including the heavy rain at Bourke and further
east. The rain-free regions in southern New South Wales
and inland Queensland also match the negative $WAA$ quite
well. As the diagnostic does not identify rainfall associated
with orographic lifting or lifting from differential vorticity
advection, one would not expect all the heavy rain in Fig. 2
to be associated with thermal advection. Furthermore, Fig.
2 shows the 24 hour rainfall accumulation, whereas Fig. 1
shows only the thermal advection distribution at a point in
time near the middle of the 24 hour period.

While Fig. 1 shows that there is little difference between
$WAA_{gr}$ and $WAA$ for a rain event near 30°S, it is worth also
comparing the two at lower latitudes and for a more rapidly
rotating storm. Figure 3 shows $WAA$ and $WAA_{gr}$ plots for
tropical cyclone Yasi at 1200 UTC on 2 February 2011, during
landfall. Once again the patterns are very similar, with the
biggest difference in magnitude only present where there is
strong flow curvature. Interestingly, the strong $WAA_{gr}$
(Fig. 3, lower panel) wraps further around the storm core
(south west quadrant) than that evident in $WAA$ (Fig. 3, upper
panel). This is because $R_o \sim 2$ calculated in the vicinity of the
$WAA$ maximum (not shown). Interestingly, $R_o \sim 5$ at the storm
centre.

It is worth discussing why the two equations yield similar
results for this highly rotating storm. In removing the
latitudinal dependent Coriolis contribution $WAA$ varies only
with the variable wind term. As argued above the variable
wind term is independent of latitude and defines the sign
of the thermal advection for a given hemisphere, which
explains why Snake Oil (Eqn. 14) has qualitative value in the
subtropics and tropics. Furthermore, setting the Coriolis term
to a constant value equivalent to 30°S effectively provides
a contribution to $R_o$ in Eqn. 12 for latitudes equatorward
of 30°S. For example, the Coriolis parameter at 30°S is
equivalent to 1.5 times the Coriolis parameter at 20°S (which
is where $WAA$ is a maximum in Fig. 3). This would give a $WAA$
value equivalent to the MTA in Eqn. 12 for a circulation with
$R_o = 0.5$. At 15°S this effective $R_o$ is approximately 1. It follows
that Eqn. 14 should have quantitative value in northern
Queensland for disturbances of order one Rossby number,
and may explain why $WAA$, used as a semi-quantitative
rainfall diagnostic, has been successful there. However, the
use of Eqn. 15 is recommended over Eqn. 14 as the ad-hoc
constant Coriolis ‘correction’ in Eqn. 14 could conceivably
introduce large magnitude errors.

<table>
<thead>
<tr>
<th>10S</th>
<th>20S</th>
<th>30S</th>
<th>40S</th>
</tr>
</thead>
<tbody>
<tr>
<td>140E</td>
<td>150E</td>
<td>160E</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Mean thermal advection (shaded) from ERA-interim reanalysis data at 1200 UTC, 13 February 2009, calculated using Eqn. 14 (upper panel) and Eqn. 15 (lower panel), with 850 hPa (black) and 500 hPa (blue) winds overlayed. Positive values, corresponding to warm air advection (or isentropic ascent) are in red (shade interval, 5 K day⁻¹). The location of Bourke is marked by the star.

(d)Goanna Oil
The preceeding discussion suggests the centrifugal parameter is especially important for Rossby number flows of order 1 and higher (e.g. Eqn. 12) such as tropical depressions and tropical cyclones, and that this term should be included in thermal advection diagnostics applied to the sub tropics and tropics. However, $V/R$ can be difficult to
equation that shows the geostrophic wind shear vector between two pressure levels can be used to estimate the strength and orientation of the mean thermal gradient between the two levels. There has been some uncertainty among the broader Australian forecasting community as to why a diagnostic based on a geostrophic relationship should work so well at low latitudes, which led to the diagnostic being dubbed ‘Snake Oil’. Since most heavy rain bearing tropical and subtropical systems would be expected to be in near gradient wind balance, the diagnostic is re-derived in this paper assuming gradient wind balance, and compared with the geostrophic equivalent diagnostic. In practice a gradient wind based thermal advection diagnostic, such as Eqn. 11, can only be applied where there is high data density or in gridded model data. An alternative to calculating the flow curvature ($R$) and tangential wind ($V$) that make up the centrifugal parameter (described in Appendix 1), is to use Eqn. 8 instead of Eqn. 11, which uses the gradient wind relationship (Eqn. 6) to avoid the need to calculate this term. This approach employs a direct calculation of the thermal gradient and does not rely on any balance relationship between the wind and the mass field.

Preliminary experimentation with a diagnostic based on Eqn. 8 was performed by Mark Hentschel from the Northern Territory Regional Forecast Office, who dubbed the diagnostic ‘Goanna Oil’ in anticipation of it being an improvement on Snake Oil. The Goanna Oil makes use of Eqn. 1 so that the impact of the static stability on vertical motion is also included in the diagnostic, and it provides physically intuitive units of vertical velocity,

$$\omega_{GO} = \frac{C}{s_p} [\nabla \cdot \nabla (\Phi_1 - \Phi_2)].$$  \hspace{1cm} (16)

Here $\omega_{GO}$ is the Goanna Oil in units of Pa s$^{-1}$.

