Recent Research on EnVar for Deterministic Prediction

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Contents

1. Scale-dependent ensemble covariance localization

2. Computing “Forecast Sensitivity Observation Impact” (FSOI) for EnVar using forecast ensembles
Scale-dependent covariance localization

Motivation

• Currently, our EnVar uses single horizontal and vertical localization length scales, very similar to our EnKF

• Comparing various EnKF studies, seems it is best to use different amount of localization depending on application:
  • convective-scale assimilation: ~10km
  • mesoscale assimilation: ~100km
  • global-scale assimilation: ~1000km – 3000km

• In the future, global systems will resolve convective scales

• Therefore, need a general approach for applying appropriate localization to wide range of scales in a single analysis procedure: Scale-dependent localization

• Relatively straightforward in EnVar, since localization acts directly on model-space cov. (not on $BH^T$ & $HBH^T$ or $R$)
Scale-dependent covariance localization

General Approach

• Ensemble perturbations decomposed with respect to a series of overlapping spectral wavebands (bandpass filter coefficients sum to 1 for each wavenumber)

• Apply scale-dependent spatial localization to the scale-decomposed ensemble covariances, both within-scale and between-scale covariances

• Keeping the between-scale covariances is necessary to maintain heterogeneity of ensemble covariances (Buehner and Charron, 2007; Buehner, 2012)

• Motivation different than spectral localization where the between-scale covariances are set to zero
Scale-dependent covariance localization
Implementation in EnVar

Current (standard) Approach

- Analysis increment computed from control vector (\(B^{1/2}\) preconditioning: \(\Delta x = B^{1/2}v\)) using:

\[
\Delta x = \sum_k e_k \circ (L^{1/2}v_k)
\]

where \(e_k\) is normalized member \(k\) perturbation

Scale-dependent Approach (Buehner and Shlyaeva, 2015, *Tellus*)

- Varying amounts of smoothing (i.e. localization) applied to same set of amplitudes for a given member

\[
\Delta x = \sum_k \sum_j e_{k,j} \circ (L_{j}^{1/2}v_k)
\]

where \(e_{k,j}\) is scale \(j\) of normalized member \(k\) perturbation
Scale-dependent covariance localization

2D Sea Ice Concentration Ensemble

- Ensemble of sea ice concentration background fields (60 members) from the Canadian Regional Ice Prediction System ensemble of 3DVar analyses experiment

Ensemble mean ice concentration

Ensemble spread
Scale separation with diffusion operator: Example of one ensemble perturbation

Original perturbation

Scale 4 (0→10km)

Scale 3 (10→30km)

Scale 2 (30→100km)

Scale 1 (100km→∞)

Work of Anna Shlyaeva
Scale separation with diffusion operator: Ensemble spread for each scale

Scale 4 (0→10km)

Scale 3 (10→30km)

Scale 2 (30→100km)

Scale 1 (100km→∞)

Full ensemble spread

Work of Anna Shlyaeva
Homogeneous correlation functions and chosen localization functions for each scales

Localization length scales: 500km, 150km, 50km, 30km (Gaussian-like functions)
Assimilation of 2 observations
One obs in area dominated by large-scale error, other in area of small-scale error

Background field and obs
30km localization
500km localization

No localization
150km localization
Scale-dep. localization
Idealized data assimilation experiment setup

- **True state:** \( x^t = x^i \) (i\(^{th}\) member) \( \Rightarrow \) \( \text{mean}(x) = x^t - e^i, \ e^i \sim N(0, B) \)
- **Background:** \( x^b = x^t + e^i \) \( \Rightarrow e^i = x^i - \text{mean}(x), \ e^i \sim N(0, B) \)
- **Observations:** observe every 4th grid point, with random gaps
- **\( e^i \) (real background error) not used in ensemble \( B \) for assimilation
Scale-dependent covariance localization
2D Sea Ice Ensemble: Conclusions

• Strong spatial variation in partition of error wrt scale leads to strong spatial variation in overall strength of localization (similarity with adaptive localization)

• Scale-dependent localization results in lowest analysis error for all scales in regions dominated by either small-scale or large-scale error – in idealized DA experiments
Application to NWP
Horizontal Scale Decomposition

Filter response functions for decomposing with respect to 3 horizontal scale ranges

- Large scale: 10000 km
- Medium scale: 2000 km
- Small scale: 500 km
Application to NWP
Horizontal Scale Decomposition

Perturbations for ensemble member #001 – Temperature at ~700hPa
Application to NWP
Horizontal Scale Decomposition

Waveband variances

Large scale
Medium scale
Small scale
All the scales

Horizontal scale-dependent localization leads to (implicit)... variable-dependent and level-dependent horizontal localization in addition to spatial dependence

6-h perturbation from 256-member EnKF
Scale-dependent covariance localization
Impact in single observation DA experiments

700 hPa T observation at the center of Hurricane Gonzalo (October 2014)

Normalized temperature increments (correlation-like) at 700 hPa resulting from various B matrices.
Scale-dependent covariance localization
Impact in single observation DA experiments

700 hPa T observation at the center of a High Pressure

Normalized temperature increments (correlation-like) at 700 hPa resulting from various B matrices.

