Creating Long Duration Areal Reduction Factors

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Abstract

The Bureau of Meteorology (BoM) provides new Intensity-Frequency-Duration (IFD) design rainfalls in the form of point values. However, as design flood estimates are often required for large catchments, areal design rainfall estimates are necessary. Therefore areal reduction factors (ARF), which are the ratios between point and areal design rainfall for a given catchment area, duration and annual exceedance probability (AEP) were produced to compensate for the effect of rainfall from large events rarely being spatially uniform.

Hypothetical circular catchments of various areas were created around BoM daily read rainfall gauges across Australia. Daily rainfall values from gauges within these catchments were weighted based on Thiessen polygon areas to determine sets of areal rainfall time series. Frequency analyses were then carried out to create design areal rainfall estimates using the same method that was used for the new IFDs. ARF values were produced for areal rainfall time series with sufficient data by dividing design areal rainfall by the Thiessen weighted point design rainfall of all gauges within a hypothetical catchment.

Sample mean ARF values were created based on combinations of catchment area, duration and AEP. A new form of equation was developed to estimate ARF values for differing catchment areas, durations and AEPs.

Spatially dependent ARF relationships were investigated by delineating regions of similar climatology. By extracting and modelling sample mean ARF values from within these regions discernible differences were identified in ARF relationships for different spatial zones.

The resultant ARFs provide an improved basis for defining ARFs across Australia that are fundamentally more compatible with Australian catchments and IFDs than the previous standards.

1. INTRODUCTION

In 2013 the Bureau of Meteorology (BoM) provided new Intensity-Frequency-Duration (IFD) design rainfalls (http://www.bom.gov.au/water/designRainfalls/ifd/) in the form of point values as part of the upcoming revision of Australian Rainfall and Runoff (ARR). In design flood estimation however, catchments of significant size are often investigated and it is therefore necessary to provide areal design rainfall values. Since areal design rainfalls relate directly to point design rainfalls, a common method used is to apply a reduction factor to point values to obtain areal design rainfalls for a given catchment area, duration and annual exceedance probability (AEP). This factor is known as the areal reduction factor (ARF) and is the ratio between the areal and design rainfall for these variables.

Depending on the area of interest in Australia, the previous edition of Australian Rainfall and Runoff (ARR87) (Pilgrim 1987) recommended ARFs that were developed for either the Chicago or Arizona areas in the US. More recently, areal reduction factors were derived on a state by state basis, in conjunction with the application of the CRC-FORGE method for deriving rare design rainfalls, using a modified Bells method (Bell, 1976) developed by Siriwardena and Weinmann (1996a).

Given the state-based and internationally derived ARFs used in the past, an Australia wide standard on ARFs, created from as much Australian data as possible would be of considerable value. These nationally consistent ARFs were produced using the same data, distribution and fitting method as the new IFDs (without regional pooling), making them more compatible and practical in their application to the new IFD values.

Section 2 describes the method used to create ARF values for the hypothetical circular catchments; Section 3 details the creation and fitting of an equation to sample mean ARF values; and Section 4 discusses the potential for regionalisation of ARF data to better represent climatically different areas.

2. METHODOLOGY FOR CREATING AREAL REDUCTION FACTORS

The method that was chosen for this study is a modified Bell's method (Bell 1976, Siriwardena & Weinmann 1996a). In particular this method is effective at determining relationships between AEPs and ARF, which is especially beneficial given the increase in accuracy of the new IFDs for low AEPs. Most importantly this method allows for the inclusion of the data that were collected and processed for the new IFDs, so vast quantities of quality data can be included with minimal effort, allowing an increase in accuracy over previously used ARFs.

These data included 15364 BoM rainfall gauging sites that were quality controlled using a combination of automated and manual methods (Green et al. 2012a). The annual maximum series (AMS) that was used in this method was also generated in the creation of the new IFDs.