A performance comparison between the Snake Oil and Goanna Oil thermal advection diagnostic is beyond the scope of this paper: to provide a theoretical explanation for why the Snake Oil diagnostic has been used successfully in the subtropics and tropics.

**Summary**

Forecasters in the Brisbane office have been using rainfall diagnostics for decades that are based on the relationship between turning winds with height and thermal advection to aid in the prediction of subtropical and tropical heavy rainfall. This relationship, as derived in many dynamic meteorology text books, is based on the thermal wind equation that shows the geostrophic wind shear vector between two pressure levels can be used to estimate the strength and orientation of the mean thermal gradient between the two levels. There has been some uncertainty among the broader Australian forecasting community as to why a diagnostic based on a geostrophic relationship should work so well at low latitudes, which led to the diagnostic being dubbed ‘Snake Oil’. Since most heavy rain bearing tropical and subtropical systems would be expected to be in near gradient wind balance, the diagnostic is re-derived in this paper assuming gradient wind balance, and compared with the geostrophic equivalent diagnostic. In
the geostrophic diagnostic the wind shear vector between two pressure levels is oriented perpendicular to the mean thermal gradient and the magnitude of the thermal gradient can be estimated from the wind shear vector. For the gradient wind diagnostic, it is the difference between the sum of the Coriolis and centrifugal acceleration vectors on the two pressure levels that defines the orientation and magnitude of the mean thermal advection. Using the observed wind shear vector alone to estimate the mean thermal gradient can lead to non-trivial errors (Forsythe 1945). However, the wind shear vector can still be used to identify the presence of thermal advection as a scalar quantity, while the inclusion of the centrifugal parameters will provide a more accurate estimate of the thermal advection magnitude.

When applying the actual winds to the thermal advection diagnostics, the difference between the geostrophic and gradient wind based diagnostics reduces to a function of the Rossby number (Eqn. 12). It follows from Eqn. 12, that for regular lows, the gradient wind based diagnostic will always diagnose greater thermal advection than the geostrophic diagnostic. Qualitatively, however, the relationship between turning winds with height and thermal advection is the same for both, i.e., winds that rotate anti-cyclonically (cyclonically) with height are associated with warm air advection (cold air advection).

The thermal advection diagnostics were shown to be composed of two variable quantities (for a given atmospheric layer), a rotation term (which incorporated the Coriolis parameter in both versions of the diagnostic, plus a centrifugal parameter in the gradient wind version) and the turning winds with height term that is identical for both equations. The only variable quantity in the semi-quantitative Snake Oil thermal advection diagnostic (Eqn. 14) is the turning winds with height term, after the variable Coriolis contribution was dropped to reduce an apparent latitude dependency on the diagnostic magnitudes and subsequent observed rainfall. This effectively increases the magnitude of the diagnostic at latitudes equatorward of 30°S, which partly compensates for any missing centrifugal acceleration contribution. The fixed Coriolis thermal advection diagnostic is shown in Figs 1 and 3 to reproduce well the magnitude only in regions of strongly curved flow. This shows that MTA estimates from individual wind soundings (where no indication of the centrifugal parameter is available) can still offer useful forecast guidance; however, in gridded data it is recommended that the rotation term should contain a variable Coriolis parameter and the centrifugal parameter (Eqn. 3). The mean layer temperature gradient in gridded data can either be calculated purely as a function of wind using the gradient wind relationship (which requires the calculation of the centrifugal parameter, Appendix 1), or it can be calculated directly from the thickness gradient, which bypasses the need for any balance relationship between the wind and mass field (e.g. the Goanna Oil thermal advection vertical motion diagnostic).

Acknowledgments

This paper is dedicated to the memory of the late Mark Hentschel, who began trialling the Goanna Oil diagnostic. The author acknowledges the years of encouragement by Jeff Callaghan, without which these ideas may never have been published. Many thanks to Michael Reeder who brought to my attention the work on thermal wind in natural coordinates presented in this paper, and to Stephen Fletcher, Jeff Kepert and an anonymous reviewer for suggestions to improve the paper structure and mathematical rigour.

References


Appendix 1: Centrifugal parameter calculation

The vertical component of relative vorticity in natural coordinates can be expressed as the sum of a shear vorticity and a curvature vorticity (e.g. Holton 2004),
\[ \zeta = -\frac{\partial V}{\partial n} + \frac{V}{R}, \] ...
(A1)

where \( n \) points 90° to the left of the wind direction, and \( V \) is the wind speed. Here, the curvature vorticity, \( V/R \), is equivalent to the centrifugal parameter (Eqn. 3). In rectangular gridded data the calculation of \( V/R \) is non-trivial.
Fortunately the other two terms in Eqn. A.1 can be calculated easily in such data. A unit vector pointing in the direction of \( n \) can be expressed as,
\[ \hat{n} = \left(-\frac{v}{\sqrt{u^2 + v^2}}, \frac{u}{\sqrt{u^2 + v^2}}\right), \] ...
(A2)

and by definition,
\[ \frac{\partial v}{\partial n} = \hat{n} \cdot \nabla V. \] ...
(A3)

It follows that,
\[ \frac{V}{R} = \zeta + \frac{\partial v}{\partial n} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \left(\frac{v}{u}\right) \frac{\partial u}{\partial x} + \left(\frac{u}{v}\right) \frac{\partial v}{\partial y}. \] ...
(A4)