- $B_{\text{ens}}$: Std hLoc
- $B_{\text{ens}}$: No hLoc
- $B_{\text{ens}}$: SD hLoc
- $B_{\text{nm}}$: No hLoc

hLoc: 1500km / 4000km / 10000km
Scale-dependent covariance localization
Forecast impact

- 2.5-month trialling (June-August 2014) in our global NWP system
- 3DEnVar with 100% $B_{ens}$ used in both experiments

1) **Control experiment** with $h_{Loc} = 2800$ km

2) **Scale-Dependent experiment** with a 3 horizontal-scale decomposition
   - I. Small scale uses $h_{Loc} = 1500$ km
   - II. Medium scale uses $h_{Loc} = 2400$ km
   - III. Large scale with $h_{Loc} = 3300$ km

Ad hoc values!

Same vertical localization (2 units of $\ln(p)$) for control experiment and every horizontal scale in scale-dependent experiment
Scale-dependent covariance localization
Forecast impact – Comparison against ERA-Interim

24h Northern E-T

U

RH

24h Southern E-T

U

RH

> Control  ➤  Scale-Dependent
Scale-dependent covariance localization
Forecast impact – Comparison against ERA-Interim

120h Northern E-T

120h Southern E-T

➤ Control  ➤ Scale-Dependent
Scale-dependent covariance localization
Forecast impact – Comparison against ERA-Interim

Time series Northern E-T

Time series Southern E-T

Std Dev for U at 250 hPa

Control ➤ Scale-Dependent
Scale-dependent localization: Summary and Conclusions

• S-D localization is feasible and straightforward to implement in EnVar, but more expensive than using single-scale localization
  – In the S-D localization experiments reported here: 3x more expensive (3 spectral bands)
  – Will explore tricks to speed it up

• Results using a horizontal scale-dependent horizontal localization indicate small forecast improvements in our global NWP system

• In terms of dynamical balance, S-D localization does not seem to present any issue for the rotational part of the analysis increments (not shown)

• Finding the optimal S-D localization setup is not straightforward
  – Initial attempt to apply objective approach resulted in very large localization length scales that degraded forecast quality
**Future Work**

- Test S-D localization in higher resolution limited-area applications
- Continue to examine automated methods for estimating optimal localization length scales
- Examine the impact of adding **vertical** scale-dependent **vertical** localization

<table>
<thead>
<tr>
<th>Horizontal scale</th>
<th>Vertical scale</th>
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<tbody>
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<td>hLoc-small</td>
<td>vLoc-large</td>
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FSOI adapted for EnVar: Motivation

- 4DVar for both global and regional DA replaced by 4D-EnVar (November 2014) at ECCC
- Development/support essentially discontinued of tangent linear and adjoint of forecast model
- Therefore, to perform FSOI in context of 4D-EnVar, requires adapting approach to avoid use of adjoint of forecast model
- Pure ensemble approach exists (e.g. as used at NCEP), but can only give impact of observations assimilated in EnKF
- At ECCC numerous observation types assimilated in 4D-EnVar **not** assimilated in EnKF (AIRS, IASI, CrIS, SSMIS, Geo-rad, GB-GPS)
- New FSOI approach combines elements of standard adjoint approach (for analysis) and pure ensemble approach (for forecast propagation)
Basic idea of FSOI

- Goal is to partition, with respect to arbitrary subsets of observations, the forecast error reduction from assimilating these observations:

\[ \Delta e^2 = (e_{t+\Delta t}^a)^T C(e_{t+\Delta t}^a) - (e_{t+\Delta t}^b)^T C(e_{t+\Delta t}^b) \]

**Scalar measure of forecast error**

![Diagram showing the relationship between forecast error and assimilation of observations.](image)

- Assimilation of obs: \( x_t^b \rightarrow x_t^a \)
- Verification analysis: \( e_{t+\Delta t}^b = M(x_t^b) - x_{t+\Delta t}^a \)

\( e_{t+\Delta t}^f = M(x_t^f) - x_{t+\Delta t}^a \)
FSOI general approach

• Forecast error reduction from assimilating all observations:

\[ \Delta e^2 = (e^{fa}_{t+\Delta t})^T C(e^{fa}_{t+\Delta t}) - (e^{fb}_{t+\Delta t})^T C(e^{fb}_{t+\Delta t}) \]

• This can be rewritten as a sum of contributions from each observation, allowing the calculation of contribution from any subset of obs:

\[ \Delta e^2 \approx \sum_i (y^o - H(x^b))_i \partial \Delta e^2 / \partial y^o_i \]