The modified Bell's method involves defining hypothetical circular catchments in areas with sufficient data and creating an areal time series by weighting data values based on Thiessen polygon areas. Frequency quantiles are created from these time series and divided by the weighted point frequency quantiles of the sites within the catchment. Once ARFs are calculated for the required catchment areas, durations and AEPs in as many locations as possible they are averaged across these attributes and an equation is fitted to model these values.

For this study ARFs were derived for catchment areas of 125 km2, 250 km2, 500 km2, 1000 km2, 2000 km2, 4000 km2, 8000 km2, 15,000 km2 and 30,000 km2, durations from 1 to 7 days and AEPs of 50%, 20%, 10%, 5%, 2% and 1%.

2.1. Defining hypothetical circular catchments

Using ArcGIS, circular buffers of the desired radii were drawn around 15,364 BoM rainfall gauging sites. In order to achieve a minimum quality in the data and to ensure acceptable levels of spatial independence, constraints were created regarding shared data, minimum number of rain gauges and the dominance of a single site within a catchment.

The first constraint, originally used by Siriwardena and Weinmann (1996b), is to adopt catchments with a minimum of 3 potential site quantiles for inclusion up to catchment areas of 500 km^2 and the requirement for an additional site for every 500 km^2 after that. The subsequent application of this method for each of the states also adhered to this constraint.

The second constraint involves the maximum number of shared stations that were allowed for both hypothetical catchments to be included. The QLD, SA and WA CRC-FORGE studies (Hargraves 2004, Hill *et al.* 2000, Durrant & Bowman 2004) all allowed up to 30% overlap between hypothetical catchments whereas NSW allowed one third (Nandakumar *et al.* 2012) and Victoria and Tasmania manually placed catchments with varying degrees of overlap (Siriwardena & Weinmann 1996b, Gamble *et al.* 1998). Considering the additional data used in this study and the focus on quality, 30% of stations were allowed to be common in this study.

The last constraint defines the maximum weighting that is allowed to be given to a single station's data within a hypothetical catchment. Following the method used for the NSW study (Nandakumar et al, 2012), the values shown in Table 1 detail the values of this constraint for the various catchment areas.

Table 1. Constraints applied to various catchment areas

Hypothetical circular catchment area (km2)	Minimum number of rain gauge quantiles	Maximum proportion of catchment area for single gauge	Number of catchments adopted
125	3	67%	163
250	3	67%	357
500	3	67%	646
1,000	4	50%	646
2,000	6	33%	490
4,000	10	33%	320
8,000	18	33%	181
15,000	32	33%	103
30,000	62	33%	54

As can be seen in Figure 1, the distribution of hypothetical catchments over Australia is far from comprehensive. While enforcing less conservative constraints would increase the number of hypothetical catchments it would not increase the data coverage enough to justify the loss in data quality. This is because rainfall data coverage over much of Australia is quite sparse, so even the most lenient of constraints would leave similar areas of no coverage.

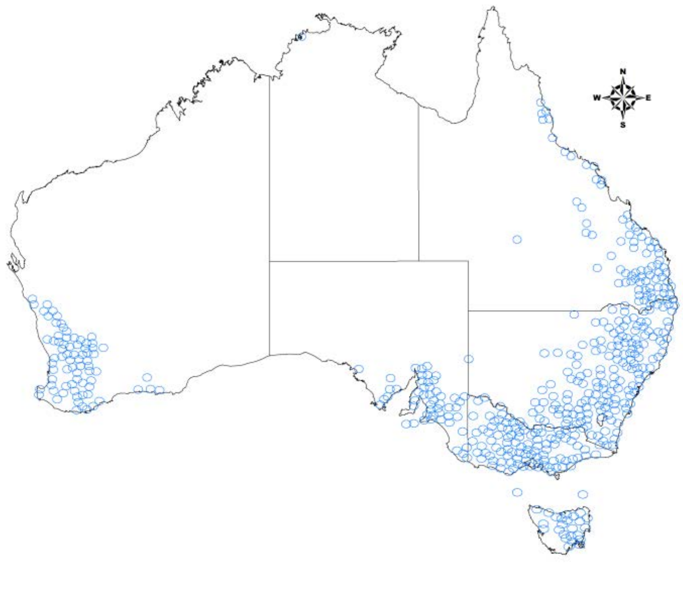

Figure 1 2000 km2 hypothetical catchments used in creation of ARFs for all of Australia

2.2. Creation of areal time series and annual maximum series

The areal time series for each hypothetical catchment was calculated on a daily basis. From the start of the earliest rain day for all sites within a catchment to the latest, days were tested to satisfy the minimum number of gauges and maximum proportion criteria detailed in Table 1. On valid days, the hypothetical catchment was split into Thiessen polygons around the sites with in-filled or non-missing data. A Thiessen weight (the proportional area of a site's polygon) was applied to each station's rainfall value and they were added together to create an areal average rainfall for that day.

AMS were extracted from these areal time series using similar criteria to that used for the new IFDs as it was proven to be effective and the consistency allowed for the inclusion of the already derived AMS from the new IFDs. The employed criteria required every valid AMS year to have a minimum of 10 months which had 75% valid data and data for 60% of the days of the year. Years that didn't satisfy this condition were still included if their AMS value was within the top 10% of values for that hypothetical catchment.

To minimize scatter from the data obtained from the rainfall frequency analysis, a minimum number of AMS years required for the inclusion of rainfall frequency quantiles had to be defined. For the new IFDs this cut-off was 30, but the IFD process involved regionalisation which compensated for the lack of data at individual sites. Therefore during the production of ARFs, both 50 and 30 AMS years were tested to see which would yield the best results.

2.3. Using the rainfall frequency analysis to derive areal reduction factors

Rainfall frequency quantiles were calculated using a GEV distribution fitted using L-moments for the new IFDs (Green *et al.* 2012b). This distribution was tested against a variety of other distributions that are commonly used for this purpose and was found to perform the best for site and regional rainfall frequency estimates. Hence the GEV distribution was chosen to be used in the production of ARFs.

Each site within a given hypothetical catchment was assigned a proportion of their catchment using Thiessen polygons, and their quantile values were weighted and combined based on their proportional areas. The quantiles for the areal time series of the hypothetical catchment were created using the same distribution and fitting method as the new IFDs. These quantiles were then divided by the combined site quantiles to produce an areal reduction factor. This was repeated for all valid hypothetical catchments of all the required sizes.