• Where the sensitivity of the change in forecast error to each observation can be written as (using the chain rule):

\[ \frac{\partial \Delta e^2}{\partial y^o} = \left( \frac{\partial x_t}{\partial y^o} \right) \left( \frac{\partial x_{t+\Delta t}}{\partial x_t} \right) \left( \frac{\partial \Delta e^2}{\partial x_{t+\Delta t}} \right) \]

\[ \text{Sens. wrt obs} = \left( \text{Adjont of DA} \right) \left( \text{Adjont of Fcst} \right) \left( \text{Sens. wrt fcst} \right) \]
New FSOI approach
Forecast step uses ensemble, DA step like variational approach

• Instead of using adjoint of forecast model, sensitivities propagated to analysis time using extended background ensemble forecasts \( \rightarrow \) requires use of 100% ensemble \( B \) in analysis step

• The analysis increment is a (spatially varying) linear combination of the background ensemble\(^*\), the propagated increment is assumed to be the same linear combination of the ensemble\(^*\) at the forecast time

• Adjoint of analysis step uses standard variational approach

\[
\text{Sens. wrt obs} = (\text{Adjoint of DA})(\text{Adjoint of Fcst})(\text{Sens. wrt fcst.})
\]

\( x_t^b \rightarrow x_t^a \)

\( \Delta t \)

\( t \)

\( t-6h \)

*actually the deviations of the ensemble members from the ensemble mean state
Formulation (Idea from A. Lorenc working paper)

Change in forecast error at time \( t \) (with respect to norm \( \mathbf{C} \)) is given by:

\[
\Delta e^2 = (\mathbf{e}_t^a)^T \mathbf{C} (\mathbf{e}_t^a) - (\mathbf{e}_t^b)^T \mathbf{C} (\mathbf{e}_t^b)
\]

Denote gradient of this with respect to any quantity as: \( \langle \cdot \rangle = \partial \Delta e^2 / \partial (\cdot) \)

Write \( \Delta e^2 \) as a sum of contributions from each obs:

\[
\Delta e^2 \approx (\mathbf{y}^o - H (\mathbf{x}^b))^T (\hat{\mathbf{y}}^o)
\]

Based on extended ensemble at forecast time

Where:

\[
\delta\mathbf{x}_t = \mathbf{C} (\mathbf{e}_t^a + \mathbf{e}_t^b) \quad \Rightarrow \text{sensitivity wrt forecast}
\]

\[
\hat{\mathbf{v}} = \mathbf{B}_t^{T/2} \delta\mathbf{x}_t \quad \Rightarrow \text{sensitivity wrt control vector}
\]

\[
\hat{\mathbf{y}}^o = \mathbf{R}^{-1} \mathbf{H} \mathbf{B}^{1/2} (\hat{\mathbf{z}} + \hat{\mathbf{v}}) \quad \Rightarrow \text{sensitivity wrt observations}
\]

And \( \hat{\mathbf{z}} \) is obtained by minimizing the cost function:

\[
J (\hat{\mathbf{z}}) = \frac{1}{2} \hat{\mathbf{z}}^T \hat{\mathbf{z}} + \frac{1}{2} \left( \mathbf{H} \mathbf{B}_0^{1/2} (\hat{\mathbf{z}} + \hat{\mathbf{v}}) \right)^T \mathbf{R}^{-1} \left( \mathbf{H} \mathbf{B}_0^{1/2} (\hat{\mathbf{z}} + \hat{\mathbf{v}}) \right)
\]
FSOI experiments with new approach

- Performed EnVar data assimilation experiment similar to operational configuration, but with 100% ensemble B
- Evaluated FSOI at 0UTC and 6UTC over 26 days in January 2015
- Forecast error measured with **dry global energy norm up to 100hPa** relative to GDPS final cycle analyses
- Computed for forecast lead times of both 12h and 24h
- For ensemble-variational (EV-FSOI) approach, computed FSOI both with and without horizontal advection of the localization (0.75 × wind)
- Compared results with using adjoint of forecast model to propagate sensitivities from forecast→analysis time (ADJ-FSOI)
- Ensemble and analysis grid has 50km grid-spacing, adjoint model has 100km grid-spacing (adjoint never run at the higher resolution)
FSOI Results

Actual and estimated change in 12h forecast error from assimilating observations at 0Z and 6Z

![Graph showing FSOI results with different error metrics and dates]
FSOI Results
Actual and estimated change in 24h forecast error from assimilating observations at 0Z and 6Z

![Graph showing error in 30h, 24h forecast, real change in error, and total FSOI with adjoint model, ensemble, and ensemble and advection.]