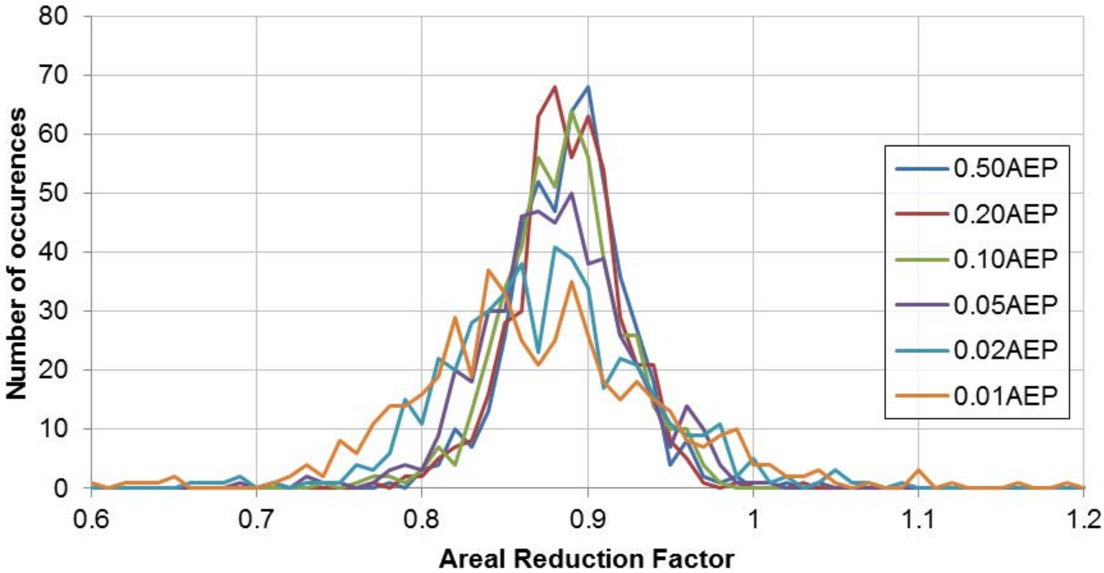

Figure 2 Histogram of areal reduction factors for all AEPs, the 1 day duration, 2000 km2 catchment area and a minimum AMS of 30 years

The results of this process, as seen in Figure 2, highlight the variability associated with ARF values.

This is especially a problem for low AEPs, as even 50 AMS years are insufficient to make accurate predictions on the 1% AEP. Thus it is necessary to average ARFs for the required catchment areas, durations and AEPs to enable ARF estimates to be made. Sample averaging was initially done for all hypothetical catchments in Australia, regardless of location, in order to test the performance of various equation forms in fitting the data.

The choice of minimum AMS years was tested to see the impact on accuracy and variability of ARF sample means. Since the accuracy of these estimates is highly dependent on sample sizes and the accuracy of the higher probability quantiles is sufficient using 30 years of AMS, a cut-off requiring 50 years of AMS was only used for the 1% and 2% AEPs. While decreased variability in ARF values for the higher AMS cut-off is evident from the standard deviation (Table 2), the decrease in sample size has considerable impact on the uncertainty of the sample mean. Given that there is such a significant decrease in sample size and increase in standard error, the AMS cut-off remained at 30 years for the 1% and 2% AEPs.

Table 2. Average statistics on ARF for all catchment sizes for the 1% and 2% AEPs

Catchment Size (km^2)		**125**	**250**	**500**	**1000**	**2000**	**4000**	**8000**	**15,000**	**30,000**
Sample size	30 AMS	163	357	646	646	490	320	181	103	54
	50 AMS	89	209	423	426	319	206	116	61	31
Standard error	30 AMS	0.0051	0.0040	0.0028	0.0028	0.0029	0.0031	0.0038	0.0050	0.0082
	50 AMS	0.0060	0.0043	0.0033	0.0028	0.0029	0.0033	0.0044	0.0062	0.0108
Standard deviation	30 AMS	0.0654	0.0749	0.0722	0.0703	0.0650	0.0559	0.0508	0.0503	0.0602
	50 AMS	0.0562	0.0623	0.0673	0.0572	0.0512	0.0470	0.0473	0.0481	0.0602

3. DEVELOPING EQUATION FORMS TO MODEL OBSERVED AREAL REDUCTION FACTORS

When Siriwardena and Weinmann (1996b) re-established ARF relationships for Victoria, they adopted the use of two equation forms (Eq.1 & Eq. 2) using the units of km^2, minutes and fractions for the area, duration and AEP terms. The main assumption with these equations is that ARFs approach unity with increasing duration and diminishing catchment area, which is conceptually true, as smaller catchments with long duration events tend to have fairly uniform rainfall in space.

$$\mathbf{ARF_{0.5} = 1 - a\left(Area^{b} - c.\log_{10}(duration)\right).duration^{-d}} \qquad (1)$$

$$\mathbf{ARF_{AEP} = 1 - a\left(Area^{b} - c.\log_{10}(duration)\right).duration^{-d} + e.Area^{f}.duration^{g}(0.3 + \log_{10}(AEP))} \qquad (2)$$

The 4 coefficient model of Eq. 1 is solved first, as it models behavior of ARFs at 50% AEP which are the values with the most certainty. Provided there is a significant relationship between AEP and ARF, and the quality of the data is good enough to highlight it, the remaining three coefficients in Eq. 2 are solved to extend ARFs relationships to different AEPs. Using a variety of evolutionary solving methods, solutions were obtained for the coefficients of both equations using sample mean ARF data for all of Australia. Due to the number of coefficients in these equations, there is significant parameter interdependence, so the coefficients are dependent upon the solution method chosen.

The results of this process provide excellent performance for both equations, with high R^2 values and a low mean absolute error (MAE) (Table 3). However for short durations and small catchment areas the model does not perform as well (Figure 3). This may be due to increased sampling error from smaller sample sizes, from large proportions of Thiessen polygon area being assigned to a single gauge or from naturally occurring differences between ARF and AEP in small durations and catchment areas. Since the differences disappear for larger durations and larger catchment sizes have the same proportions of area assigned to a single gauge it is more likely this feature is naturally occurring.

To obtain optimal model performance, multiple equation forms were tested, with Eq. 3 showing the best results. The advantage of the new form is that it allows flexible fitting to the small catchment, short duration data without compromising the good fit to the more reliable large catchment data. With the addition of the third term the ARF equation no longer approaches one, but instead a number very close to one for small catchment areas. However this will not be an issue as using this relationship for catchment areas below 10 km^2 cannot be justified without further investigation.

$$\begin{aligned} ARF_{AEP} = {} & 1 - a\left(Area^{b} - c.\log_{10}(duration)\right).duration^{-d} \\ & + e.Area^{f}.duration^{g}(0.3 + \log_{10}(AEP)) \\ & + h.10^{i.Area.\frac{duration}{1440}}(0.3 + \log_{10}(AEP)) \end{aligned} \quad (3)$$

Table 3. ARF model performance indicators for the new equation form

Equation form	No. data points ARF0.5	No. data points ARFAEP	ARF0.5 R2	ARFAEP R2	MAE ARFAEP
Eq. 2	63	378	0.9987	0.9976	0.00223
Eq. 3	63	378	0.9987	0.9981	0.00209

The increase in model performance is small when observing the differences in R^2 alone (Table 3), but the area of analysis with the highest residuals is being modelled much better (Figure 3). For this reason Eq. 3 was chosen as the basis for ARFs in Australia, using Eq.1 or Eq.2 if the data in a particular region supports it.