- Error in 30h forecast
- Error in 24h forecast
- Real change in error
- Total FSOI with adjoint model
- Total FSOI with ensemble
- Total FSOI with ensemble and advection
FSOI Results

Number of assimilated observations

Ordered by 12h ADJ-FSOI

- Total over the 52 cases: 0Z+6Z during 26 day period
Overall, similar results between using ensemble or adjoint model

Advection always increases apparent impact

Some obs types more affected by advection (e.g. Aircraft, Raob, AMSU-A)

Surface, GEO-wind and aircraft obs have relatively larger impact when using ensemble

FSOI Results
Daily average impact on 0Z+6Z 12h forecasts

Ordered by ADJ-FSOI

ADJ-FSOI
EV-FSOI
EV-FSOI with advection
Relative to 12h forecasts:

- Overall, results differ more between using ensemble or adjoint model.
- Apparent impact of advection is greater.
- Only in-situ surface obs have significantly larger apparent impact when using ensemble.
- Satellite radiances and GPS-RO have lower apparent impact with ensemble than adjoint.

FSOI Results

Daily average impact on 0Z+6Z 24h forecasts

Ordered by ADJ-FSOI

ADJ-FSOI
EV-FSOI
EV-FSOI with advection

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FSOI Results
Impact per observation on 0Z+6Z 24h forecasts

Ordered by total ADJ-FSOI

ADJ-FSOI
EV-FSOI
EV-FSOI with advection
FSOI Results: 24h forecasts
Average daily impact for Land, Ship (+buoy) and Raob

ADJ-FSOI  EV-FSOI  EV-FSOI with advection
FSOI Results: 24h forecasts
Average daily impact for Raob as function of pressure

ADJ-FSOI  EV-FSOI  EV-FSOI with advection

Temperature

Humidity

Wind components

Environment and Climate Change Canada
Environnement et Changement climatique Canada
FSOI Results: 24h forecasts
Daily average impact for each AMSU-A channel

• Reminder: Forecast error measured only from sfc to 100hPa
• Channel 8 peaks around 150hPa, channel 9 around 70hPa
• Apparent impact of lowest peaking channel lower with ensemble approach (propagation limited by vertical localization?)

ADJ-FSOI
EV-FSOI
EV-FSOI with advection
FSOI Results: 24h forecasts
Daily average impact for each AIRS channel

ADJ-FSOI
- near-sfc temperature
- tropospheric humidity and temperature
- near-sfc temperature and humidity
- tropospheric temperature
- stratospheric temperature

EV-FSOI with advection
- near-sfc temperature
- tropospheric humidity and temperature
- near-sfc temperature and humidity
- tropospheric temperature
- stratospheric temperature
FSOI Results: 24h forecasts
Daily average impact for each IASI channel

ADJ-FSOI

- near-sfc temperature and humidity
- tropospheric humidity and temperature
- near-sfc temperature and humidity
- tropospheric temperature
- stratospheric temperature

J/kg

EV-FSOI with advection

- near-sfc temperature and humidity
- tropospheric humidity and temperature
- near-sfc temperature and humidity
- tropospheric temperature
- stratospheric temperature

J/kg
FSOI Results: 24h forecasts
Daily average impact for Geo-Radiances in 5°x5° boxes

- Detailed spatial impact of geostationary radiances (1 channel per instrument) is generally similar between approaches.
FSOI Conclusions

• Results with new FSOI approach adapted for use with EnVar seem promising and qualitatively similar to results using adjoint model.
• Introducing a simple flow-following covariance localization increases apparent impact and improves similarity with adjoint model results.
• Significant differences remain for some obs types (e.g. in-situ sfc, lower peaking AMSU-A), more detailed analysis needed:
  – Due to vertical ensemble localization?
  – Due to nonlinear ensemble vs. linear adjoint (with simplified physics) propagation?
  – Due to use of multi-physics approach in ensemble?
• Current approach limited to measuring forecast impact in EnVar analysis using 100% ensemble B, examining approach for hybrid B.
• Routine (operational) application requires extension of 256-member background ensemble forecasts.
Extra slides follow
Scale separation of ensemble perturbations with diffusion operator

- Apply diffusion with increasing length scales to the original ensemble perturbations.
- Decompose into different scales by taking differences between perturbations before and after each level of diffusion.
- Example: \( e \) – original ensemble perturbation; \( D_n \) – diffusion with lengthscale \( n \)

\[
\begin{align*}
  e_1 &= D_{10\text{km}}(e) \\
  e_2 &= D_{30\text{km}}(e_1) \\
  e_3 &= D_{100\text{km}}(e_2)
\end{align*}
\]

- Scale 4 (smallest): \( e - e_1 \)
- Scale 3: \( e_1 - e_2 \)
- Scale 2: \( e_2 - e_3 \)
- Scale 1 (largest): \( e_3 \)

- Scale-decomposed perturbations sum up to the original \( e \)