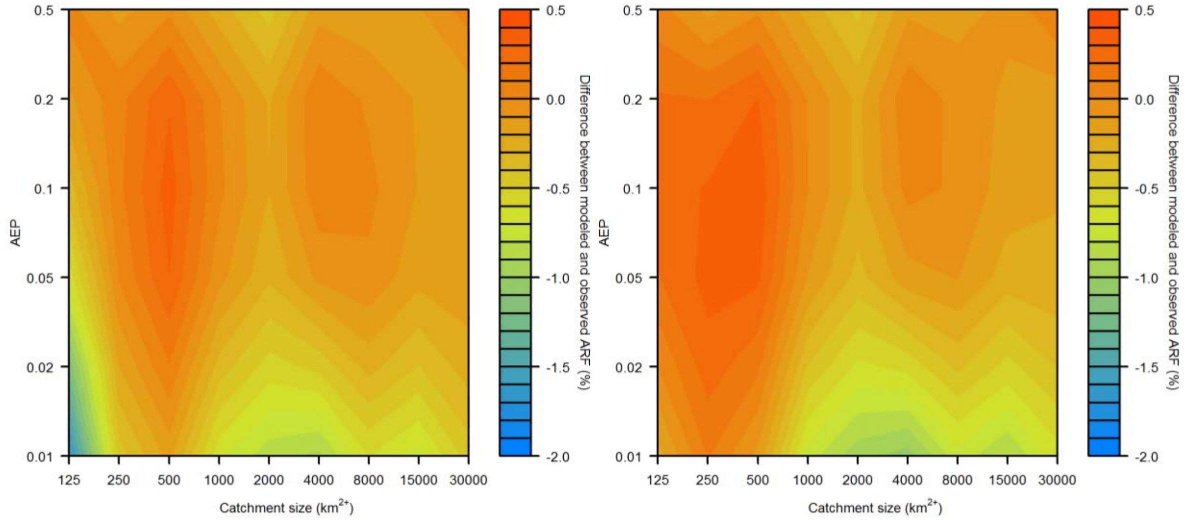

Figure 3 Sample mean ARFs residuals (original model) for the 1 day duration (left) and sample mean ARFs residuals ARF (new form) for the 1 day duration (right)

4. REGIONALISATION

The previous basis for ARFs used in ARR87 and in the Siriwardena and Weinmann derived values assumed changes in ARF relationships for different climatic regions. Therefore changes in ARF relationships were tested by delineating regions that are vastly climatologically different (Figure 4). The "South Low Rain" region is characterised by low yearly rainfall that predominantly falls in winter (http://www.bom.gov.au/jsp/ncc/climate_averages/climate-classifications/index.jsp?maptype=seasb#maps) and the "East Coast" region has high yearly rainfall totals with many events driven by topography. The sample means that are used to model ARF values were then calculated from the sub-set of hypothetical catchments that fell within these regions. Using the form of equation developed in the previous section, a model was fitted to the data to determine the quality of the data and relationship within the region.

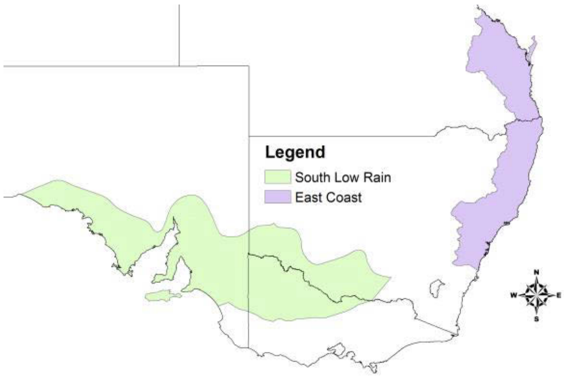

Figure 4 Regions for testing spatial variation in ARFs

The two regions show significantly different ARF relationships in both sample mean and modelled outputs (Figure 5). Creating sub-sets of data for regionalisation increases the scatter of sample mean data and hence decreases the overall model performance (Table 5). Considering the vast differences in ARF relationship from region to region, it is probable that more accuracy can be gained by trading model performance for more region specific ARF relationships.

Since ARFs differ within the defined spatial regions, the variation of ARF with various climatic and geographic indicators was also investigated. Some indicators, such as elevation and mean annual rainfall (MAR) showed small, yet significant correlations to ARF (Table 4). To test the viability of using these weak relationships as a form of regionalisation, the strongest of the indicators, MAR, was divided into 5 ranges and sample mean ARF values were calculated from catchments that had average MAR values that fell within the designated range.

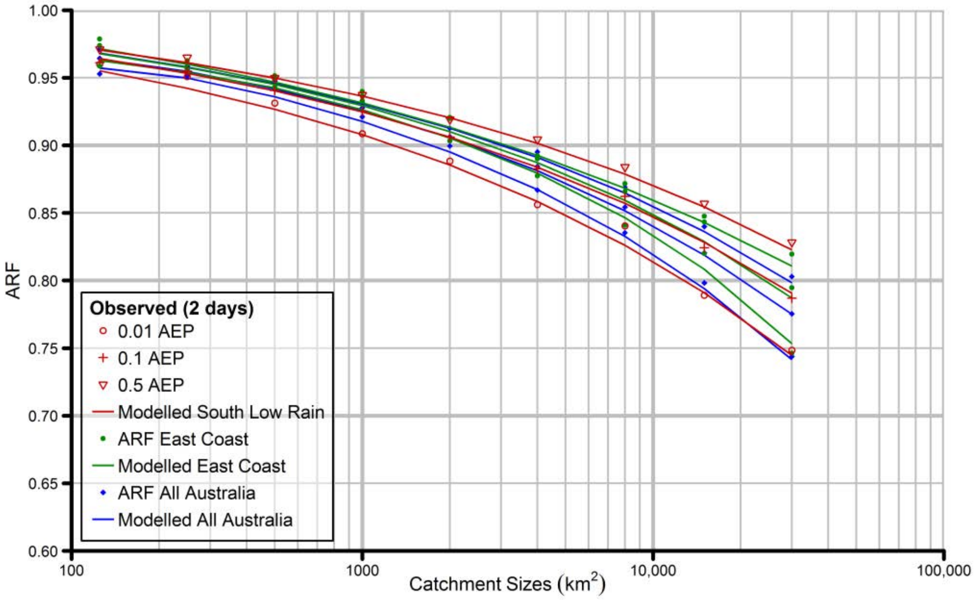

Figure 5 Comparison between ARF relationships for the 2 day duration and the "East Coast" and "South Low Rain" regions

Table 4. Pearson correlation of various climatic and geographic indicators to ARF

Latitude	Longitude	MAR	Elevation	Slope	Aspect	Catchment area	Duration	AEP
0.0466	0.0195	0.123	-0.0754	0.0172	-0.0726	-0.492	0.209	0.0798

Regional differences between the various MAR ranges were evident, particularly for lower AEPs. However, the confidence limits for these lower AEPs were much wider, with some catchment sizes exhibiting inverse relationships between ARF and AEP to other catchment sizes. This is highlighted by the larger differences between the 0.5 AEP and the all AEP R^2 values compared to the other regionalising method (Table 5). Due to the weaker relationships with AEP and higher uncertainty, regionalisation is not being done based solely on climatic and geographic indicators, but instead by intelligently choosing spatial regional zones that exhibit similar climatic characteristics such as seasonal rainfall, thunderstorm frequency, MAR and topography.

Table 5. Indicators of model performance for various sub-regions and all of Australia

Region title	ARF0.5 R2	ARFAEP R2	MAE ARFAEP	Number of hypothetical catchments use
All of Australia	0.9987	0.9981	0.00209	2960
East Coast	0.9933	0.9900	0.00459	564
South Low Rain	0.9973	0.9914	0.00412	465
MAR 1	0.9977	0.9889	0.00417	596
MAR 3	0.9970	0.9875	0.00521	591
MAR 5	0.99547	0.9897	0.00520	589

5. CONCLUSION

As the ARFs provided in ARR87 were based on values created for Chicago and Arizona in the US, the derivation of ARFs for the entirety of Australia from local data is necessary. ARF relationships were created on a state-by-state basis, in conjunction with the application of the CRC-FORGE method to derive rare design rainfalls, using a modified Bell's method. Using this established method with the much more extensive data base used for the new IFDs, ARF values were derived that are more accurate, current and compatible with the new IFDs to which they will be applied.

ARF values calculated in this study highlighted divergence between modelled and sample mean ARF values for small catchment areas and shorter durations. The increased accuracy of the ARFs provided the confidence to refine the equation used to model ARFs, yielding a better relationship between ARF and AEP for small catchment areas. This creates a better foundation for the definition of ARF values for all locations in Australia.

Investigation into spatial differences in ARF values highlighted the value of region specific ARF values that can be defined across state boundaries. This will permit ARFs to be climatologically specific, regardless of the state they are in, which will increase the accuracy of design flood estimates and be the basis for further study. The new equation form and the discoveries on the potential value of regionalisation will allow the production of the most accurate predictions on ARFs for Australia to date. These predictions will be the ARFs recommended for design flood estimation in the new edition of ARR.

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